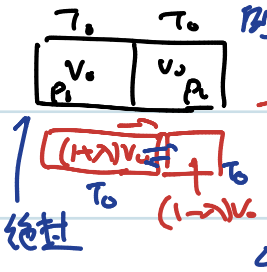


例108. 左右导温 左右绝热

$\Downarrow \Delta S=0$

$\Delta U=0$. $T_L=T_R=T_0$ $\frac{C_V+R}{C_V}$

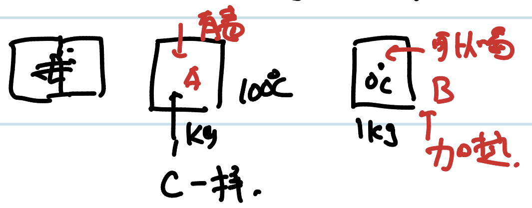
$\Delta S>0$.

$$\Delta S=nC_V\ln\frac{V^{\gamma-1}T}{V^{\gamma-1}T}+\square$$

$$=nC_V(\gamma-1)\ln\frac{V_0(1+\lambda)}{V_0}+nC_V(\gamma-1)\ln(1-\lambda)$$

$$=nR\ln\left[1-\lambda^2\right]$$

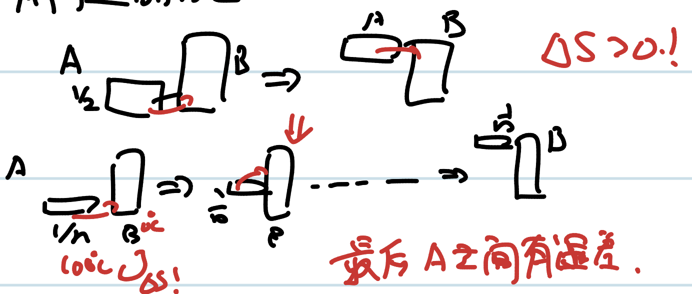
例107. 与外界绝热，不可以导热而



C一样。

热传导方式

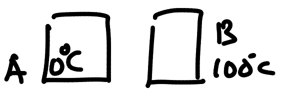
用导热的方法 $A\to B$. 问 $T_{B\max}>70°C$?

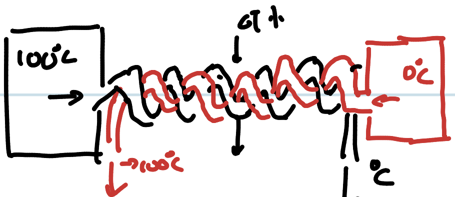


最后 A之间有温差.

理论最大值? 末态. A的温度差为0

则整体 $\Delta S_{min}=0$

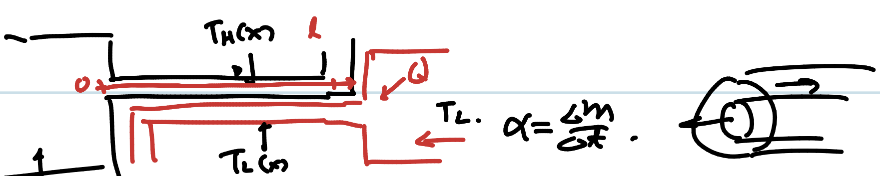




在 dt 内. 有 dm 的物体交换 在 T 到 $T+dT$ 之间.

交换 $C\,dT\,m$ 热量.

$$\Delta S_{产}=\frac{\delta Q}{T}-\frac{\delta Q}{T+dT}=\frac{\delta Q\,dT}{T^2}\to 0$$

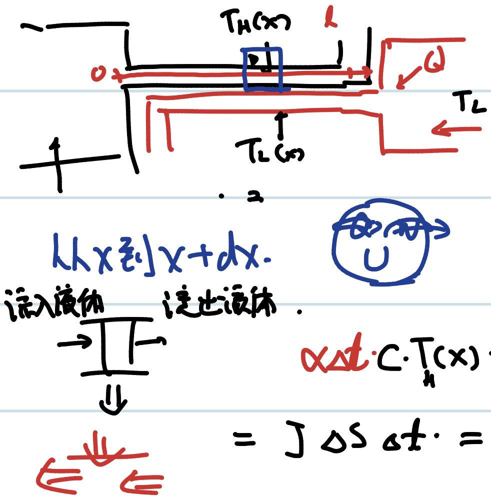


已是 100°C, T_H. 流速 $\alpha=\frac{dm}{dt}$, 每单位长度有 $\Delta S=2\pi r\cdot dl$ 面积

导热. 热流 $J\,dt\cdot\Delta S=\delta Q$, $J=k\Delta T$.

