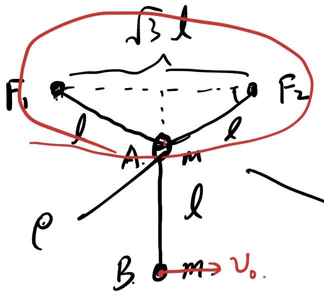
例：




求简振模. 小振动. 光滑.

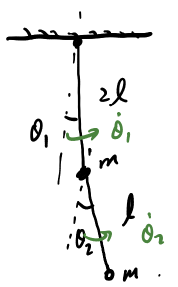
轨道. 椭.

$2a = 2l \Rightarrow a = l$

$b = \frac{l}{2}$. $\rho = \frac{a^2}{b} = 2l$

$\cos\theta = 1 - \frac{\theta^2}{2}$

$\theta_1, \theta_2 \ll 1$




$\Rightarrow E_p = -mg\cdot 2l\cos\theta_1 - mg\cdot(2l\cdot\cos\theta_1 + l\cdot\cos\theta_2)$

$E_p = -mg\cdot 2l + \frac{1}{2}mg\cdot 2l\cdot\theta_1^2 - mg(2l+l) + \frac{1}{2}mg\cdot 2l\cdot\theta_1^2 + \frac{1}{2}mg\cdot l\cdot\theta_2^2$

$5mgl$

$E_p + \square = \frac{1}{2}(4mgl)\theta_1^2 + \frac{1}{2}(mgl)\theta_2^2$ ①

$E_k = \frac{1}{2}\cdot m\cdot(2l\cdot\dot\theta_1)^2 + \frac{1}{2}\cdot m\cdot(2l\dot\theta_1 + l\dot\theta_2)^2$

$= \frac{1}{2}\cdot m\cdot 4l^2\dot\theta_1^2 + \frac{1}{2}\cdot m 4l^2\dot\theta_1^2 + \frac{1}{2}ml^2\dot\theta_2^2 + \frac{1}{2}m\cdot 4l^2\dot\theta_1\dot\theta_2$

$E_k = \frac{1}{2}\cdot(8ml^2)\dot\theta_1^2 + \frac{1}{2}ml^2\dot\theta_2^2 + \frac{1}{2}\cdot(4ml^2)\dot\theta_1\dot\theta_2$

第一步. 令. $x = 2\theta_1$ $y = \theta_2$ $\Rightarrow \dot x = 2\dot\theta_1, \dot y = \dot\theta_2$

$\Rightarrow \begin{cases} E_p = \frac{1}{2}\cdot(mgl)x^2 + \frac{1}{2}(mg\cdot l)\cdot y^2. \\ E_k = \frac{1}{2}\cdot(2ml^2)\dot x^2 + \frac{1}{2}ml^2\cdot\dot y^2 + \frac{1}{2}(2ml^2)\dot x\dot y \end{cases}$

第二步: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix}$.

$\Rightarrow \dot x = \cos\theta\cdot\dot\xi + \sin\theta\cdot\dot\eta$, $\dot y = -\sin\theta\dot\xi + \cos\theta\cdot\dot\eta$.

$\Rightarrow E_p = \frac{1}{2}\cdot mgl\cdot[\cos^2\theta\cdot\xi^2 + \sin^2\theta\eta^2 + \cancel{2\sin\theta\cos\theta\,\xi\cdot\eta} + \sin^2\theta\cdot\xi^2 + \cos^2\theta\cdot\eta^2 - \cancel{2\sin\theta\cos\theta\,\xi\cdot\eta}]$

$= \frac{1}{2}mgl[\xi^2 + \eta^2]$.

$E_k = \frac{1}{2}\cdot(2ml^2)(\cos^2\theta\dot\xi^2 + \sin^2\theta\dot\eta^2 + 2\cos\theta\sin\theta\dot\xi\dot\eta)$
$+ \frac{1}{2}ml^2(\sin^2\theta\cdot\dot\xi^2 + \cos^2\theta\cdot\dot\eta^2 - 2\sin\theta\cos\theta\cdot\dot\xi\dot\eta)$
$+ \frac{1}{2}(2ml^2)\cdot(-\sin\theta\cdot\cos\theta\cdot\dot\xi^2 + (\cos^2\theta - \sin^2\theta)\dot\xi\dot\eta + \sin\theta\cos\theta\cdot\dot\eta^2)$.

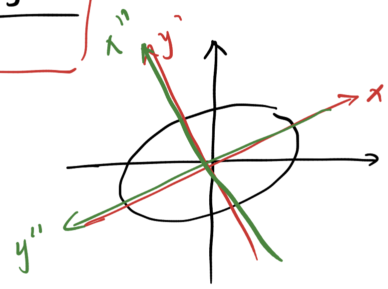
$= \frac{1}{2}\cdot ml^2(2\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta)\cdot\dot\xi^2$
$+ \frac{1}{2}ml^2\cdot(2\sin^2\theta + \cos^2\theta + 2\cdot\sin\theta\cos\theta)\dot\eta^2$
$+ \frac{1}{2}ml^2(4\sin\theta\cos\theta - 2\sin\theta\cos\theta + 2(\cos^2\theta - \sin^2\theta))\dot\xi\dot\eta$

$4\sin\theta\cos\theta - 2\sin\theta\cos\theta + 2(\cos^2\theta - \sin^2\theta) = 0$.

$\Rightarrow 2\tan\theta + 2 - 2\tan^2\theta = 0$.

$\Rightarrow \tan^2\theta - \tan\theta - 1 = 0$.

$\Rightarrow \boxed{\tan\theta = \frac{1\pm\sqrt{5}}{2}}$ → 任取一个都行.



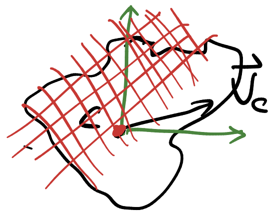

$E_p = \frac{1}{2}mgl\xi^2 + \frac{1}{2}mgl\eta^2$

$\Rightarrow E_k = \frac{1}{2}\alpha\dot\xi^2 + \frac{1}{2}\beta\dot\eta^2$.

$\Rightarrow \omega_1 = \sqrt{\frac{mgl}{\alpha}}$, $\omega_2 = \sqrt{\frac{mgl}{\beta}}$ $\Rightarrow$ … ✓.

$\vec L = \vec L_c + \vec L_r$.

质心角动量. 质心系(平动)




O

$\vec L = \sum m_i\cdot\vec r_i\times\vec v_i$ $\begin{cases}\vec r_i = \vec r_c + \vec r_r \\ \vec v_i = \vec v_c + \vec v_r\end{cases}$

$= \sum m_i\cdot(\vec r_c + \vec r_r)\times(\vec v_c + \vec v_r)$

$= \boxed{\sum m_i(\vec r_c\times\vec v_c)} + \sum m_i\vec r_c\times\vec v_r \leftrightarrow \vec r_c\times\boxed{\sum m_i\vec v_r}$ 0

$+ \sum m_i\vec r_r\times\vec v_c + \sum m_i(\vec r_r\times\vec v_r)$

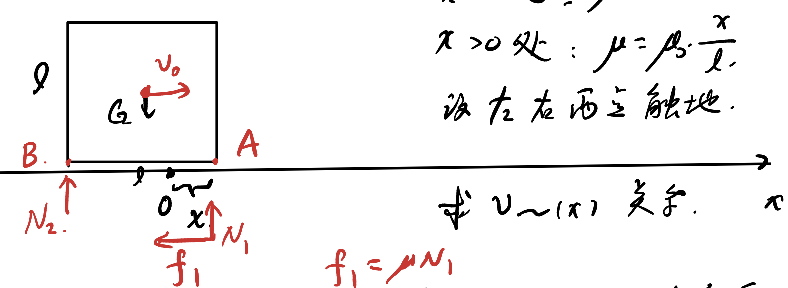
$(\sum m_i\vec r_r)\times\vec v_c$
$=0$

$\vec L = \vec L_c + \vec L_r$

$E_{k总} = E_c + E_r$.

