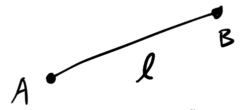
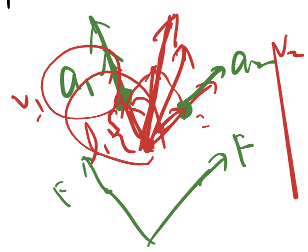
绳长不变. ⇒ 绳相关坐标下表示



$\Rightarrow (\vec r_A-\vec r_B)\cdot(\vec r_A-\vec r_B)=l^2$

$\frac{d}{dt}\downarrow$ $2(\vec V_A-\vec V_B)(\vec r_A-\vec r_B)=2\cdot l\cdot\dot l$

$\Rightarrow (\vec V_A-\vec V_B)\cdot l\hat l=l\cdot\dot l$

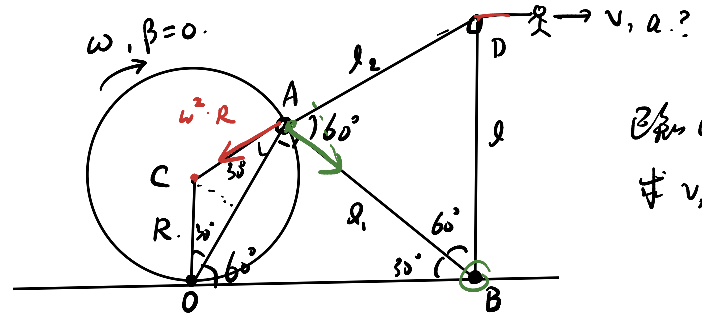


导 $\Rightarrow \vec V_{A\parallel}-\vec V_{B\parallel}=\dot l$ ✓

$\Rightarrow$ …… $(\vec a_A-\vec a_B)\cdot\hat l+\frac{(V_{A\perp}-V_{B\perp})^2}{l}=\ddot l$

$a_{A\parallel}-a_{B\parallel}$　向心　绳长变化

例: $\omega,\beta=0$. V, a?



已知 ω. 纯滚

求 v,a.

解: $v=-\dot l_1-\dot l_2$

$a=-\ddot l_1-\ddot l_2$

$\Rightarrow V_A=\omega\cdot\sqrt3R$. 方向 ⊥ OA.

↘ 方向 //AB.

$\Rightarrow \dot l_1=V_{A//AB}=-V_A=-\sqrt3\omega R$ ①

$\dot l_2=V_{A//AD}=-V_A\cdot\cos60°=-\frac12\sqrt3\omega R$ ②

$\Rightarrow V=-(\dot l_1+\dot l_2)=\frac32\sqrt3\omega R$ ✓

$a_{A\parallel}-a_{B\parallel}=\ddot l-\frac{(V_{A\perp}-V_{B\perp})^2}{l}$

$a_{相\parallel}=\ddot l-\frac{V_{相\perp}^2}{l}$

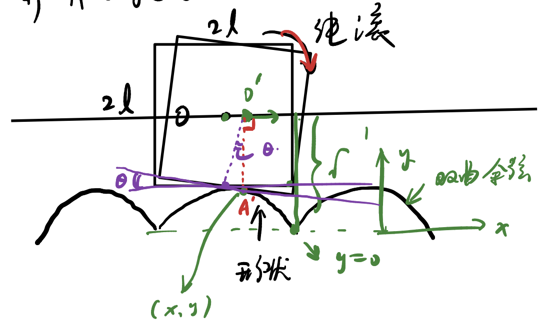
$\Rightarrow$ 对AD: $a_{AD\parallel}=\omega^2R=\ddot l_2-\frac{(V_A\cdot\sin60°)^2}{l}$

$\Rightarrow \ddot l_2=\omega^2R+\frac{\frac34\cdot3\cdot\omega^2R^2}{l}=\cdots$

相似: 求出 $\ddot l_1=\cdots$

$\Rightarrow a=-(\ddot l_1+\ddot l_2)$ ⇒ ✓.

例: 奇异的轮子.



要求 O 高度不变.

求 形状.

① V_0 水平.

② 接触点速度瞬心.

⇒ A'O' 竖直.

$\Rightarrow y+A'O'=常=\sqrt2 l$ ①

$A'O'=l/\cos\theta$ ⇐

$\tan\theta=斜率=\frac{dy}{dx}$

$\rightarrow \cos\theta=\frac{dx}{\sqrt{(dx)^2+(dy)^2}}=\frac{1}{\sqrt{1+(y')^2}}$

$\Rightarrow y+l\cdot\sqrt{1+(y')^2}=\sqrt2 l$

$\Rightarrow l^2\cdot(1+(y')^2)=(\sqrt2 l-y)^2$

$\frac{dy}{dx}=y'=\sqrt{(\sqrt2-\frac yl)^2-1}$

$\Rightarrow \int\frac{dy}{\sqrt{(\sqrt2-\frac yl)^2-1}}=\int dx$

令 $\sqrt2-\frac yl=ch\alpha$　$ch^2\alpha-1=sh^2\alpha$

$\Rightarrow -\frac{dy}{l}=sh\alpha\cdot d\alpha$

$\int\frac{-l\cdot sh\alpha\cdot d\alpha}{sh\alpha}=\int dx$

$\Rightarrow -l(\alpha-\alpha_0)=x$

$\Rightarrow \alpha=-\frac xl+\alpha_0 \Rightarrow \sqrt2-\frac yl=ch(-\frac xl+\alpha_0)$

$\Rightarrow y=\cdots$

令 $x=i\theta$.

$chx=\frac{e^x+e^{-x}}{2}$

$shx=\frac{e^x-e^{-x}}{2}$

$thx=\frac{sh}{ch}$

$\Rightarrow sh'x=chx$

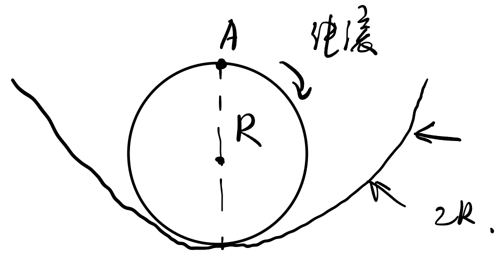
$ch'x=shx$.

$ch^2x-sh^2x=1$.

$\sin'\theta=\cos\theta$

$\cos'\theta=-\sin\theta$.

$\cos^2\theta+\sin^2\theta=1$



要求 A 高度不变.

形状? 圆.

2. 静力学. 化简 思路.

正交 $\sum \vec{F}=0$

三力 → { 三力组成矢量△ ; 三力汇交 ($\sum \vec{M}=0$) } ⇒ n力

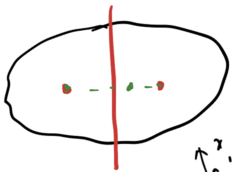
内力 → 没.

⊥ 丑力.

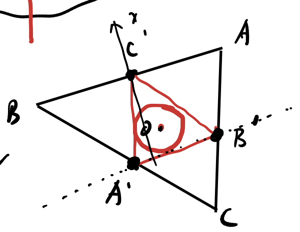
$\sum \vec{M}=0$ ⇒ { 丑力 过 轴 ; 丑力 // 轴 }

2.1. 重力. 弹力. 摩擦力.