求 $T_H(x)$. $T_L(x)$ 分布. 单位时间内熵产生率.

稳定.



从 x 到 $x+dx$.

流入液体 流出液体.

$$\alpha\,dt\cdot C\cdot T_H(x)-\alpha\,dt\cdot C\,T_H(x+dx)$$

$$=J\,\Delta S\,dt=k(T_H-T_L)\cdot\Delta x\cdot 2\pi r\cdot dt$$

$$\begin{cases}-\alpha C\frac{dT_H}{dx}=k\cdot 2\pi r\cdot(T_H-T_L). & ①\\ -\alpha C\frac{dT_L}{dx}=k\cdot 2\pi r\,(T_H-T_L) & ②\end{cases}$$

① − ②: $-\alpha C\frac{d(T_H-T_L)}{dx}=0$

$T_H-T_L=\Delta T$. 常数.

① + ②: $-\alpha C\frac{d(T_H+T_L)}{dx}=4\pi rk\cdot\Delta T$.

$$T_H+T_L=-\frac{4\pi rk\Delta T}{\alpha C}x+T_0.$$

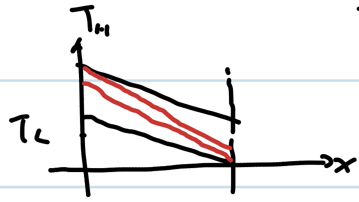
边界: $x=0$. $T_H(0)=T_{H,0}$, $T_L(0)=T_{H,0}-\Delta T$. $x=L$. $T_H(l)=T_{L0}+\Delta T$. $T_L(l)=T_{L0}$

$$-\frac{4\pi r}{\alpha C}k\Delta T\cdot l=-2T_{H,0}+2\Delta T+2T_{L0}$$

$$\Delta T=\frac{T_{H,0}-T_{L,0}}{-1+\frac{2\pi r}{\alpha C}kl}$$; 可令 $\Delta T\to 0$.

$l\to\infty$

$$\frac{2\pi rkl}{\alpha C}>>1.$$

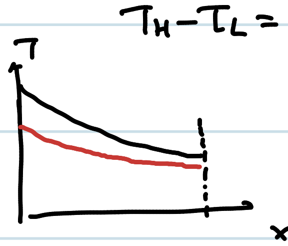


设:
$$\begin{cases}-\alpha C_1\frac{dT_H}{dx}=k\cdot 2\pi r\cdot(T_H-T_L). & ①\\ -\alpha C_2\frac{dT_L}{dx}=k\cdot 2\pi r\,(T_H-T_L) & ②\end{cases}$$

代入

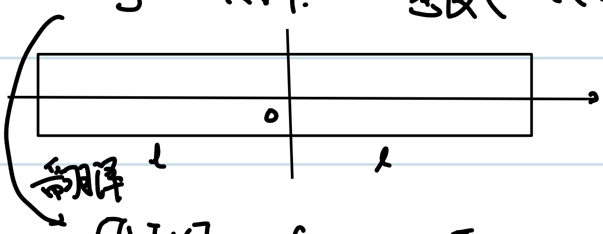
$$\frac{①}{\alpha_1 C}-\frac{②}{\alpha_2 C}\quad \frac{d(T_H-T_L)}{dx}=-\left(\frac{1}{\alpha_1 C}-\frac{1}{\alpha_2 C}\right)k2\pi r(T_H-T_L)$$

$\alpha_1\ne\alpha_2$. $T_H-T_L=T_0+A\cdot e^{-\left(\frac{1}{\alpha_1 C}-\frac{1}{\alpha_2 C}\right)k2\pi r\cdot x}$.



例110 $t<0$, $T(x)=T_0\left(2-\frac{|x|}{l}\right)$. 面积$S$. 热导率 K.