质心. 相对

例：




$x<0$ 处: $\mu = 0$.

$x>0$ 处: $\mu = \mu_0\cdot\frac{x}{l}$.

设左右两边触地.

求 $v\sim(x)$ 关系.

$f_1 = \mu N_1$

力平衡: 竖直: $N_1 + N_2 = G$. ①

力矩: 质心为参考.

$\Rightarrow N_2\cdot\frac{l}{2} + f_1\cdot\frac{l}{2} = N_1\cdot\frac{l}{2}$ ② $\Rightarrow N_1 = \frac{mg}{2 - \frac{\mu_0 x}{l}}$

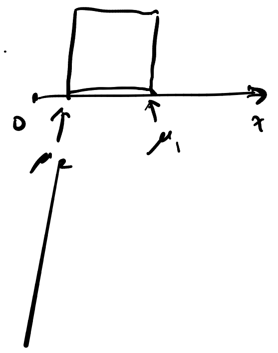
$\Rightarrow \boxed{f = \mu N_1 = \frac{\mu mg\cdot l}{2l - \mu_0 x} = \frac{mg\cdot\mu_0\cdot x}{2l - \mu_0 x}}$

牛二:$\left(m\ddot{x}=f=\frac{\mu_0 mg x}{2l-\mu_0 x}\right)\cdot dx$.

$\Rightarrow m\frac{d\dot x}{dt}\cdot dx=\frac{\mu_0 mg x}{2l-\mu_0 x}dx$

$\Rightarrow \int m\cdot\dot x\, d\dot x=\int\frac{\mu_0 mgx}{2l-\mu_0 x}dx\Rightarrow\cdots$

当 $x=l$ 时, $v>0$. ⇒ 再看受力.



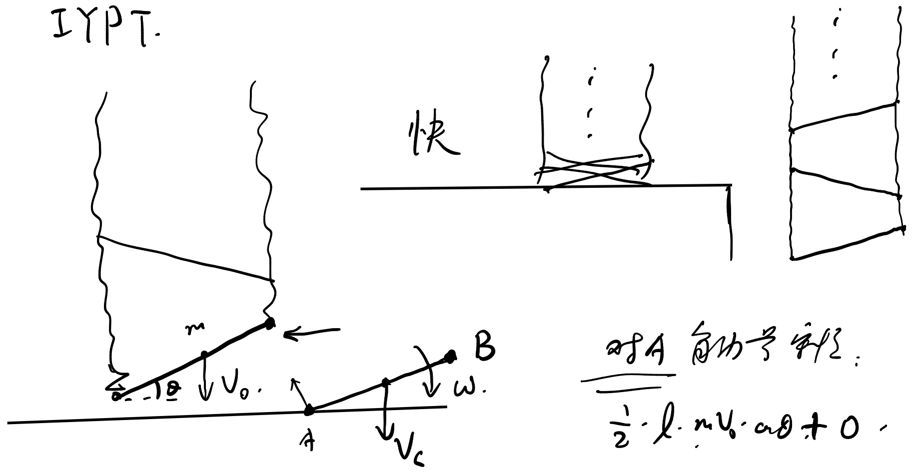
或.

$f=\frac{\mu_0 mg x}{2l-\mu_0 x}$

⇒ 解: $\int\Delta E_k=\int -f\cdot dx\Rightarrow\cdots$

IYPT.

快



对A 角动量守恒:

$\frac12\cdot l\cdot mV_0\cdot\cos\theta+0$

$=\frac12 l\cos\theta\cdot mV_c+\frac1{12}ml^2\cdot\omega$.

$V_A=0\Rightarrow V_c-\omega\frac l2\cos\theta=0$

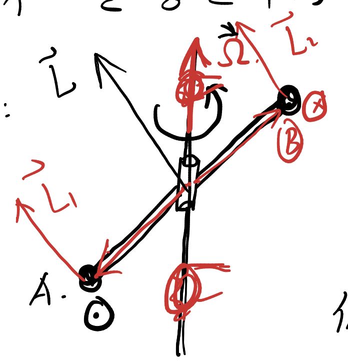
$\Rightarrow \frac12 ml V_0\cos\theta=\frac12 m l V_c\cos\theta+\frac1{12}ml^2\frac{2V_c}{l\cos\theta}$

$\Rightarrow V_c=\frac{V_0\cos\theta}{\cos\theta+\frac{1}{3\cos\theta}}$

$\Rightarrow V_B=V_c+\omega\frac l2\cos\theta=2V_c=\boxed{\frac{2\cos\theta}{\cos\theta+\frac1{3\cos\theta}}}V_0$.

$\vec L$ 不一定总是平行于 $\vec\omega$.

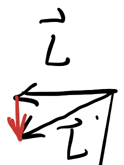
例:



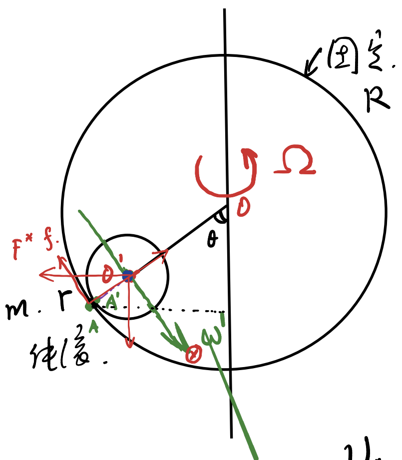
$\vec L=\sum m_i\vec r_i\times\vec v_i$

$\frac{d\vec L}{dt}=\vec M$.

俯:



例:



$\frac{2\pi}{\Omega}$. 已知 $\Omega$

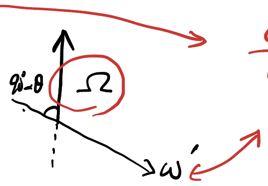
已知公转系中 $\vec\omega'$

⊥于 $OO'$面

⇒ 求 $f=?$

公转系中(转动系). $\begin{cases}V_A=\Omega\cdot R\cdot\sin\theta\\V_A'=\omega'\cdot r\end{cases}\Rightarrow\omega'=\frac{R\sin\theta}{r}\Omega$.

$SO'$-红平动系:



$\frac{d\vec L}{dt}=\vec M$.

$\frac{d\vec L}{dt}=\vec\Omega\times I\cdot\vec\omega'$

$=\Omega\cdot\omega'\cdot I\cdot\cos\theta$. ⊗ $I=\frac25 mr^2$.

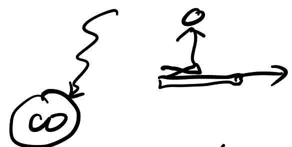
$\frac{d\vec L}{dt}=\vec M\Rightarrow M=f\cdot r$

$f=\frac{\Omega\omega' I\cos\theta}{r}$

相对论: (1) 事件 (2) 世界线的语言描述.

$A(x,y,z,ct)\longleftrightarrow A(x',y',z',ct')$.

事件:



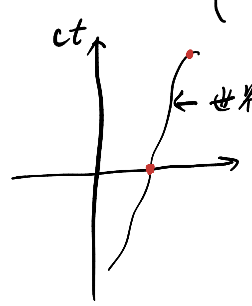
$S'$

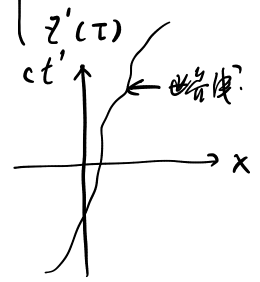
$S$ $(x,y,z,ct)$. $S'$ $(x',y',z',ct')$.