质点系: $m_i, \vec{r}_i, \Rightarrow \vec{r}_c = \frac{\sum m_i \vec{r}_i}{\sum m_i}$



例:



材质.

三角形框. 重心:

中点连线△. 内接圆心.

⇒ O到A'B'距离·$M_总$ = C'到A'B'距离·M_C

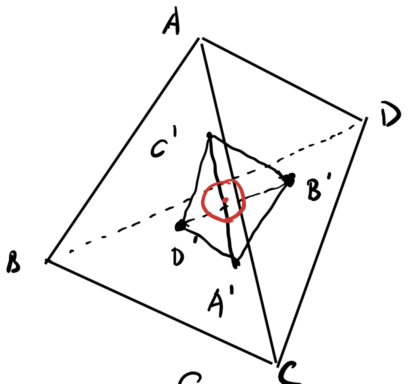
O到A'B'距离·总长·线密度 = C'到A'B'距离·A'B'密度

⇒ O到A'B'距离 = $\frac{S_面}{总长}$ } ⇒ 即A'B'C'内心

O到B'C'距离 =

A'C'距离 =

例:



板 ⇒ 四面体. 重

证明 重心中点连线

四面体. 内接球心.

$$\frac{S_{A'B'C'}}{S_{ABC}} = \frac{S_{B'C'D'}}{S_{BCD}} = \cdots$$

重心. O点. O到A'B'C'的距离: h_O.

D'到A'B'C'的距离: $h_{D'}$

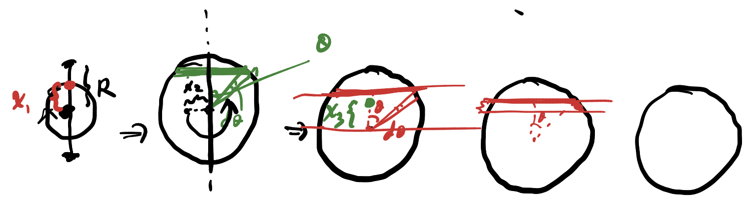
$$\Rightarrow h_O \cdot M_总 = h_{D'} M_{D'} + \underset{0}{h_{A'}} m_{A'} + \underset{0}{h_{B'}} m_{B'} + \underset{0}{h_{C'}} m_{C'}$$

$$h_O \cdot \sigma \cdot S_{面总A'B'C'D'} = h_{D'} \cdot \sigma \cdot S_{A'B'C'}$$

$$\Rightarrow h_O = \frac{V_{A'B'C'D'}}{S_{面总A'B'C'D'}}$$ ⇒ 证毕.

n维半球. 质心. 巴普斯.

预备:



1D. 2D 3D 4D 5D

体积 $V^1\ 2R$ $V^2\ \pi R^2$ $V^3\ \frac{4}{3}\pi R^3$ $V^4\ \frac{\pi^2}{2}R^4$

表面积 2. $2\pi R$ $4\pi R^2$ $2\pi^2 R^3$

$$dV^n = S^n\, dr. \qquad V^2 = \int_0^\pi V^1_{(R\sin\theta)} \cdot R\,d\theta \sin\theta$$

$$\Rightarrow S^n = \frac{dV^n}{dr} \qquad = \int_0^\pi 2R\sin\theta \cdot R\sin\theta\, d\theta$$

$= \cdots$ ✓

$$V^n = \int_0^\pi V^{n-1}_{(R\sin\theta)} \cdot R\,d\theta \cdot \sin\theta \Rightarrow \cdots$$

1维半球 以圆心为轴转 2π ⇒ 2维球.

⇒ $2\pi \cdot x_1 (V^1/2) = \pi R^2 \Rightarrow 2\pi \cdot x_1 \cdot R = \pi R^2 \Rightarrow x_1 = \frac{R}{2}$.

2维半球. ⇒ 3维球.

$$\frac{V^2}{2} \cdot 2\pi \cdot x_2 = \frac{4}{3}\pi R^3 \Rightarrow \frac{\pi R^2}{2} \cdot 2\pi \cdot x_2 = \frac{4}{3}\pi R^3$$

$$\Rightarrow x_2 = \frac{4R}{3\pi}$$

3维半球 ...... ⇒ 4维球.

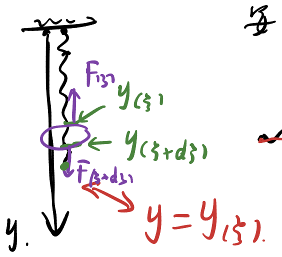
$$\frac{V^3}{2} \cdot 2\pi \cdot x_3 = V^4 = \frac{\pi^2}{2} R^4$$

$$\Rightarrow \frac{2}{3}\cancel{\pi} R^3 \cdot 2\cancel{\pi} \cdot x_3 = \frac{\cancel{\pi^2}}{2} \cdot R^4 \Rightarrow x_3 = \frac{3}{8} \cdot R \cdots$$

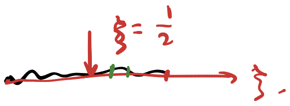
4维半球 ...... ⇒ 5维球 体积.

⇒ $x_4 = \cdots$ ✓

例. 有质量的弹簧. m, l_0. k.



要点: 用参数描述 连续体 原长时状态



$\xi = \frac{1}{2}$ $\quad \xi \in [0,1]$.

取一个位子 $\xi \to \xi + d\xi$ 一段

$$dm = d\xi \cdot m, \quad k_1 = \frac{k}{d\xi}$$

$$F_{(\xi)} = k_1 (dy_{(\xi)} - l_0 d\xi) \Rightarrow F_{(\xi)} = \frac{k}{d\xi}(dy_{(\xi)} - l_0 d\xi) = k(y'_{(\xi)} - l_0)$$

$$F_{(\xi)} = dm\, g - F_{(\xi+d\xi)}$$

$$\Rightarrow dF_{(\xi)} = -dm\, g$$

$$dF_{(\xi)} = -m\, d\xi\, g \Rightarrow \frac{dF_{(\xi)}}{d\xi} = -mg.$$

$$\Rightarrow k y''_{(\xi)} = -mg \Rightarrow \int y''_{(\xi)} = \int -\frac{mg}{k} = 常量$$

$$\Rightarrow y_{(\xi)} = -\frac{mg}{2k}\xi^2 + a\xi + b \leftarrow 形态$$

边界: $\xi = 0 \Rightarrow y = 0$. $\quad F_{(\xi)}\big|_{\xi=1} = 0$.

$$\Rightarrow y_{(0)} = b = 0, \quad F_{(1)} = k\left(-\frac{mg}{k} + a - l_0\right) = 0$$

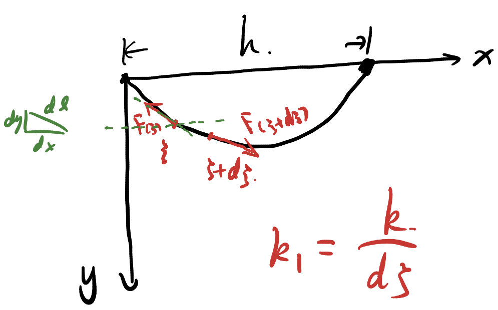
$$\Rightarrow b = 0, \quad a = l_0 + \frac{mg}{k}$$

$$\Rightarrow y_{(\xi)} = -\frac{mg}{2k}\xi^2 + \left(l_0 + \frac{mg}{k}\right)\xi.$$

