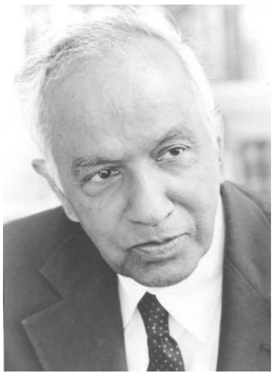
Chandrasekhar Limit



S. Chandrasekhar
(1910-1995)

In a famous work carried out in 1930, the Indian Physicist Prof Subrahmanyan Chandrasekhar (1910-1995) studied the stability of stars. The problem will help you to construct a simplified version of his analysis.

You may find the following symbols and values useful.

Speed of light in vacuum	$c = 3.00 \times 10^{8}\,\mathrm{m.s^{-1}}$
Planck's constant	$h = 6.63 \times 10^{-34}\,\mathrm{J.s}$
Universal constant of Gravitation	$G = 6.67 \times 10^{-11}\,\mathrm{N.m^2.kg^{-2}}$
Rest mass of electron	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Rest mass of proton	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$

II.1. Consider a spherical star of uniform density, radius R and mass M. Derive an expression for its gravitational potential energy (E_G) due to its own gravitational field (gravitational self energy). **[1.0 point]**

II.2. We assume that the star is made up of only hydrogen and that all the hydrogen is in ionized form. We consider the situation when the star's energy production due to nuclear fusion has stopped. Electrons obey the Pauli exclusion principle and their total energy can be computed using quantum statistics. You may take this total electronic energy (ignoring the protonic energy) to be

$$E_e = \frac{\hbar^2 \pi^3}{10 m_e 4^{2/3}} \left(\frac{3}{\pi}\right)^{7/3} \frac{{N_e}^{5/3}}{R^2}$$

where N_e is the total number of electrons and $\hbar = h/2\pi$. Obtain the equilibrium condition of the star relating its radius (R_{wd}) to its mass. This radius is called the 'White Dwarf' radius. **[2.0 points]**

II.3. Numerically evaluate R_{wd} given that mass of the star is the same as the solar mass ($M_S = 2.00 \times 10^{30}\,\mathrm{kg}$). **[1.5 points]**

II.4. Assuming that the electron distribution is homogeneous, obtain an order of magnitude estimation of the average separation (r_{sep}) between electrons if the radius of the star is R_{wd} as obtained in part (II.3). **[1.0 point]**

II.5. Let us estimate the speed of electrons. For this purpose, assume each electron to form a standing wave in a one-dimensional box of length r_{sep}. Estimate the speed of electron (v) in the lowest energy state using de-Broglie hypothesis **[1.0 point]**

II.6. Consider now a modification of the analysis in part (II.2). If we take electrons in the ultrarelativistic limit ($E = pc$), a similar analysis yields

$$E_e^{rel} = \frac{\pi^2}{4^{4/3}} \left(\frac{3}{\pi}\right)^{5/3} \frac{\hbar c}{R} N_e^{4/3}$$

Obtain the expression for the mass for which, the star can be in equilibrium in terms of the constants provided at the beginning of the question. We call this the critical mass (M_c). **[1.5 points]**

II.7. If the mass M of the star is greater than the critical mass M_c obtained in part (II.6), state whether the star will expand or contract. Tick in appropriate box.

[0.5 point]

II.8. Calculate a numerical estimate of this critical mass in units of solar mass (M_S).

(Note: Your answer may differ from Chandrasekhar's famous result because of the approximations made in this analysis)

[1.5 points]