$(x,y,z,ct)$. $S'$ $(x',y',z',ct')$.

世界线. $S$: $\begin{cases}t(\tau)\\x(\tau)\\y(\tau)\\z(\tau)\end{cases}$ 固有时. $S'$ $\begin{cases}t'(\tau)\\x'(\tau)\\y'(\tau)\\z'(\tau)\end{cases}$

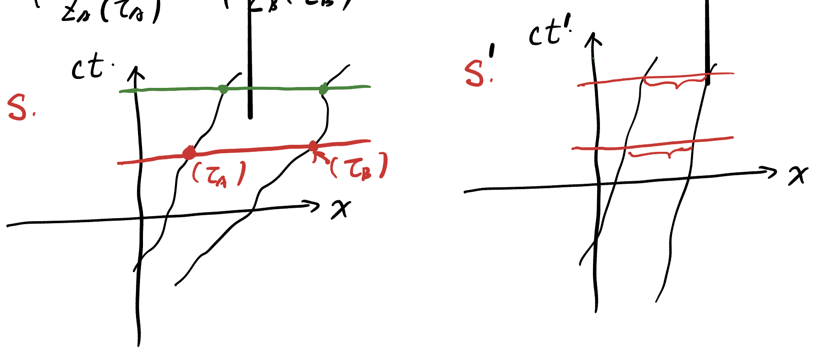
质点.



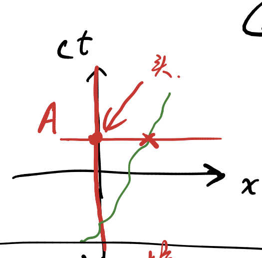
例：长度？ 某系中 同时测 左、右端 ⇒ 长度

$S: \begin{cases} t_A(\tau_A) \\ x_A(\tau_A) \\ y_A(\tau_A) \\ z_A(\tau_A) \end{cases}, \begin{cases} t_B(\tau_B) \\ x_B(\tau_B) \\ y_B(\tau_B) \\ z_B(\tau_B) \end{cases}$ $\quad S': \begin{cases} t_A'(\tau_A) \\ x_A'(\tau_A) \\ y_A'(\tau_A) \\ z_A'(\tau_A) \end{cases}, \begin{cases} t_B'(\tau_B) \\ x_B'(\tau_B) \\ y_B'(\tau_B) \\ z_B'(\tau_B) \end{cases}$

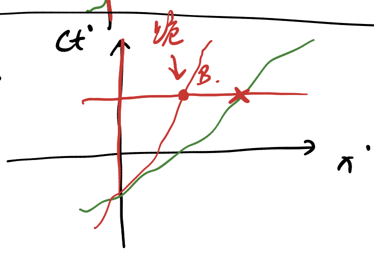



"同时"？

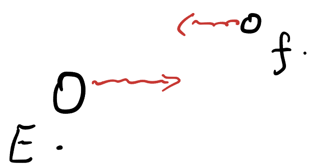
黄系： { 同时. ⇒




孙系： { 同时 ⇒




双生子.



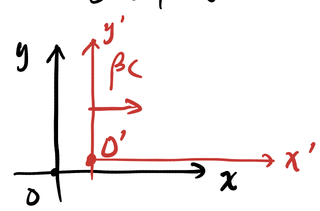

地球系中 f-S-E 相距为 $l$ 时
飞船发出电报.

A. 飞船发光时.
B. 地球接收时
C. 飞船再次收信时.

在飞船系中 3个时间
在地球系中 3个时间.
飞船系中看地球坐标
地球系中看飞船坐标

$$\begin{cases} ct' = \gamma(ct - \beta x) \\ x' = \gamma(x - \beta ct) \\ y' = y \\ z' = z \end{cases}$$

S系. 事件A $(x, y, z, ct)$.
S'系. 事件A $(x', y', z', ct')$




地系：

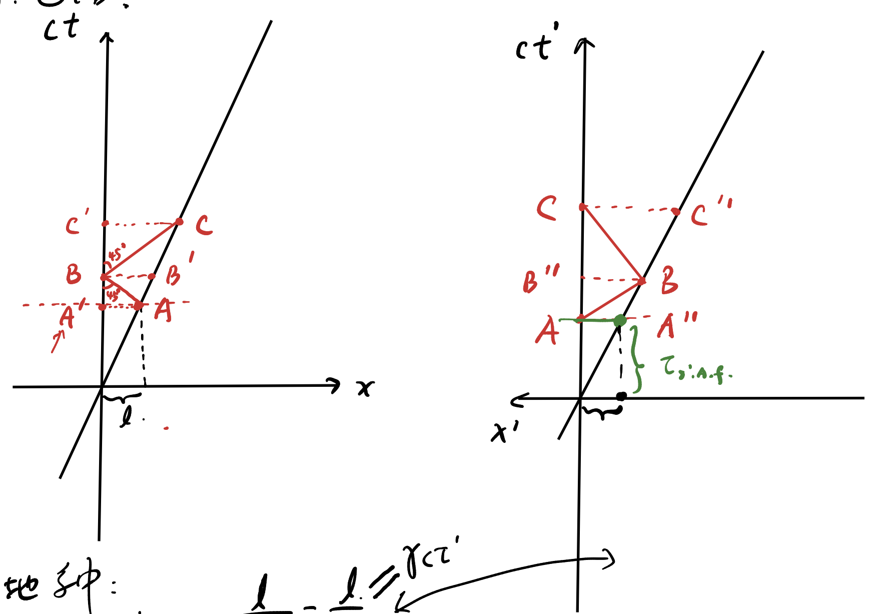
S. 地球：$\begin{cases} ct = c\tau \\ x = 0. \end{cases}$ ← 地球固有时. $\quad$ S'系. 飞船：$\begin{cases} ct' = c\tau' \\ x' = 0. \end{cases}$ ← 固有时.

S. 飞船. $\begin{cases} ct = \gamma(ct' + \beta x') = \gamma c\tau' \\ x = \gamma(x' + \beta ct') = \beta\gamma c\tau' \end{cases}$ $\quad \begin{cases} ct' = \gamma(ct - \beta x) = \gamma c\tau. \\ x' = \gamma(x - \beta ct) = -\beta\gamma c\tau \end{cases}$

事件A：飞船出发. ($t=0$) 事件B. 煮 hotpot. ($t$) $\tau$ $\quad$ 事件A：地球出发，B：火 ⇒

S. ①信： $\quad \dfrac{ct}{x} = \beta$ $\quad$ S' ①信.

S. ②信： $\quad$ S' ②信：




地系：

A：$\begin{cases} ct_A = c \cdot \dfrac{l}{\beta c} = \dfrac{l}{\beta} \neq \gamma c\tau' \\ x_A = l. \end{cases}$

S系. A事件发生时，飞船的固有时.
$c\tau_{S.A.f} = \dfrac{l}{\gamma\beta}$

S'系. A事件. 飞船固有时
$c\tau_{S'.A.f} = \dfrac{l}{\gamma\beta}$

S系. A事件发生时. 地球的固有时.
$c\tau_{S.A.e} = \dfrac{l}{\beta}$

S'系. A事件. 地球固有时.
$c\tau_{S'.A.e} = \dfrac{l}{\gamma^2\beta}$

S系. A事件发生时. f. 在：
$x_{S.A.f} = l$.

S'系. A事件. 地球位置
$x'_{S'.A.e} = -\dfrac{l}{\gamma}$

问：$\boxed{\text{飞船看. 飞船发信时地球的时间}}$
{ 地球看. 飞船发信时飞船时间.
地球看. 飞船发信时. 地球时间.