令 $\xi = 1$ $\quad y_{(\xi)} = -\frac{mg}{2k} + l_0 + \frac{mg}{k} = l_0 + \frac{mg}{2k}$

$$\Delta = \frac{mg/2}{k}$$

例: 弹簧悬链线. 重. 软. 弹性体. $l_0 \ll \frac{m_0 g}{k}$ ← 求形状



$\xi \in [0,1]$ $\quad x_{(\xi)} \leftrightarrow y_{(\xi)}$

$$k_1 = \frac{k}{d\xi}$$

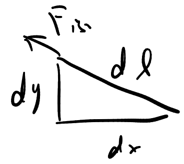
$$dl = \sqrt{dx^2 + dy^2} = \sqrt{x'^2_{(\xi)} + y'^2_{(\xi)}}\, d\xi$$

$$F_{(\xi)} = k_1(dl - l_0 d\xi) \approx \frac{k}{d\xi} dl.$$

对 $\xi \to \xi + d\xi$: 平衡: 水平方向:

$$F_{(\xi)x} = F_{(\xi)} \frac{dx}{dl} = k_1\, dx = k \cdot x'_{(\xi)} \cancel{d\xi}$$

$$\Rightarrow dF_{(\xi)x} = k \cdot x''_{(\xi)} = 0.$$



$$F_{(\xi)y} = F_{(\xi)} \cdot \frac{dy}{dl} = k_1 dl \cdot \frac{dy}{dl} = k \cdot y'_{(\xi)} \cancel{d\xi}$$

$$F_{(\xi)y} = dm\, g + F_{(\xi+d\xi)y} \Rightarrow -dF_{(\xi)y} = dm\, g.$$

$$\Rightarrow k y''_{(\xi)} d\xi = -d\xi\, mg \Rightarrow k y''_{(\xi)} = -mg.$$

$$\begin{cases} x''_{(\xi)} = 0 \Rightarrow x_{(\xi)} = a_1 \xi + b_1 \\ y''_{(\xi)} = -mg/k \Rightarrow y_{(\xi)} = -\frac{mg}{2k}\xi^2 + a_2\xi + b_2. \end{cases}$$

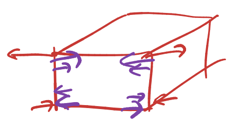
$\Rightarrow$ 边界. $\quad \xi = 0$: $x = 0$, $y = 0$. $\quad \xi = 1$: $x = h$, $y = 0$.

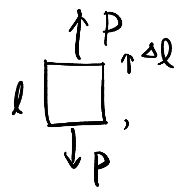
$$\Rightarrow \begin{cases} x_{(0)} = b_1 = 0 \\ x_{(1)} = a_1 + b_1 = h \end{cases} \quad \begin{cases} y_{(0)} = b_2 = 0 \\ y_{(1)} = -\frac{mg}{2k} + a_2 = 0. \end{cases}$$

$$\Rightarrow x_{(\xi)} = h \cdot \xi, \quad y_{(\xi)} = -\frac{mg}{2k}\xi^2 + \frac{mg}{2k}\xi$$

$\Rightarrow$ 抛物线.

杨氏模量.

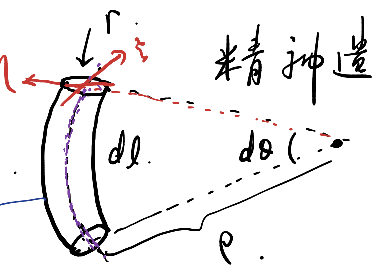




$$P = Y \cdot \frac{\Delta l}{l}$$

应力 应变

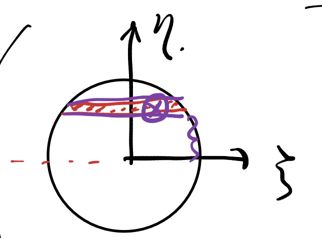
例: 精神遗产.



$\eta \in [-r, r]$. $\quad K = \frac{1}{\rho}$ $\quad \rho \gg r$

$$P_{(\eta)} = Y \cdot \frac{[(\rho + r)d\theta - \rho d\theta] \cdot \frac{\eta}{r}}{dl} = Y \cdot \frac{r \cdot \frac{dl}{\rho} \cdot \frac{\eta}{r}}{dl} = \frac{\eta}{\rho} \cdot Y.$$

$$M = \int ds \cdot P_\eta \cdot \eta = \frac{Y}{\rho} \cdot \int \eta^2 ds$$



$$\Rightarrow I_\xi = \int dm\, \eta^2 = \sigma \int ds\, \eta^2.$$

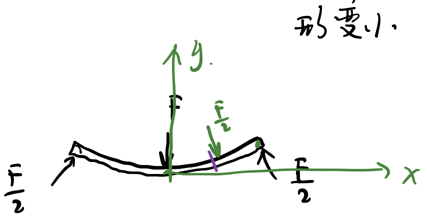
$$M = \frac{Y}{\rho} \cdot \frac{S}{m} \cdot I = KY \frac{IS}{m}.$$

特例. 圆形梁: $I = \frac{1}{4} mR^2$

$$\Rightarrow M_{圆} = K \cdot Y \cdot \frac{1}{4} \cdot \frac{mR^2 \cdot \pi R^2}{m} = \frac{1}{4} K \cdot Y \cdot \pi R^4.$$

例：手掰筷子，求形状　K很小.

形变小.



看，$x \to \frac{l}{2}$ 段 $(x>0)$

$$M = \frac{F}{2}\cdot\left(\frac{l}{2}-x\right)$$

$$M = \frac{1}{4}kY\cdot\pi R^4$$

$$\Rightarrow k = \frac{1}{\rho} = \frac{2F}{\pi R^4}\left(\frac{l}{2}-x\right)$$

$$\rho = \frac{(1+y'^2)^{\frac{3}{2}}}{|y''|} \approx \frac{1}{|y''|} \Rightarrow k = y''$$

$$\Rightarrow y'' = \frac{2F}{\pi R^4}\left(\frac{l}{2}-x\right)$$

$$\Rightarrow y = -\frac{F}{3\pi R^4}x^3 + \frac{Fl}{2\pi R^4}\cdot x^2 + a\cdot x + b.$$

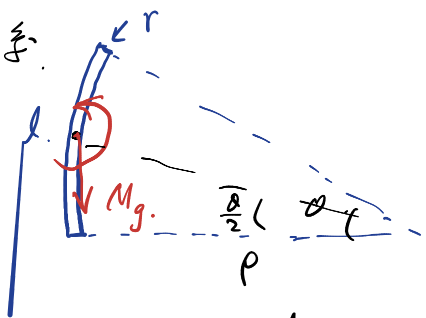
边界：$x=0,\ y=0,\quad x=0,\ y'=0$

$$\Rightarrow y_{(0)} = b = 0.\quad y'_{(0)} = a = 0.$$

$$\Rightarrow y_{(x)} = - \ \cdots \ \checkmark$$

树的高度与半径之间关系

均匀弯曲不大.