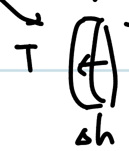
$\vec{J}=-K\nabla T$. 密度ρ. 比热c. $t\to 0^+$



① $J(x)=?$

② $t\to 0^+$ $\frac{\partial T}{\partial t}(x)=?$

③ $\frac{\partial s}{\partial t}=?$



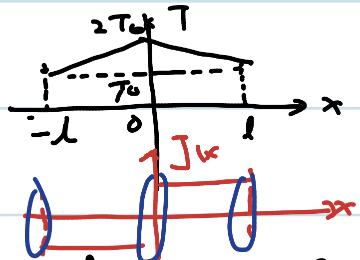
$\frac{\Delta Q}{\Delta t}=\Delta S\cdot\frac{\Delta T}{\Delta h}\cdot K \Big/ T$.

$s=\frac{\Delta S}{\Delta V}$ 熵密度.

④ $\omega(x)$. 单位体积,单位时间产生熵.

⑤ $J_s(x)$. 熵流.

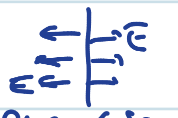
(1). $J(x)=\begin{cases}+\frac{T_0}{l}K\\ -\frac{T_0}{l}K\end{cases}$



(2) $\Rightarrow\square\Rightarrow$ $[J(x)-J(x+\Delta x)]S\cdot\Delta t=\rho c\cdot S\cdot\Delta x\cdot\Delta T$.

x $x+\Delta x$

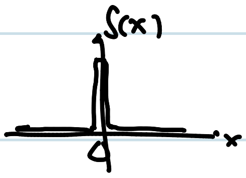
$-\frac{\partial J}{\partial x}=\rho c\cdot\frac{\partial T}{\partial t}$. $\to E$ $\to\rho$.



$\rho(x)=\sigma\cdot\delta(x)$

$\frac{\partial T}{\partial t}=0$ $x\neq 0$, $x\neq\pm l$处.

$\delta(x)=\begin{cases}0 & x\neq 0\\ +\infty & x=0\end{cases}$



$\int_{-\infty}^{\infty}\delta(x)dx=1$

$f(x)=\begin{cases}\frac{1}{2a}. & |x|<a\\ 0 & |x|>a\end{cases}$

$\boxed{\lim_{a\to 0}}$ $f(x)=\delta(x)$

$\frac{\partial T}{\partial t}=-A\delta(0)+B\delta(l)+C\delta(-l)$.

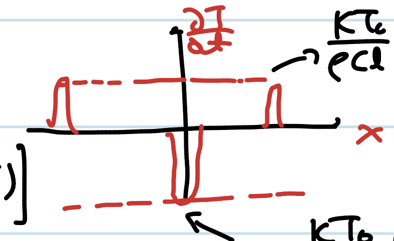
$\int_{-\varepsilon}^{+\varepsilon}\rho c\frac{\partial T}{\partial t}dx=-\int_{-\varepsilon}^{\varepsilon}\frac{\partial J_x}{\partial x}dx$

高斯定理

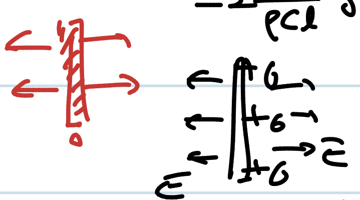
$=-[J_x(0^+)-J_x(0^-)]$

$=-2\cdot K\frac{T_0}{l}$.

$\frac{\partial T}{\partial t}=-\frac{2KT_0}{\rho cl}\delta(x)$



$\frac{\partial T}{\partial t}$ $\frac{KT_0}{\rho cl}\delta(l)$ x $-2\frac{KT_0}{\rho cl}\delta(x)$



$\rho(x)=?$

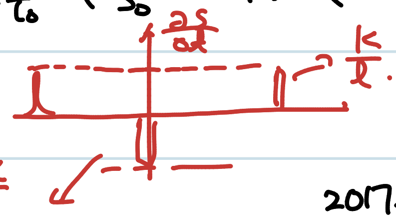
(3) 默认. P不变 $c\to C_p$.

$dQ=C_p\,dT=dU+pdV$ $S(T)$ p不变.

$S(T)-S(T_0)=\int_{T_0}^{T}\frac{dQ}{T}=C_p\ln\frac{T}{T_0}$

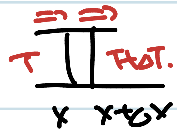
$s=\frac{S}{V}=\rho c\cdot\ln\frac{T}{T_0}+s_0$ ← 熵密度.