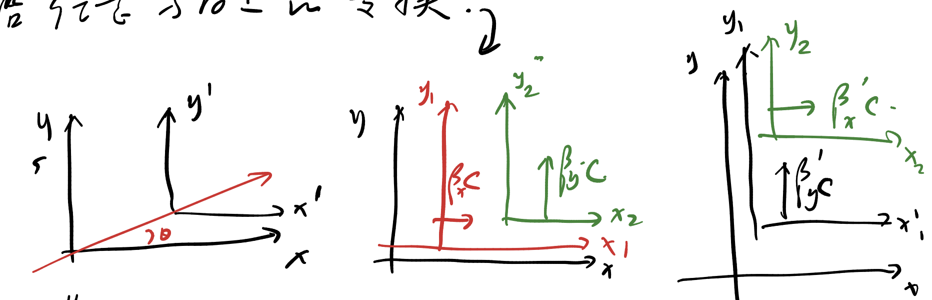

地系：

S. 地球：$\begin{cases} ct = c\tau \\ x = 0. \end{cases}$ ← 地球固有时. $\quad$ S'系. 飞船：$\begin{cases} ct' = c\tau' \\ x' = 0. \end{cases}$ ← 固有时.

S. 飞船. $\begin{cases} ct = \gamma(ct' + \beta x') = \gamma c\tau' \\ x = \gamma(x' + \beta ct') = \beta\gamma c\tau' \end{cases}$ $\quad \begin{cases} ct' = \gamma(ct - \beta x) = \gamma c\tau. \\ x' = \gamma(x - \beta ct) = -\beta\gamma c\tau \end{cases}$

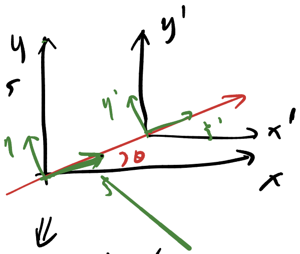
事件A：飞船出发. ($t=0$) 事件B. 煮 hotpot. ($t$) $\tau$ $\quad$ 事件A：地球出发，B：火 ⇒

沿任意方向上的变换。



第一种变换. $\neq$ 第二种. $\neq$ 第三种.

$a\cdot b=b\cdot a$. $\vec a\cdot\vec b=\vec b\cdot\vec a$. $\vec a\times\vec b\neq\vec b\times\vec a$.



第一种变换.

$\xi=\cos\theta\cdot x+\sin\theta\, y$

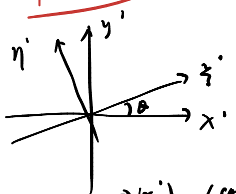
$\eta=-\sin\theta\, x+\cos\theta\, y$

在 $x', y'$ 系中测到的 $y$ 坐标长啥样?

S 系: $\begin{cases}t=t_0\\ x=0\\ y=l\end{cases}$

S'系.

$$\begin{cases}ct'=\gamma(ct-\beta\cdot\overset{\xi}{r_\parallel})\\ \xi'=\gamma(\xi-\beta\cdot ct)\\ \eta'=\eta\end{cases}$$



$$\begin{pmatrix}x'\\y'\end{pmatrix}=\begin{pmatrix}\cos&-\sin\\ \sin&\cos\end{pmatrix}\begin{pmatrix}\xi'\\ \eta'\end{pmatrix}$$

$\Rightarrow x'=\cos\theta\cdot\xi'-\sin\theta\,\eta'$

$y'=\sin\theta\,\xi'+\cos\theta\,\eta'$

$$\Rightarrow\begin{cases}ct'=\gamma(ct-\beta\cdot x\cos\theta-\beta y\sin\theta)\\ \xi'=\gamma\cdot(x\cos\theta+y\cdot\sin\theta-\beta\cdot ct)\\ \eta'=-\sin\theta\, x+\cos\theta\cdot y\end{cases}$$

$\Rightarrow ct'=$

$x'=\cos\theta\cdot\gamma(x\cos\theta+y\sin\theta-\beta ct)-\sin\theta\cdot(-x\sin\theta+y\cos\theta)$

$=(\gamma\cos^2\theta+\sin^2\theta)\,x+(\gamma-1)\sin\theta\cos\theta\cdot y-\gamma\beta\cos\theta\cdot ct$

$y'=\sin\theta\,\gamma(x\cdot\cos\theta+y\cdot\sin\theta-\beta ct)+\cos\theta\cdot(-\sin\theta\, x+\cos\theta\, y)$

$=(\gamma-1)\sin\theta\cos\theta\, x+(\gamma\sin^2\theta+\cos\theta)\,y-\sin\theta\cdot\gamma\beta\cdot ct$

$x=0$

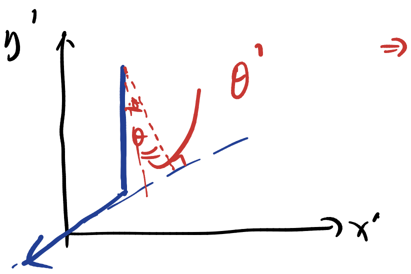
$ct'=\gamma(ct-\beta x\cos\theta-\beta y\cdot\sin\theta)=ct_0'=0$

$\Rightarrow\begin{cases}\boxed{ct=\beta\cdot y\cdot\sin\theta}\\ x=0\\ y=l\end{cases}$

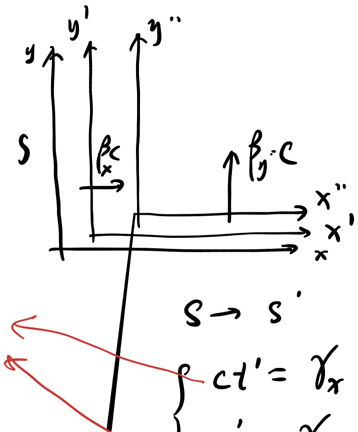
$$\begin{cases}x'=(\gamma-1)\sin\theta\cos\theta\cdot l-\gamma\beta^2\cdot\cos\theta\sin\theta\cdot l\\ y'=(\gamma\sin^2\theta+\cos\theta)\,l-\sin^2\theta\,\gamma\beta^2\, l\end{cases}$$

令 $\theta=0$. ✓

$\Rightarrow \tan\theta'=\dfrac{l\cdot\sin\theta\cdot\frac{1}{\gamma}}{l\cdot\cos\theta}$



先沿 x. 再沿 y.



问. y 轴在 S'' 系.

$S'\to S''$

$$\begin{cases}ct''=\gamma_y(ct'-\beta_y y')\\ x''=x'\\ y''=\gamma_y(y'-\beta_y ct')\end{cases}$$

$S\to S'$

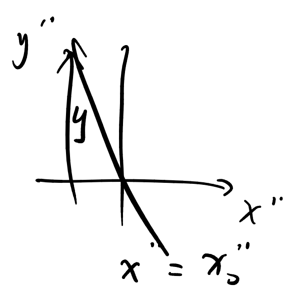
$$\begin{cases}ct'=\gamma_x(ct-\beta_x x)\\ x'=\gamma_x(x-\beta_x ct)\\ y'=y\end{cases}$$

$t''=t_0''=0$.   y 轴: $\begin{cases}x=0\\ y=l\\ t=t\end{cases}$

$$\begin{cases}ct''=\gamma_y(\gamma_x(ct-\beta_x x)-\beta_y y)\\ x''=\gamma_x(x-\beta_x ct)\\ y''=\gamma_y(y-\beta_y\gamma_x(ct-\beta_x x))\end{cases}\quad\boxed{S\to S''}$$