$$M = \frac{1}{4}\cdot kY\cdot\pi R^4.$$

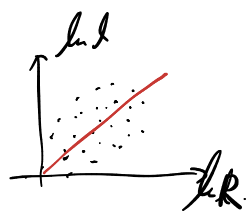
$$\theta = k\cdot l.$$

$$M = mg\cdot\frac{1}{k}\cdot\left(1-\cos\frac{\theta}{2}\right)$$

$$= \rho\cdot\pi R^2 l\cdot g\cdot\frac{1}{k}\cdot\left(\frac{1}{2}\cdot\frac{\theta^2}{2}\right)$$

$$\Rightarrow \frac{1}{4}\cancel{k}\,Y\cdot\pi R^4 = \rho\cdot\pi R^2\cdot l g\cdot\frac{1}{\cancel{k}}\cdot\frac{1}{8}\cdot k^{\cancel{2}}\,l^2$$

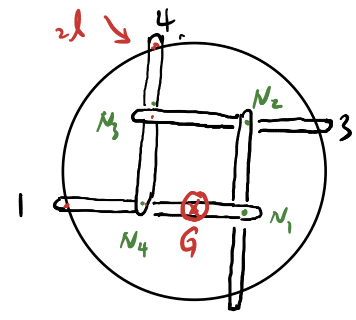
$$R^2 \propto l^3 \Rightarrow l \propto R^{\frac{2}{3}} \Rightarrow$$

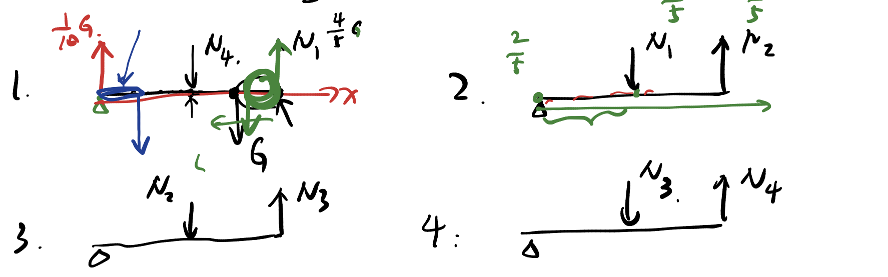


例：筷子受最大力矩为$\underline{M}$.

问：哪根最先断.

断时 $G=?$

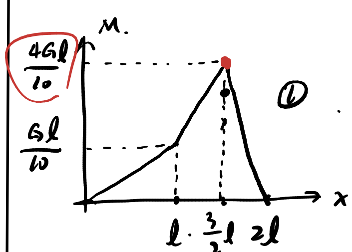




$$\Rightarrow N_4 = \frac{1}{2}N_3 = \frac{1}{4}N_2 = \frac{1}{8}N_1 \Rightarrow 3.\ 4.\ 不先断$$

1：力矩平衡：$-N_4\cdot l + N_1\cdot 2l = G\cdot\frac{3}{2}l$

$$\Rightarrow N_4 = \frac{1}{10}G \Rightarrow N_1 = \frac{4}{5}G.$$

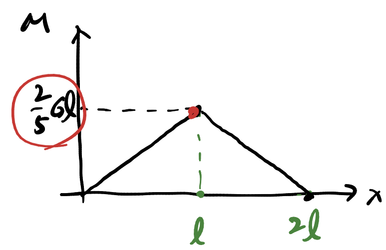


$0\sim x$：力矩：$\frac{1}{10}G\cdot x$

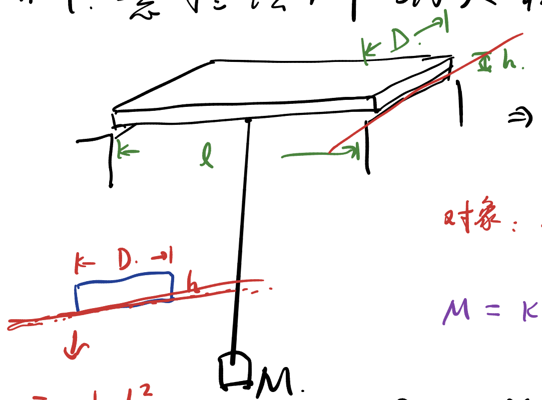
$\Rightarrow$ 1. 2 同时断

且断时 $\frac{2}{5}G\cdot l = M$

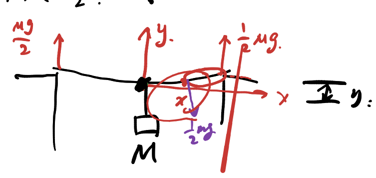
$$\Rightarrow G = \frac{5M}{2l}$$



例：悬挂法测钢尺杨氏模量.　形变小



$I = \frac{1}{3}h^2\cdot m$



对象：$x \to \frac{l}{2}$.　$M = \frac{1}{2}Mg\cdot\left(\frac{l}{2}-x\right)$

$$M = kY\cdot\frac{IS}{m} = kY\cdot\frac{\frac{1}{3}h^2\cdot m\cdot hD}{m}$$

$$\Rightarrow k\cdot Y\cdot\frac{1}{3}h^3 D = \frac{1}{2}Mg\left(\frac{l}{2}-x\right)$$

$$k = \frac{1}{\rho} = \frac{y''}{(1+y'^2)^{3/2}} \approx y'' \Rightarrow y'' = \frac{3Mg}{2Yh^3D}\left(\frac{l}{2}-x\right)$$

$$\Rightarrow y = -\frac{Mg}{4Yh^3D}x^3 + \frac{3Mgl}{8Yh^3D}\cdot x^2 + a\cdot x + b$$

$\Rightarrow$ $x=0,\ y=0,\quad x=0,\ y'=0 \Rightarrow a=b=0.$

当 $x=l$. $y = -\frac{Mg}{4Yh^3D}l^3 + \frac{3Mg}{8Yh^3D}l^3 \propto Mg.$

$$\Rightarrow y = \delta\cdot Mg.\quad \delta \propto \frac{1}{Y} \Rightarrow Y.\checkmark$$

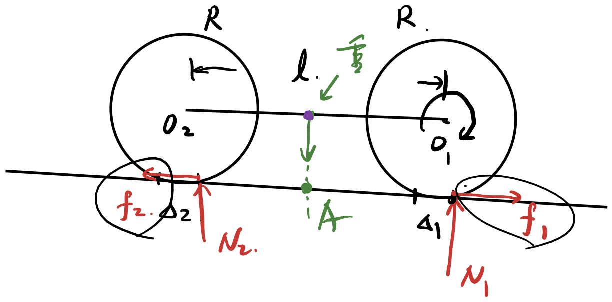
例：滚动摩擦； 车（轮轴） M.

→前. 重心：r高

$\Delta = k\cdot N \ll R$

简化.

前轮驱动.



(1) m匀速 v_0 行驶时. 功率 P.

(2) 求最小启动力矩 ← $a=0$ ⇒ 匀速.

(3) m 匀力矩启动. 求加速度.