$\frac{\partial s}{\partial t}=\frac{\rho c}{T}\cdot\frac{\partial T}{\partial t}$



$\frac{\partial s}{\partial t}$ $\frac{k}{l}$. $=\frac{k}{l}$

(4) 单位体积产生熵.

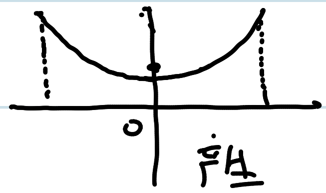


Δt内 $\Delta Q=K\cdot\frac{\partial T}{\partial x}\cdot S\cdot\Delta t$.

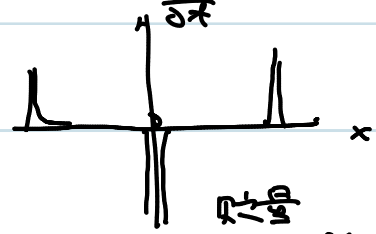
产生 $\Delta S=\frac{\Delta Q}{T}-\frac{\Delta Q}{T+\Delta T}=\frac{\Delta T\cdot\Delta Q}{T^2}$.

$\frac{\Delta S}{\Delta t\Delta V}=\frac{1}{T^2}\cdot\frac{\Delta T\quad K\cdot\frac{\partial T}{\partial x}\cancel{S}\cancel{\Delta t}}{\Delta x\cdot\cancel{S}\quad\cancel{\Delta t}}$

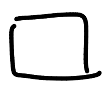
$=\frac{1}{T^2}K\cdot\left(\frac{\partial T}{\partial x}\right)^2$



ω x. 0 产生

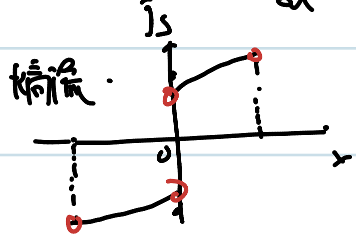


$\frac{\partial s}{\partial t}$ x 贮量



$\frac{\partial s}{\partial t}=\omega+j_{流入}-j_{流出}$. $\int\left(\frac{\partial s}{\partial t}-\omega\right)dV=-\oint\vec{j}\cdot d\vec{S}$

$\frac{\partial s}{\partial t}=\omega-\frac{dj_s}{dx}$ → 熵流. $\frac{dj_s}{dx}=\omega-\frac{\partial s}{\partial t}$.

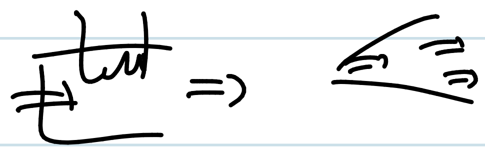


熵流.

$j_x=\begin{cases}\frac{1}{T}\cdot K\left(\frac{\partial T}{\partial x}\right)^2+A & x\in(0,l)\\ \frac{1}{T}K\left(\frac{\partial T}{\partial x}\right)^2+B & x\in(-l,0)\end{cases}$

$x\to 0^+$, $x\to 0^-$ $A-B=\frac{K}{l}$.

j_s $\frac{\partial s}{\partial t}<0$



5.3 相变.

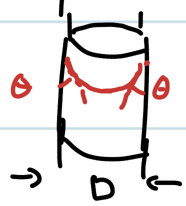
相变潜热 λ.

定P. 定T. 从一个相→另一个相. ΔQ.

$\Delta U=-W_{外}+Q$ $Q=\Delta U+p\Delta V$

焓 令$H=U+pV$. P不变. $\Delta H=\Delta Q$.

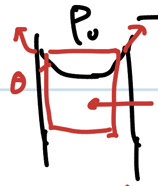
例111 对不同D 内水的相变温度不同.



$T=T_0+f(D)$ 求 $f(0)=?$

$D\to\infty$

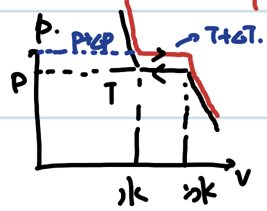
θ, σ, λ. $\rho_{水}$. $\rho_{气}$



$\Sigma F=0$

$P_0 - \delta P \Rightarrow 2\pi r \frac{P\sigma}{r}\cos\theta = \Delta P\cdot\pi\frac{D^2}{4}$

$\delta P = \frac{4\sigma}{D}\cos\theta$.