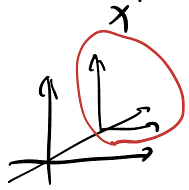
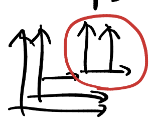
$t''=0\Rightarrow\gamma_x ct-\beta_y y=0\Rightarrow\begin{cases}ct=\dfrac{\beta_y}{\gamma_x}\cdot y\\ x=0\\ y=l\end{cases}$

$$\begin{cases}x''=\gamma_x\left(-\beta_x\dfrac{\beta_y}{\gamma_x}\cdot l\right)=-\beta_x\beta_y l\\ y''=\gamma_y\left(l-\beta_y\gamma_x\cdot\dfrac{\beta_y}{\gamma_x}\cdot l\right)=\gamma_y(1-\beta_y^2)\cdot l\\ y''=\dfrac{1}{\gamma_y}\cdot l\end{cases}$$



$\Rightarrow$ S''系测到的 y 轴的斜率:

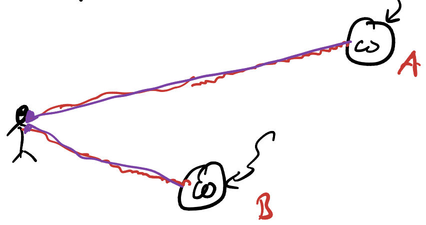
$$k''=\frac{y''}{x''}=\frac{-\gamma_x}{\gamma_y\gamma_x\beta_x\beta_y}=-\frac{1}{\gamma_y\beta_x\beta_y}$$

对比:  和  不一样. S系.

再用旋转变换 使 $\begin{cases}\beta_x=\beta\cdot\cos\theta\\ \beta_y=\dfrac{\beta\cdot\sin\theta}{\sqrt{1-\beta^2\cos^2\theta}}\end{cases}$

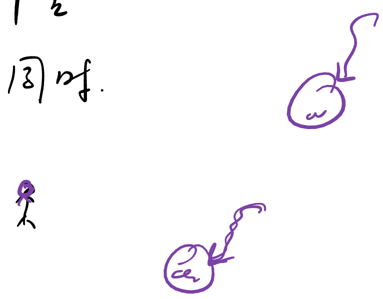
视觉形象.

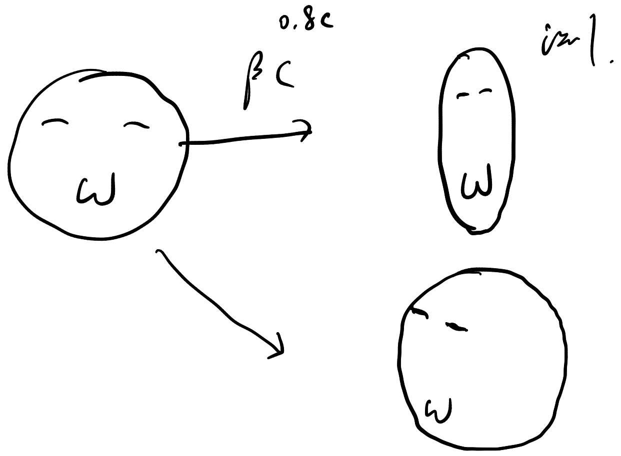
光同时入眼.




测量

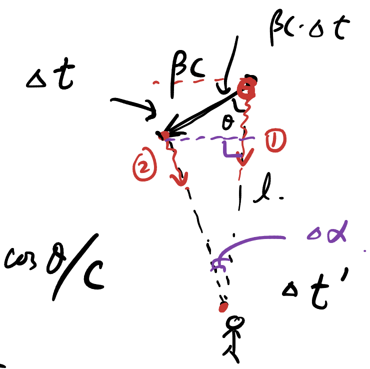
同时.






超光速景象.

地系.




$$\Rightarrow \Delta t' = \Delta t - \beta c\,\Delta t\cos\theta/c$$

$$\Delta x' = \beta c\,\Delta t\sin\theta$$

"看到速度": $\dfrac{\Delta x'}{\Delta t'} = \dfrac{\beta c\,\cancel{\Delta t}\sin\theta}{1-\beta c\frac{\cos\theta}{c}}\cdot c$

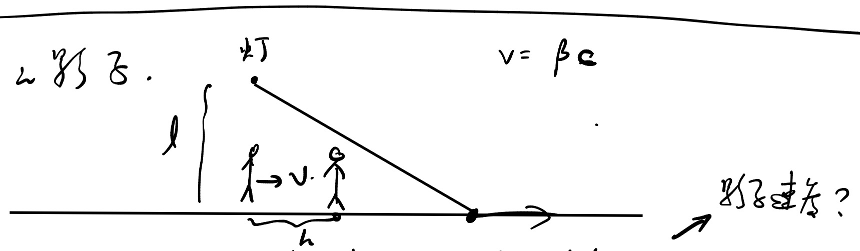
$$= \boxed{\frac{\beta\sin\theta}{1-\beta\cos\theta}}\cdot c$$ → 可能大于1.

$\beta = 0.8$

$\theta = 45^\circ$

$\Rightarrow \dfrac{0.8\frac{1}{\sqrt2}}{1-0.8\frac{1}{\sqrt2}} > 1.$

人与影子.

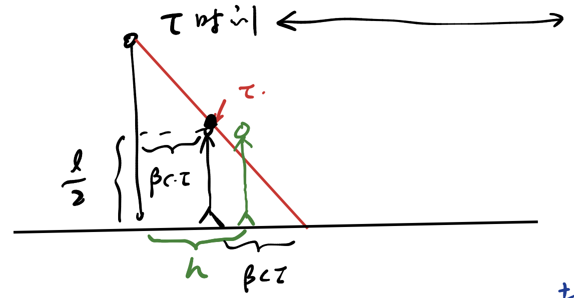



(1). 地系中, 人走到 h 处. 影子在何处 (地系同时)

(2) 人系中 灯 ← h处时. 影子在哪儿? 影子速度?

(1) 设 τ时刻. 有一光子. 从头顶掠过. 于t时刻 落地 x.

t时刻. 影子. x处.




影子位置:

$$\begin{cases} x = 2\beta c\tau & ① \\ t = \tau + \sqrt{(\frac{l}{2})^2+(\beta c\tau)^2}/c & ② \end{cases}$$

地系看到的影子世界线

人在h处时. ⇒ 地系时间 ⇒ $t = \dfrac{h}{\beta c}$.

$\Rightarrow$ ② $\Rightarrow \dfrac{h}{\beta c} = t = \tau + \sqrt{(\frac{l}{2})^2+(\beta c\tau)^2}/c$

$$\Rightarrow \frac{h^2}{(\beta c)^2} + \tau^2 - \frac{2h\tau}{\beta c} = \left(\frac{l}{2c}\right)^2 + (\beta\tau)^2.$$

$$\Rightarrow (1-\beta^2)\tau^2 - 2\frac{h}{\beta c}\tau + \left(\frac{h}{\beta c}\right)^2 - \left(\frac{l}{2c}\right)^2 = 0$$

$$\Rightarrow \tau = \frac{2\frac{h}{\beta c} \pm \sqrt{\left(\frac{2h}{\beta c}\right)^2 - 4(1-\beta^2)\left[\left(\frac{h}{\beta c}\right)^2-\left(\frac{l}{2c}\right)^2\right]}}{2}$$

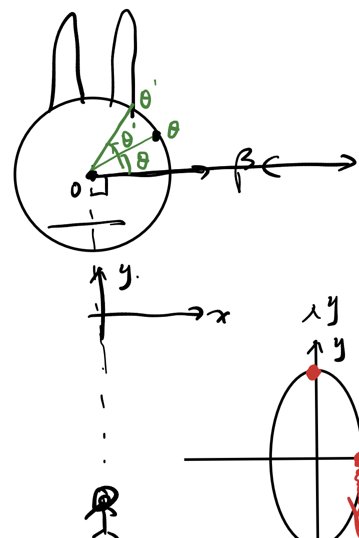
① ⇒ $x = 2\beta c\,\tau$. ← 人在h处时. 地系同时 ⇒ 影子.