(1) $P = F\cdot v = M\cdot\omega$ $\omega = \frac{v_0}{R}$

对前轮 O_1 力矩：$M = N_1\cdot\frac{\Delta}{2} + f_1\cdot R$ ⊛

求N：整体：$N_1+N_2=G$ ①

整体以A为轴：力矩：$N_1(\frac{l}{2}+\frac{\Delta}{2}) = N_2(\frac{l}{2}-\frac{\Delta}{2})$ ②

⇒ $N_1=N_2=\frac{G}{2}$

匀速：$f_1=f_2$

后轮：以 O_2 为轴 ⇒ $N_2\cdot\frac{\Delta_2}{2} = f_2\cdot R$

⇒ $f_1\cdot R = f_2\cdot R = N_2\cdot\frac{\Delta_2}{2} = N_1\cdot\frac{\Delta_1}{2}$

⊛ ⇒ $M = \frac{G}{2}\cdot k\cdot\frac{G}{2}/2 \times 2 = \frac{kG^2}{4}$

⇒ $P = M\cdot\omega = \frac{kG^2}{4}\cdot\frac{v_0}{R}$

(2) $a\to 0$. $v\to$ 不变 ⇒ $M=\frac{kG^2}{4}$

(3) 前轮：O_1 为轴：$M' = N_1\cdot\frac{\Delta_1}{2}+f_1\cdot R$ ⊛'

整体：质心所在面质心为轴 力矩：

$f_2\cdot R + N_2\cdot(\frac{l}{2}-\frac{\Delta_2}{2}) = f_1\cdot R + N_1\cdot(\frac{l}{2}+\frac{\Delta_1}{2})$ ①

整体竖直：$N_1+N_2=G$ (2)

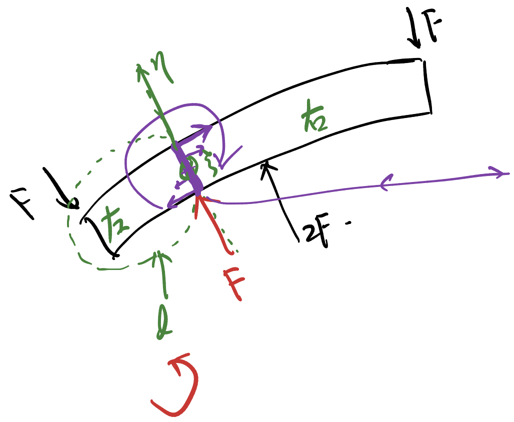
整体水平：$f_1-f_2 = m\cdot a$ (3)

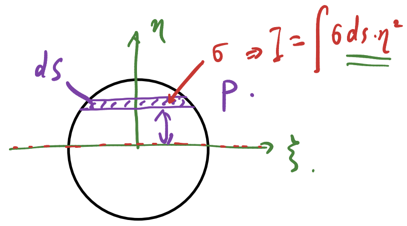
后轮 力矩平衡：O_2：$N_2\cdot\frac{\Delta_2}{2} = f_2\cdot R$ (4)

⇒ 解出……

令 $f_1=\mu N_1$ ⇒ 矛盾 ⇒ $f_1<\mu N$ ⇒ 静.

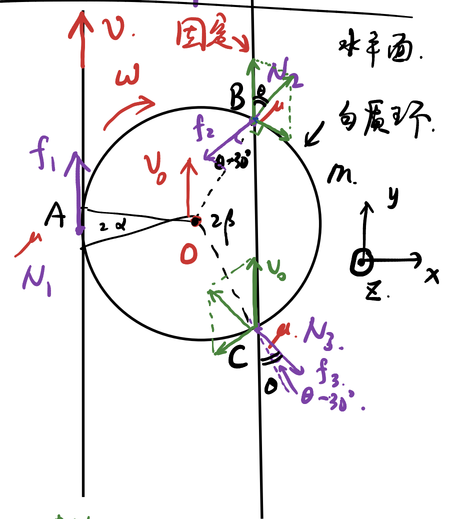
令 $v_0+\omega R = v$ ⇒ 矛盾 ⇒ $f_1=\mu N$ ⇒ 动.





$M = \int p\cdot ds\cdot\eta = \int \sigma\cdot\eta^2 ds$

例1. $2\alpha \ll 1$ $\beta=\frac{\pi}{3}$



Z方向平衡：

$G = N_1+N_2+N_3$.

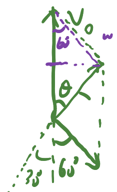
过O点且//x轴力矩：

$N_2=N_3$

过O点且//y轴力矩，

$N_1\cdot R = 2\cdot N_2\cdot R\cdot\cos 60°$

⇒ $N_1=N_2=N_3=\frac{1}{3}G$. ①



⇒ $\cos\theta = \frac{v_0-\omega R\frac12}{\sqrt{v_0^2+\omega^2R^2-v_0\omega R}}$ $\sin\theta = \frac{\omega R\frac{\sqrt3}{2}}{\sqrt{v_0^2+\omega^2R^2-v_0\omega R}}$

y方向上力平衡：$f_1 = f_2\cdot\cos\theta + f_3\cdot\cos\theta$ ②

$f_2=\mu N_2$, $f_3=\mu N_3$ ⇒ $f_2=f_3=\mu\frac{G}{3}$

力矩平衡：过O.且//z轴 ⇒

$f_1 R = f_2\cdot\sin(\theta-30°)\cdot R\times 2$. ③

② ⇒ $f_1 = \frac{2\mu G}{3}\cdot\frac{v_0-\omega R\cdot\frac12}{\sqrt{\cdots}}$ ②'

③ ⇒ $f_1\cdot R = \frac{2\mu G\cdot R}{3}\cdot(\sin\theta\cdot\frac{\sqrt3}{2} - \cos\theta\cdot\frac12)$.

$f_1\cdot\cancel{R} = \frac{2\mu G\cancel{R}}{3}\cdot\frac{\frac{\sqrt3}{2}\cdot\frac{\sqrt3}{2}\omega R - \frac12(v_0-\frac12\omega R)}{\sqrt{\cdots}}$ ③'

②'=③' ⇒ $v_0-\omega R\cdot\frac12 = \frac34\omega R - \frac12 v_0+\frac14\omega R$

⇒ $\frac32 v_0 = \frac32\omega R$ ⇒ $v_0 = \omega R$ ✓

①. 若 动摩擦 ⇒ $f_1=\mu N_1 = \mu\cdot\frac{G}{3}$

⇒ ②'：$\frac{\cancel{\mu G}}{\cancel{3}} = \frac{2\cancel{\mu G}}{\cancel{3}}\cdot\frac{v_0-\omega R\cdot\frac12}{\sqrt{\ }}$

⇒ $\sqrt{v_0^2+\omega^2R^2-v_0\omega R} = 2v_0-\omega R$

⇒ $v_0^2+\cancel{\omega^2R^2}-v_0\omega R = 4v_0^2+\cancel{\omega^2R^2}-4v_0\omega R$ $(2\omega R-v_0)$

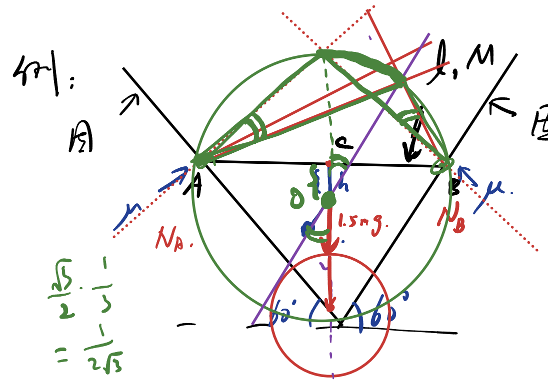
⇒ $v_0=0$ 或 $v_0=\omega R$.