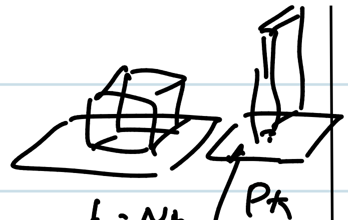
$\eta_{max}=\frac{W}{Q}=\frac{\delta T}{T+\delta T}=\frac{\delta T}{T}$

$\frac{\delta T}{T}=\frac{\delta P\cdot(V_2-V_w)}{\lambda}$

λ常数. V_2. V_w 常数

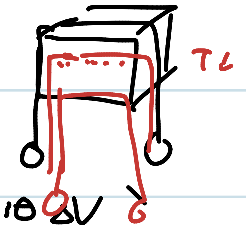
$\Delta T=\frac{T}{\lambda}\cdot\delta P\cdot(V_2-V_w)\simeq\frac{T_0}{\lambda}\frac{4\sigma}{D}\cos\theta\,\delta V$

$\ln\frac{T+\delta T}{T}=\frac{\delta P\cdot\delta V}{\lambda}$.



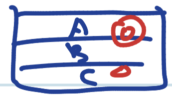
$f=N_f$　$P_大$

T↓　液态

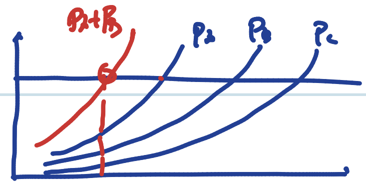


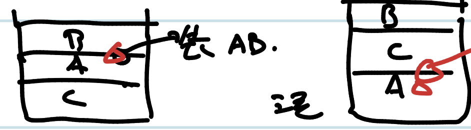
T↓

三层:

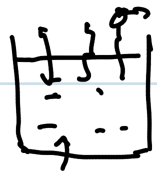


先AD界面





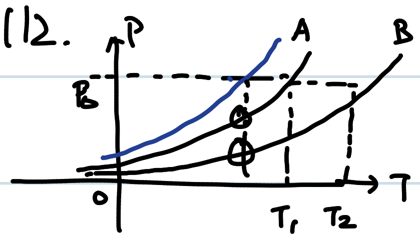
三层　先AD.　二层　先AC.



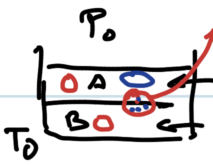
①共沸,　P_{H_2O}, $P_{苯}$: P---　一定.

可求

例112.



$P_A(T)$ $P_B(T)$ 给表.

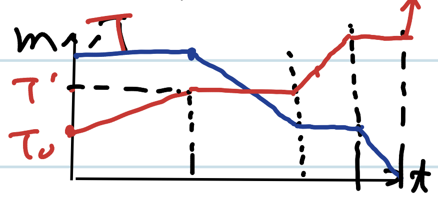


$P_{AS}+P_{BS}$.

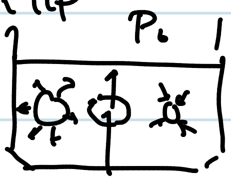
m_A　λ_A　C_A

m_B.　λ_B. C_B

初T_0. 以恒定功率加热液体　求 $m_液-t$.



共沸



$P_S>P_0$　P_S　$P_S<P_0$

平衡　$P_S=P_0+\frac{2\sigma}{r}$　　$P_S(T_0)=P_0$时 不沸

一般 r_0　$P_S(T_0+\delta T)=P_0+\frac{2\sigma}{r_0}$　沸　迅速长大.

$r\uparrow$　P_S比 $P_0+\frac{2\sigma}{r}$大

T'　$P_A(T')+P_B(T')=P_0$. 界面沸腾. 绝热.

上升. 来不及 变成 B.

A.B共同沸腾.

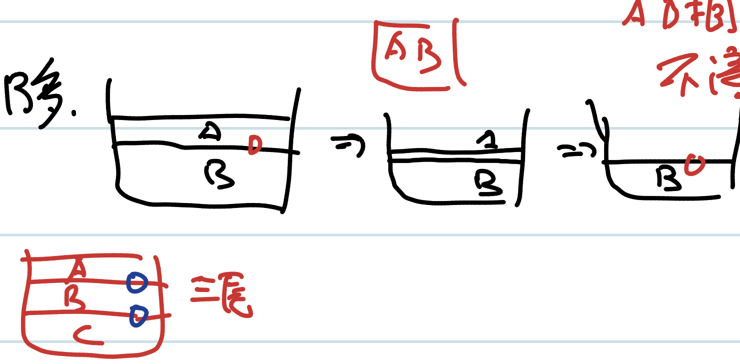
$\Delta n_A:\Delta n_B=P_A(T'):P_B(T')$　固定比例.