⇒ 影子速度, $v = \dfrac{dx}{dt}$ ← $dx = 2\beta c\,d\tau$. $dt = \cdots d\tau$.

$v = \cdots \Rightarrow v > c$.

例: 兔子.

人在运动. 以βc 向垂直地看. 问看到的形状?




测量

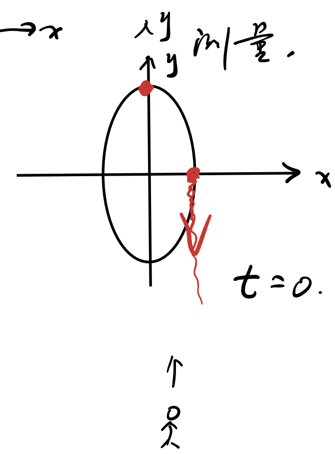
兔本征系中. 点的标记.

$l_{x0} = R\cdot\cos\theta.$

$l_{y0} = R\cdot\sin\theta.$

水平宽度: $l_x = \frac{1}{\gamma}\cdot R\cdot\cos\theta.$

竖直高度: $l_y = R\cdot\sin\theta.$




地系: (S系).

$$\begin{cases} x = \frac{1}{\gamma}\cdot R\cos\theta + \beta c t \\ y = R\cdot\sin\theta. \end{cases}$$

地系中. θ点发的光要多跑. $R\cdot\sin\theta$ 的距离.

⇒ 先发 $\dfrac{R\cdot\sin\theta}{c}$

⇒ $t_{发} = -\dfrac{R\cdot\sin\theta}{c}$

$\Rightarrow x_{发} = x_{发} = \frac{1}{\gamma}R\cdot\cos\theta - \beta R\cdot\sin\theta$

$x_{快} = x_{发} = \frac{1}{\gamma}R\cdot\cos\theta - R\cdot\sin\theta$

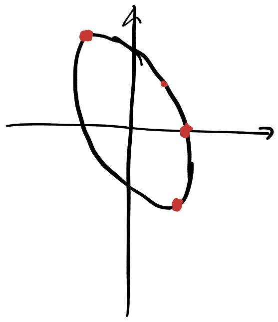
$= R\left(\underbrace{\sqrt{1-\beta^2}}_{\cos u}\cos\theta - \underbrace{\beta}_{\sin u}\sin\theta\right)$

$= R\cdot(\cos u\cos\theta - \sin u\sin\theta)$

$= R\cdot\cos(\theta+u)$.

兔子没瘦　　　　转 $u$.

同时刻的发光点位置 ⇒



看.兔子.烤熟

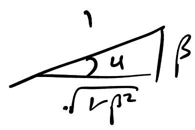
⇓ 本征系中.均匀.匀速地被烤熟.

问.看到的兔子是怎么熟的?

等熟线

本征系中: $\tau$ ← 可以表示成熟程度.

地系: $y = -ct$.



$r\cdot\sin\theta + \gamma(c\tau + \beta\cdot r\cdot\cos\theta) = 0$.

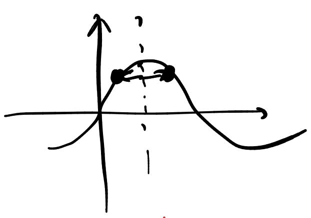
$\Rightarrow \tau = -\frac{1}{\gamma}\cdot r\cdot\sin\theta - \beta r\cdot\cos\theta = -r\cdot(\sqrt{1-\beta^2}\cdot\sin\theta + \beta\cdot\cos\theta)$

$= -r\cdot(\sin\theta\cos u + \cos\theta\cdot\sin u)$

$= -r\cdot\sin(\theta+u)$.

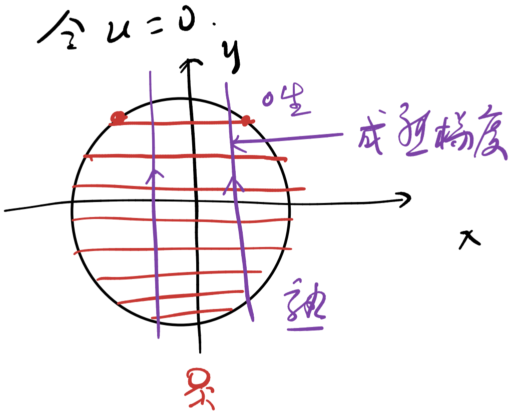
等 $\tau$ 线即是等熟线.

$\theta + u = \frac{\pi}{2} \pm \varphi$

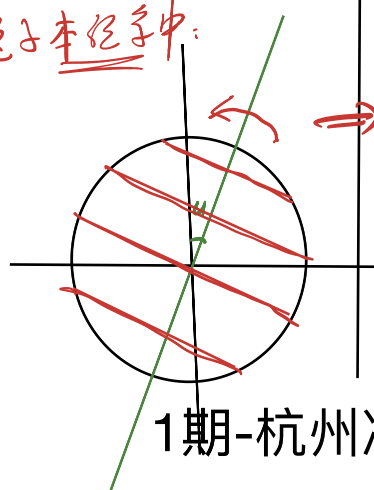


$\Rightarrow \theta = \frac{\pi}{2} - u \pm \varphi$.

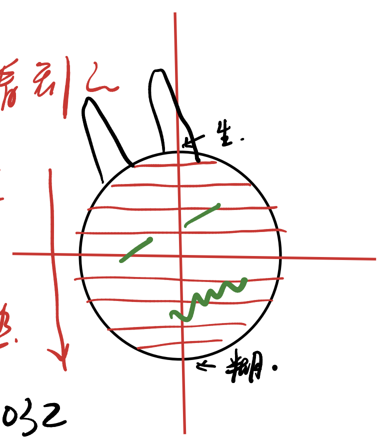
令 $u = 0$.



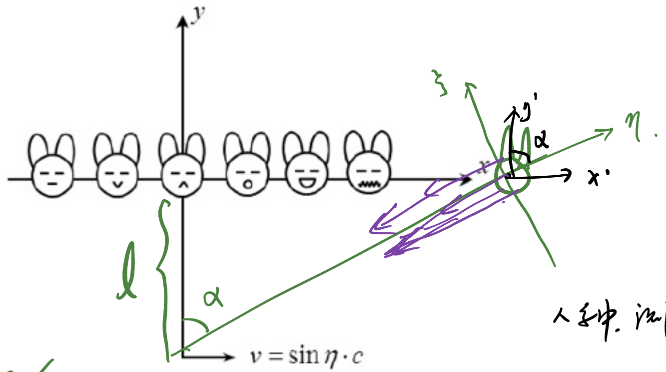

兔子本征系中:



⟹ 人看到的




如图在 $x$ 轴上有很长一列兔子。在距离 $x$ 轴 $l$（$l >>$ 兔子间隔）的地方有一个人以速度 $v = \sin\eta\cdot c$ 向 $x$ 轴正方向运动。




人系:

$\Rightarrow \begin{cases} \eta = x'\sin\alpha + y'\cos\alpha \\ \xi = -x'\cos\alpha + y'\sin\alpha \end{cases}$

$\eta = 0$ 时, $t' = 0$

$\eta \Rightarrow t'_{发} = -\frac{\eta}{c}$.

$\Rightarrow \eta = -ct'_{发}$.

人系中测得的兔子: $\begin{cases} ct' = ct' \\ y' = r\cdot\sin\theta \\ x' = \frac{1}{\gamma}\cdot r\cdot\cos\theta - \beta ct' \end{cases}$

（1）人向 $y$ 轴正方向看去，由于人不能区分兔子的不同部分距离他有多远，只能看清楚光从哪里射过来，于是只能脑补兔子的形象。此时他会人为兔子不是变瘦了，而是转了一个角度。求出这个角度 $\upsilon_0$。⟹ $\eta$.