③'：$\frac{\cancel{\mu G}}{\cancel{3}} = \frac{2\cancel{\mu G}}{\cancel{3}}\cdot\frac{\frac34\omega R-\frac12 v_0+\frac14\omega R}{\sqrt{\ }}$

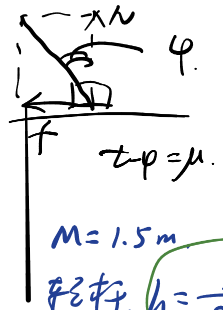
⇒ $\cancel{v_0^2}+\omega^2R^2 - v_0\omega R = 4\omega^2R^2+\cancel{v_0^2}-4v_0\omega R$.

⇒ $\omega R=0$ 或 $\omega R = v_0$. ⇒ 动摩擦. $\boxed{v_0=\omega R} < \frac{v}{2}$.

利用摩擦角的化简.



$\frac{\sqrt{3}}{2}\cdot\frac{1}{3}$
$=\frac{1}{2\sqrt{3}}$

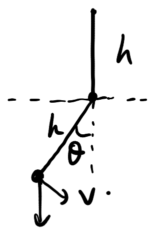


$\tan\varphi=\mu$.

$M=1.5m$

摆杆, $h=\frac{l}{2\sqrt{3}}$.

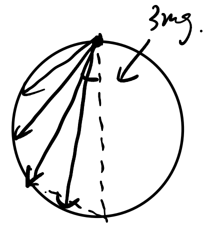
求 μ_{min}.



$\Rightarrow$ $T-mg\cdot\cos\theta=\frac{mv^2}{h}$ ① $\Rightarrow T=3mg\cos\theta$.

解: $\frac{1}{2}mv^2=mg\cdot h\cdot\cos\theta$.

$\Rightarrow \tan\varphi=\tan15°=\mu_{min}$.



例: 平面图形.

OA 竖直, 固定.

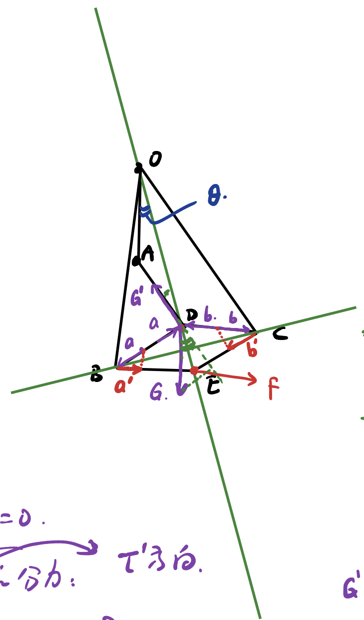
$|OA|=|AD|=|BD|=|CD|=$
$=|BE|=|CE|=a$.

$|OB|=|OC|=b$.

铰接.

为使系统平衡.

求 $F_{min}=?$



$\Rightarrow \frac{a}{b}=\frac{a'}{b'}$

$\frac{a}{a'}=\frac{b}{b'}$

D 平衡: $\vec{G}+\vec{G'}+\vec{a}+\vec{b}=0$. τ' 方向.

⇓ 垂直于 $\vec{G'}$ 方向的分力:

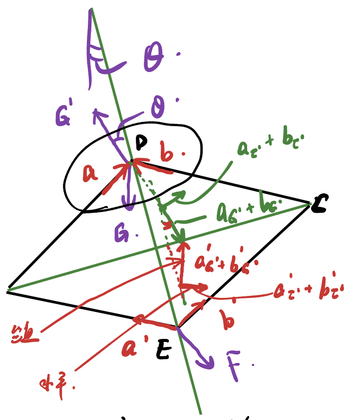
$\Rightarrow G_{\tau'}+a_{\tau'}+b_{\tau'}=0$.

$\Rightarrow |a_{\tau'}+b_{\tau'}|=G_{\tau'}=G\cdot\sin2\theta$.

$|a_{G'}+b_{G'}|=$ 任意

B. C. 受力平衡 $\Rightarrow \frac{a}{a'}=\frac{b}{b'}$

E 受力: $\vec{F}+\vec{a'}+\vec{b'}=0$



BCDE: 沿 BC 镜面对称.

大小: 方向 $\begin{cases} a.\ a' \text{方向对称} \\ b.\ b' \text{方向对称} \\ \frac{a}{a'}=\frac{b}{b'} \end{cases}$

$\vec{a}+\vec{b}$, $\vec{a'}+\vec{b'}$ $\Rightarrow$ 对称

$\Rightarrow \frac{\vec{a}+\vec{b}}{\vec{a'}+\vec{b'}}=\frac{a}{a'}=\frac{b}{b'}$

$a'_{水平}+b'_{水平}=$ 确定.

$a'_{竖直}+b'_{竖直}=$ 任意.

$\Rightarrow F_{min}=F_{水平}=a'_{水平}+b'_{水平}=G\cdot\sin2\theta\cdot\frac{a'}{a}$

第二部分: 几何.

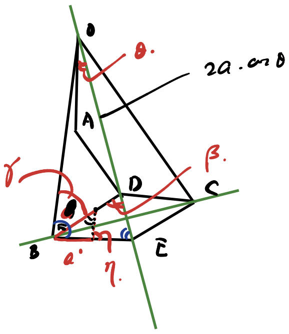
在红△中, 正弦定理:

$\frac{a'}{a}=\frac{\sin\gamma}{\sin\eta}$ ①.

△OBD 中:

$\frac{\sin\gamma}{2a\cdot\cos\theta}=\frac{\sin\beta}{b}$ ②

△OBE 中: $\frac{\sin\eta}{2a\cdot\cos\theta+2a\cdot\cos\beta}=\frac{\sin\beta}{b}$ ③



$\frac{②}{③}\Rightarrow \frac{\sin\gamma}{\sin\eta}=\frac{2a\cdot\cos\theta}{2a\cdot\cos\theta+2a\cdot\cos\beta}$

求 $\cos\beta$

在 △OBD 中 余弦定理:

$a^2+(2a\cdot\cos\theta)^2+2a\cdot2a\cdot\cos\theta\cdot\cos\beta=b^2$.

$\Rightarrow \cos\beta=\frac{b^2-a^2-4a^2\cos^2\theta}{4a^2\cdot\cos\theta}$.

$\Rightarrow \frac{\sin\gamma}{\sin\eta}=\frac{2a\cdot\cos\theta}{2a\cdot\cos\theta+\frac{b^2-a^2-4a^2\cos^2\theta}{2a\cdot\cos\theta}}$

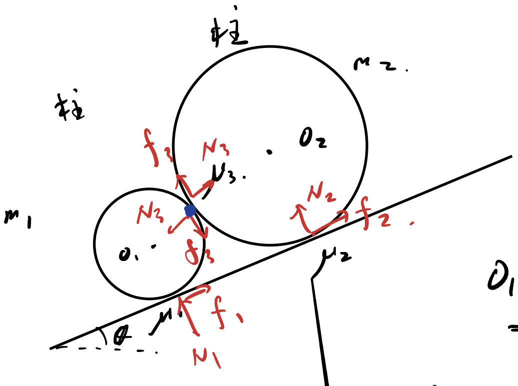
$=\frac{4a^2\cos^2\theta}{b^2-a^2}$

$\Rightarrow F_{min}=G\cdot\sin2\theta\cdot\frac{4a^2\cos^2\theta}{b^2-a^2}$

选取合适的支点 $\Rightarrow$ 使问题化简.

