

### 3. To Commemorate the Centenary of Rutherford's Atomic Nucleus: the Scattering of an Ion by a Neutral Atom

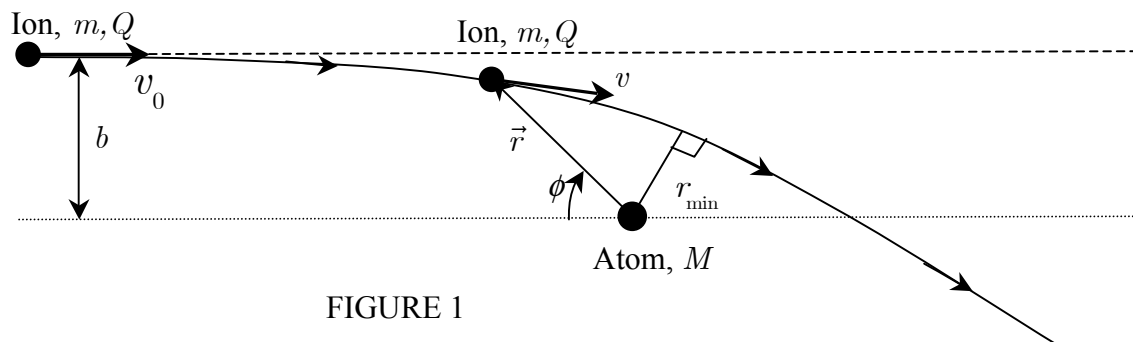
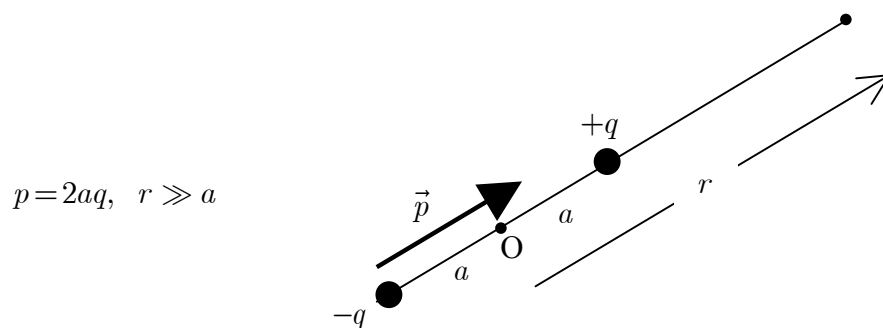


FIGURE 1

An ion of mass  $m$ , charge  $Q$ , is moving with an initial non-relativistic speed  $v_0$  from a great distance towards the vicinity of a neutral atom of mass  $M \gg m$  and of electrical polarisability  $\alpha$ . The impact parameter is  $b$  as shown in Figure 1.

The atom is instantaneously polarised by the electric field ( $\vec{E}$ ) of the in-coming (approaching) ion. The resulting electric dipole moment of the atom is  $\vec{p} = \alpha \vec{E}$ . Ignore any radiative losses in this problem.

**3.1** Calculate the electric field intensity  $\vec{E}_p$  at a distance  $r$  from an ideal electric dipole  $\vec{p}$  at the origin O along the direction of  $\vec{p}$  in Figure 2. **[1.2 points]**



$$p = 2aq, \quad r \gg a$$

FIGURE 2

3.2 Find the expression for the force  $\vec{f}$  acting on the ion due to the polarised atom. Show that this force is attractive regardless of the sign of the charge of the ion.

**[3.0 points]**

3.3 What is the electric potential energy of the ion-atom interaction in terms of  $\alpha, Q$  and  $r$ ?

**[0.9 points]**

3.4 Find the expression for  $r_{\min}$ , the distance of the closest approach, as shown in Figure 1.

**[2.4 points]**

3.5 If the impact parameter  $b$  is less than a critical value  $b_0$ , the ion will descend along a spiral to the atom. In such a case, the ion will be neutralized, and the atom is, in turn, charged. This process is known as the “charge exchange” interaction. What is the cross sectional area  $A = \pi b_0^2$  of this “charge exchange” collision of the atom as seen by the ion?

**[2.5 points]**