（2）人向 $y$ 轴偏向 $x$ 轴方向向 $\alpha$ 角方向看过去，由于相同的原因，人还是只能脑补兔子形象，于是他发现兔子仍然是转了一个角度 $\upsilon$，同时被横向放大了一个系数 $\lambda$。求出 $\upsilon, \lambda$。这一问中取 $\eta = \pi/3$，$\alpha$ 分别取 $\alpha_1 = 0.4$，$\alpha_2 = 0.8$ 计算出对应的 $\upsilon_1, \lambda_1, \upsilon_2, \lambda_2$。

$\Rightarrow x' = \frac{1}{\gamma}\cdot r\cdot\cos\theta - \beta\cdot\eta$.　　$y' = r\cdot\sin\theta$

$= \underbrace{\frac{1}{\gamma}}_{\cos u}\cdot r\cdot\cos\theta - \underbrace{\beta}_{\sin u}\cdot(x'\cdot\sin\alpha + y'\cdot\cos\alpha)$.

$\Rightarrow x' = \frac{r\cdot\cos u\cdot\cos\theta - r\cdot\sin\theta\cdot\sin u\cdot\cos\alpha}{1+\beta\cdot\sin\alpha}$.

$\Rightarrow \xi = -\frac{r\cdot\cos u\cdot\cos\theta - r\cdot\sin\theta\cdot\sin u\cdot\cos\alpha}{1+\beta\cdot\sin\alpha}\cos\alpha + r\cdot\sin\theta\cdot\sin\alpha$.

$= -\frac{\cos u\cdot\cos\theta - \sin u\cos\alpha\sin\theta - \sin\alpha\sin\theta - \sin u\sin^2\alpha\cdot\sin\theta}{1+\sin u\cdot\sin\alpha}\, r$

$= -\frac{a\cos\theta - b\sin\theta}{1+\sin u\cdot\sin\alpha}\cdot r$

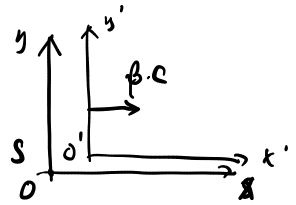
$= \lambda\cdot r\cdot\cos(\theta - u')$.　转角 $u'$.

$\lambda = \frac{\sqrt{a^2+b^2}}{1+\sin u\sin\alpha}$　放大率 $= \lambda$.

1期-杭州决-032

4D 矢量. 洛伦兹变换.

S: S'



$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

记 $x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$ $\mu = 0, 1, 2, 3.$

$x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z.$

某个. $A^\mu$ ⇒. 横方在换变时不变: ⇒ 四矢量.

某个: $B$ ⇒ 换系时 不变. ⇒ 标量.

$m_0$, $\tau$, $q$. 固有寿命. 固有能量.

相位: 标量: (波峰 ← 数峰).

标量·矢量 ⇒ 矢量. ⇒ 可构造一系列矢量.

$\dfrac{d\,矢量}{d\,标量}$ ⇒ 矢量. ⇒

$(ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2$

⇒ $x^\mu_{(\tau)}$ 矢量 ⇒

$\dfrac{dx^\mu}{d\tau} \Rightarrow u^\mu$ ← 四速度 矢量.

$\dfrac{d^2x^\mu}{d\tau^2} \Rightarrow a^\mu$ ← 四加速度. 矢量.

$m_0 \cdot u^\mu \Rightarrow p^\mu$ ← 四动量. 矢量.

$\dfrac{dp^\mu}{d\tau} \Rightarrow F^\mu$ ← 四维力 矢量.

S系: $v_x = \dfrac{dx}{dt}$

---

固有电荷密度: $\rho_0$.

$\rho_0 \cdot u^\mu = j^\mu$ 四电流密度 ⇒ 矢量.

波: $u = u_0 \cdot \cos(\omega t - k_x \cdot x - k_y \cdot y - k_z \cdot z)$.

$\phi = k^\mu x_\mu$ 标量.

$k^\mu = \begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix}$.

$\phi(x, y, z, ct) = \phi(x', y', z', ct')$

$= (\omega' t' - k_x' x' - k_y' y' - k_z' z')$

$= (\omega' \gamma (t - \beta \frac{x}{c}) - k_x' \gamma (x - \beta c t) - k_y' y - k_z' z)$

$= [\gamma(\omega' + \beta c \cdot k_x') t - \gamma(k_x' + \beta \frac{\omega'}{c}) x - k_y' y - k_z' z]$

$\phi(x, y, z, ct) = \phi(x', y', z', ct')$

$= [\gamma(\omega' + \beta c \cdot k_x') t - \gamma(k_x' + \beta \frac{\omega'}{c}) x - k_y' y - k_z' z]$

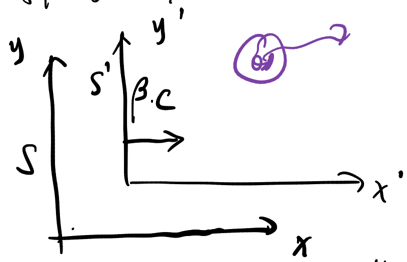
$= (\omega t - k_x \cdot x - k_y \cdot y - k_z \cdot z)$

$$\Rightarrow \begin{cases} \omega/c = \gamma(\omega'/c + \beta k_x') \\ k_x = \gamma(k_x' + \beta \cdot \omega'/c) \\ k_y' = k_y \\ k_z' = k_z \end{cases} \Leftrightarrow A'^\mu = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} A^\mu$$

---

粒子速度变换.

$S: x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$, $S': \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$



$\dfrac{dt}{d\tau} = \gamma_1$.

S系中: $u^\mu = \dfrac{dx^\mu}{d\tau} = \dfrac{dx^\mu}{dt} \dfrac{dt}{d\tau} = \gamma_1 \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix}$.

S'系中: $u'^\mu = \dfrac{dx'^\mu}{d\tau} = \dfrac{dx'^\mu}{dt'} \dfrac{dt'}{d\tau} = \gamma_2 \begin{pmatrix} c \\ v_x' \\ v_y' \\ v_z' \end{pmatrix}$

$\dfrac{dt'}{d\tau} = \gamma_2$

$$\Rightarrow u'^\mu = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} u^\mu$$

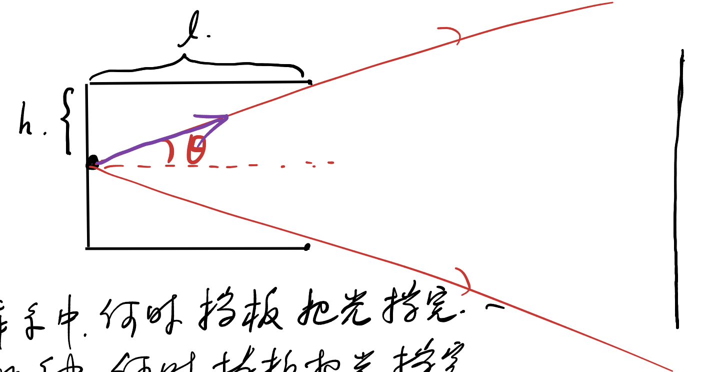
$$\Rightarrow \begin{cases} \gamma_2 c = \gamma_1 (\gamma c - \beta\gamma \cdot v_x) & ① \\ \gamma_2 v_x' = \gamma_1 (-\beta\gamma \cdot c + \gamma \cdot v_x) & ② \\ \gamma_2 v_y' = \gamma_1 v_y & ③ \\ \gamma_2 v_z' = \gamma_1 v_z & ④ \end{cases}$$

$\dfrac{②}{①} \Rightarrow v_x' = \dfrac{v_x - \beta c}{1 - \beta v_x/c}$ ← 速度变换.