一个方程 → 一个力.

例:



平衡. 求 $\mu_1/\mu_2/\mu_3$ 各自至少为多少.

$f=\mu N$.

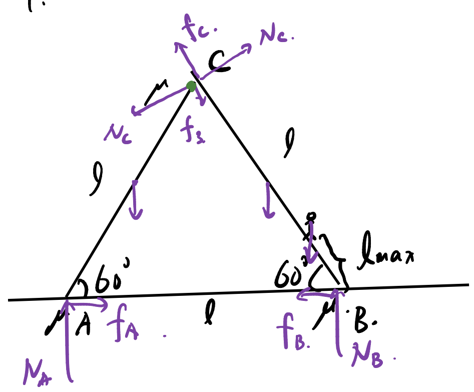
O_1 与 O_2 都是 ~~柱~~ 柱.

$\Rightarrow f_1=f_3=f_2$.

整体: 沿斜面: $f_1+f_2=2f=(m_1+m_2)\cdot g\cdot\sin\theta$.

N_1: 一个方程 → 解出一个力.

例:



杆: m, (均匀)　人 m

$\mu < \frac{1}{\sqrt{3}}$　求 l_{max}

平衡时. f_A. f_B. f_C.

$f_A \le \mu N_A$,　$f_B \le \mu N_B$.

$f_C \le \mu N_C$

$\frac{f}{N}$ 最大处最容易滑

对整体. 以 B 为轴:

$\Rightarrow N_A \cdot l =$ 重力力矩. $\Rightarrow N_A =$ ✓.

整体竖直: $N_A + N_B = 3mg$. $\Rightarrow N_B =$ ✓.

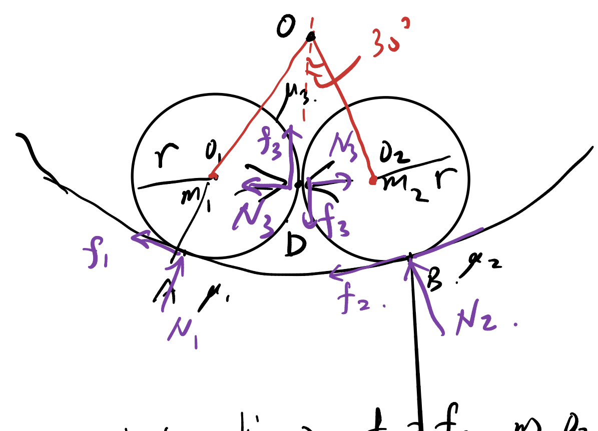
整体, 水平力 $\Rightarrow f_A = f_B$.

以 C 为轴. 看右杆 $\Rightarrow f_A = \cdots$ ✓ $\Rightarrow f_B = \cdots$ ✓.

以 B 为轴. 看右杆 $\Rightarrow N_C = \cdots$

$f_C = \cdots$ ✓

例:



求平衡.

$\mu_1 \mu_2 \mu_3$ 分别至少多少?

以 O_1 为轴 左球. $\Rightarrow f_1 = f_3$.　以 O_2 为轴. 看右球. $f_2 = f_3$.

$\Rightarrow f_1 = f_2 = f_3 = f$.

$\Rightarrow$ 整体. 以 O 为轴 力矩: $m_1 g \cdot 2R \cdot \frac{1}{2} - m_2 g \cdot 2R \cdot \frac{1}{2}$

$= f_1 \cdot 3R + f_2 \cdot 3R$.

$\Rightarrow f = \frac{1}{6}(m_1 - m_2) g$.

以 D 为轴. 左球. 力矩 $\Rightarrow N_1 \Rightarrow$ ✓.

$\Rightarrow N_2 \Rightarrow$ ✓

$\Rightarrow N_3 \Rightarrow$ ✓.

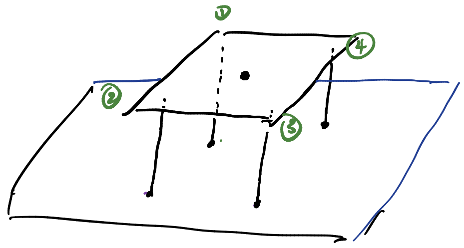
$\Rightarrow \mu_1 \mu_2 \mu_3$ 都有 ...

静不定与解除.

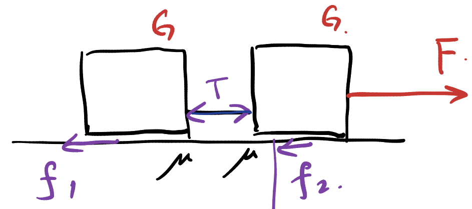
模型化时.

刚性 ⟷ 形变信息

加信息: $\begin{cases} \text{刚性杆} \Rightarrow \text{弹簧杆}. \Rightarrow ✓ \\ \text{临界情况} \Rightarrow \end{cases}$



例:



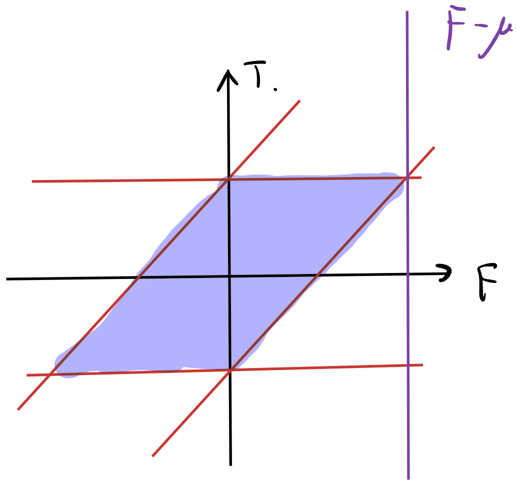
$-\mu G < f_1 < \mu G$.　　$-\mu G < f_2 < \mu G$.

左: $T = f_1$　　右手: $T + f_2 = F$

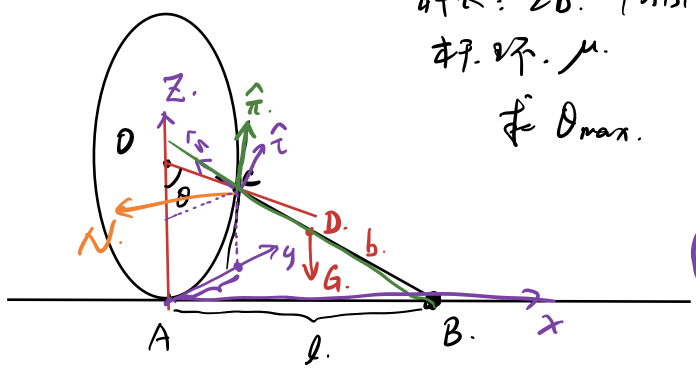
$\Rightarrow T = F - f_2$

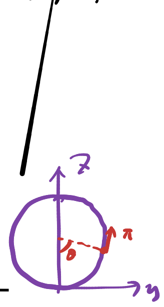
$-\mu G < T < \mu G$

$F - \mu G \le T \le F + \mu G$



例: 杆长: 2b. $|AB|=l$, 环 r.
杆.环. μ.
求 θ_{max}.





$\vec{N}$: $\perp BC$ 且 $\perp$ 于过C的.环的切线 $\Rightarrow \hat{\pi}$.

$\widehat{BC}$

$\vec{f} \perp \vec{N}$. $\vec{f}$: 在 BC 与 $\hat{\pi}$ 所在平面内.

$\hat{n}$: 沿 $\vec{BC}$. $\hat{\tau}$: $\perp \vec{BC}$.

$\vec{f} = \vec{f}_n + \vec{f}_\tau$

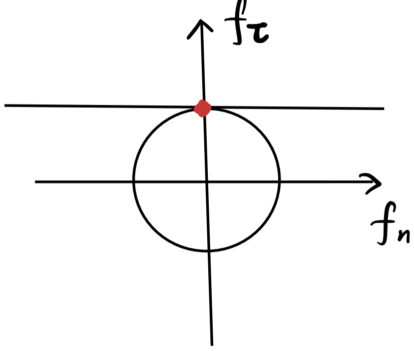
约束: 由 B. 对杆力矩.

$$\left(\vec{BD}\times\vec{G} + \vec{BC}\times\vec{N} + \vec{BC}\times(\vec{f}_n+\vec{f}_\tau) = 0\right)\cdot\hat{N}$$

$$\Downarrow \underbrace{(\vec{BD}\times\vec{G})\cdot\hat{N}}_{\text{确定}} + \underbrace{\vec{BC}\times\vec{N}\cdot\hat{N}}_{0} + \underbrace{\vec{BC}\times\vec{f}_n\cdot\hat{N}}_{0} + \vec{BC}\times\vec{f}_\tau\cdot\hat{N} = 0$$

$\Rightarrow (\vec{BD}\times\vec{G})\cdot\hat{N} + \vec{BC}\times\vec{f}_\tau\cdot\hat{N} = 0$

$\Rightarrow f_\tau$ = 确定. $\Rightarrow f_n$ = 任意



$f^2 = f_n^2 + f_\tau^2 \le \mu N$.

$\Rightarrow$ 临界时. $f_n = 0$

$\Rightarrow \vec{f} = \vec{f}_\tau$

$\Rightarrow$ 解平衡. 即可. 解析几何.

$\vec{BC} = (-l,\ R\cdot\sin\theta,\ R(1-\cos\theta))$.

$\hat{\pi} = (0,\ 1\cdot\cos\theta,\ 1\cdot\sin\theta)$.

$\hat{N} \parallel$: $\vec{BC}\times\vec{\pi}$.

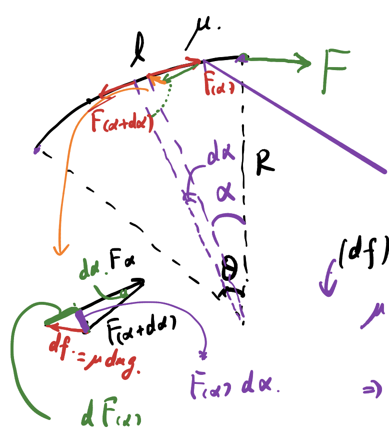
$\vec{N}_1 = (R\sin^2\theta - R\cos\theta(1-\cos\theta),\ 0 + l\sin\theta,\ -l\cos\theta - 0)$.

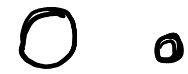
$\Rightarrow \hat{N} = \dfrac{\vec{N}_1}{|\vec{N}_1|} = \cdots$ ✓

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$\hat{i}\times\hat{j} = \hat{k}$.
$\hat{j}\times\hat{k} = \hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$

例: 绳长 l. m,





$\theta = \dfrac{l}{R}$.

一小段: $\alpha \sim \alpha + d\alpha$.

$(df)^2 = (dF(\alpha))^2 + (F(\alpha)d\alpha)^2$

$\mu dmg = \mu\cdot\dfrac{d\alpha}{\theta}\cdot m\cdot g$

$\Rightarrow \mu^2 m^2 g^2 \cdot \dfrac{1}{\theta^2}(d\alpha)^2 = (dF(\alpha))^2 + (F(\alpha)d\alpha)^2$.

$\Rightarrow \dfrac{\mu^2 m^2 g^2}{\theta^2} = F'^2_{(\alpha)} + F^2_{(\alpha)}$

$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = C$.

$\dfrac{d}{d\alpha} \Rightarrow 0 = 2F'(\alpha)F''(\alpha) + 2F(\alpha)F'(\alpha)$

$\Rightarrow F''(\alpha) + F(\alpha) = 0$. $\boxed{\ddot{x} + \omega^2 x = 0}$ 谐振

$F(\alpha) = A\cdot\cos(\alpha+\alpha_0)$.

$\theta < 90°$

① $\alpha = 0$. $F(0) = A\cos(\alpha_0) = F$. $\to A = \dfrac{F}{\sin\theta}$

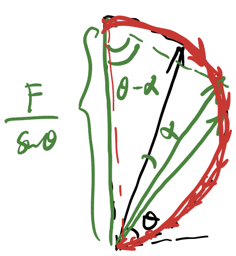
② $\alpha = \theta$ 时. $F(\theta) = A\cos(\theta+\alpha_0) = 0$.

$\Rightarrow \theta + \alpha_0 = \dfrac{\pi}{2}$. $\alpha_0 = \dfrac{\pi}{2} - \theta$.

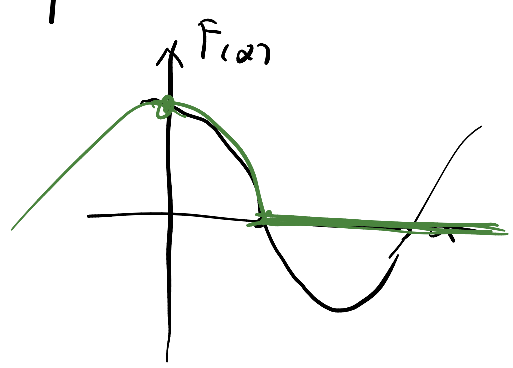
$F(\alpha) = \dfrac{F}{\sin\theta}\cdot\cos\left(\dfrac{\pi}{2} - \theta + \alpha\right) = \boxed{\dfrac{F}{\sin\theta}}\sin(\theta-\alpha)$

$F'(\alpha) = -\dfrac{F}{\sin\theta}\cdot\sin\left(\dfrac{\pi}{2} - \theta + \alpha\right)$

$\Rightarrow F^2_{(\alpha)} + F'^2_{(\alpha)} = \dfrac{F^2}{\sin^2\theta} = \dfrac{\mu^2 m^2 g^2}{\theta^2} \Rightarrow F = \dfrac{\mu mg\cdot\sin\theta}{\theta}$.



$\Rightarrow$.



$\theta > 90°$. $\Rightarrow$

$F(\alpha) = \dfrac{F}{\sin 90°}(\cos\alpha)$

$\dfrac{\frac{\pi}{2}}{\theta}\cdot m\cdot\mu g = \dfrac{F}{\sin 90°} \Rightarrow F = \dfrac{\frac{\pi}{2}\mu mg}{\theta}$