$\dfrac{③}{①} \Rightarrow v_y' = \dfrac{v_y}{\gamma(1 - \beta v_x/c)}$

$\dfrac{④}{①} \Rightarrow v_z' = \dfrac{v_z}{\gamma(1 - \beta v_x/c)}$

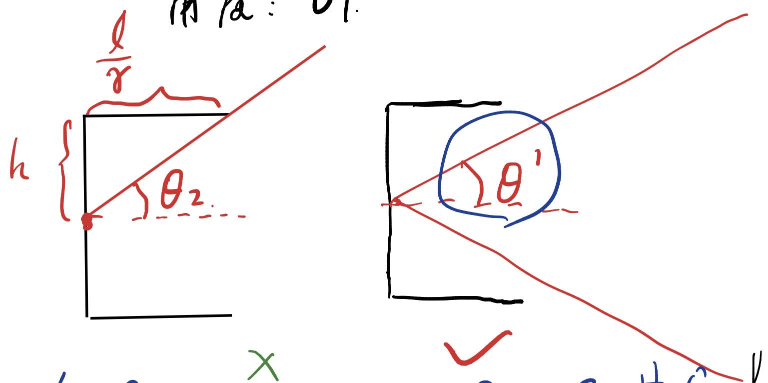
例:



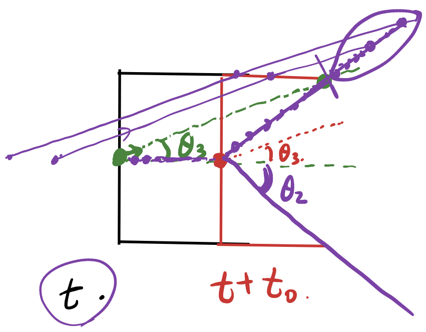
① 车系中,何时挡板把光挡完.
② 地系中,何时挡板把光挡完.

车系中: $\tan\theta = \frac{h}{l}$ ✓

地系: 在车系中,以$\theta$发出光,地系的角度: $\theta_1$.



$\theta' = ?$ A: $\theta_1$. B = $\theta_2$. C. 其它.



相对论四维动量守恒定理 快度

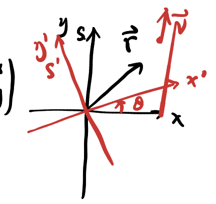
$$m_0 U^\mu = P^\mu$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \operatorname{ch}\theta & -\operatorname{sh}\theta \\ \operatorname{sh}\theta & \operatorname{ch}\theta \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

欧氏:

S: $\vec{r} = (x, y) \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$

S': $\vec{r}' = (x', y') \Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix}$



$\Rightarrow x' = \cos\theta \cdot x + \sin\theta \cdot y$

$y' = -\sin\theta \cdot x + \cos\theta \cdot y$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$(\vec{v}') = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} (\vec{v})$$

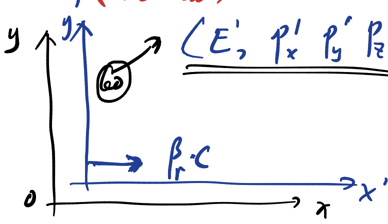
$x^\mu = (ct, x, y, z)$

$S \to S'$:
$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$P^\mu = \begin{pmatrix} m_0\gamma c \\ m_0\gamma U_x \\ m_0\gamma U_y \\ m_0\gamma U_z \end{pmatrix} = \begin{pmatrix} E/c \\ P_x \\ P_y \\ P_z \end{pmatrix}$$

$\gamma m_0 c^2$

$\gamma m_0 v_x = P_x$



$(E', P_x', P_y', P_z')$

$S \to S'$

$$\Rightarrow \begin{pmatrix} E'/c \\ P_x' \\ P_y' \\ P_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & & 0 \\ -\beta\gamma & \gamma & & \\ & 0 & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} E/c \\ P_x \\ P_y \\ P_z \end{pmatrix}$$

$$(\ )^2 = E^2/c^2 - (P_x^2 + P_y^2 + P_z^2) = E_0^2/c^2 \quad \text{标量.}$$

物体组: 地系: $E_总$, $P_{总x}$, $P_{总y}$, $P_{总z}$.

总可以找一个系 看到 物体组总动量为0.
⇒ 零动量坐标系. (质心系)

$E_{总0}$

~~多物体~~

S. 地系中: $E_总$, $P_{总x}$, $P_{总y}$, $P_{总z}$.

质心系: S'

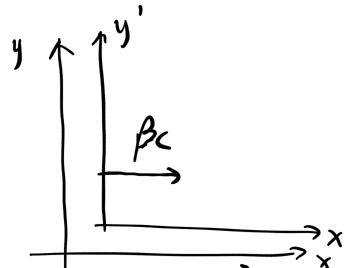
$E_{总0}$, $P = 0$

$$\begin{pmatrix} E_总/c \\ P_{总x} \\ P_{总y} \\ P_{总z} \end{pmatrix} \qquad \begin{pmatrix} E_{总0}/c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} E_{总0}/c \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & \\ & 0 & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} E_总/c \\ P_{x总} \\ P_{y总} \\ P_{z总} \end{pmatrix}$$

$$0 = -\gamma\beta \cdot E_总/c + \gamma \cdot P_{总x}$$

$$\Rightarrow \beta = \frac{P_{总x}}{E_总/c}$$



$$\Rightarrow (E_{总0}/c)^2 = (E_总/c)^2 - P_总^2$$

$$\Rightarrow E_总^2 = P_总^2 c^2 + E_{总0}^2$$

总 质心 相对

$\frac{1}{2} M V_c^2$ $\frac{(M \cdot V_c)^2}{2M}$

⇒ 低速情况下 相似于: $E = \frac{P^2}{2m_0} + E_r$.

令 $\frac{v}{c} \ll 1$

总 质心 相对

例：$m_{10}$ $\xrightarrow{}$ ($\beta_1 c$, $\gamma_1$)　$m_{20}$ o　$\Rightarrow$　$m_{30}$ $\xrightarrow{v_3}$　$m_{40}$ $\xrightarrow{v_4}$　$x$

问：$\beta_1 c$ 至少为多少才有可能碰撞。

碰前：总能：

$$\begin{cases} E_总 = \gamma_1 m_{10} c^2 + m_{20} c^2 \\ P_{总x} = \gamma_1 m_{10} \beta_1 c + 0 \end{cases}$$

碰后：$E_总$，$P_{总x}$

设质心系中看到的能量为 $E_{总0}$

四维不变量：$E_总^2 - P_{总x}^2 c^2 = E_{总0}^2 = (m_{30}c^2 + m_{40}c^2)^2$

质心系：$m_{30}$ O　　$m_{40}$ O　$p'$

$$E_{总0} = \sqrt{m_{30}^2 c^4 + p'^2 c^2} + \sqrt{m_{40}^2 c^4 + p'^2 c^2}$$

$$(\gamma_1 m_{10} c^2 + m_{20} c^2)^2 - (\gamma_1 m_{10} \beta_1 c)^2$$

$$= \gamma_1^2 m_{10}^2 c^4 + m_{20}^2 c^4 + 2\gamma_1 m_{10} c^4 m_{20} - \gamma_1^2 m_{10}^2 \beta_1^2 c^2$$

$$= m_{10}^2 c^4 + m_{20} c^4 + 2\gamma_1 m_{10} m_{20} c^4$$

$\Rightarrow$ 质心系中 $p' = 0$ $\Rightarrow$ $\checkmark$

$\Rightarrow$ $\checkmark$.