① $\frac{P_A(T')}{P_B(T')}\frac{\mu_A}{\mu_B}>\frac{m_A}{m_B}$.　B多出

② $<$　A多出.

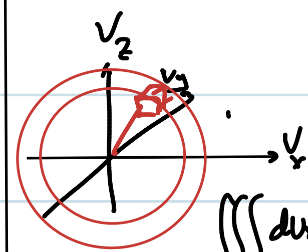
AB相溶　不溶 → 搅拌一样.

B多.



三层

5.5 Maxwell速度分布率



$v_x\sim v_x+dv_x$, $v_y\sim v_y+dv_y$

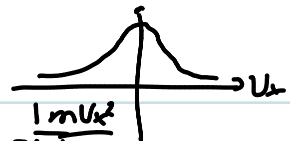
$v_z\sim v_z+dv_z$.

$dp=f(v_x,v_y,v_z)\,dv_xdv_ydv_z$

$\iiint dv_xdv_ydv_z\cdot f(v_x,v_y,v_z)=1$.

热平衡.

$f(v_x,v_y,v_z)=\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{mv^2}{2kT}}$



$dp=f(v_x)dv_x$;　$f(v_x)=\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}e^{-\frac{1}{2}\frac{mv_x^2}{kT}}$

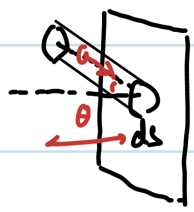
$|\vec{v}|$. 在v到v+dv之间.

$4\pi v^2dv\cdot f(v_x,v_y,v_z)=g(v)\,dv$ → p

$g(v)=4\pi v^2\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{1}{2}mv^2/kT}$

分布 → 热力学量（对于粒子体系可算）

例113. 状态方程.　p.g.e.　N.R 非相对论



取. v~v+dv. θ. dΩ. dS. dt.

$dn=n_0\cdot v\cos\theta\,dt\cdot dS\cdot\frac{d\Omega}{4\pi}\cdot f(v)dv$

数密度

$P_压.=\frac{dF}{dSdt}=\frac{\int 2mv\cos\theta\,dn}{dS\cdot dt}$.

$J_{粒子流}=\frac{\int 1\cdot dn}{dS\,dt}$.

$\varepsilon_{能流}=\frac{\int\frac{1}{2}mv^2dn}{dS\,dt}$.　$\int_{-1}^{1}x^2dx$

$P_压.=\int_0^{\pi}\frac{2mv\cos\theta\cdot n_0 v\cos\theta\cdot\cancel{2\pi}\sin\theta d\theta}{\cancel{4\pi}\ 2}f(v)dv$

$=\frac{2}{3}n_0\int_0^{\infty}\frac{1}{2}mv^2f(v)dv=\frac{2}{3}n_0\left\langle\frac{1}{2}mv^2\right\rangle=n_0kT$

$$\int_0^\infty u^n e^{-\alpha u^2} du = G_n(\alpha)$$

$$\frac{dG_n(\alpha)}{d\alpha} = -G_{n+2}(\alpha).$$

$$G_1(\alpha) = \int_0^\infty \frac{1}{\alpha} e^{-\alpha u^2} d\frac{u^2\alpha}{2} = \frac{1}{2\alpha}.$$

$$[2G_0(\alpha)]^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx dy\, e^{-\alpha(x^2+y^2)}$$

$$= \int_0^\infty 2\pi r dr\, e^{-\alpha r^2} = 2\pi \cdot G_1 = 2\pi \cdot \frac{\pi}{2\alpha}$$

$$G_0(\alpha) = \frac{\sqrt{\pi}}{2\sqrt{\alpha}} \to G_2(\alpha) = \frac{\sqrt{\pi}}{4\alpha^{3/2}}$$

$$\int_{-\infty}^{\infty} A e^{-\frac{1}{2}mv^2/kT} dv. \quad A\cdot\sqrt{\frac{\pi}{\frac{1}{2}m/kT}} = 1. \quad A = \sqrt{\frac{m}{2\pi kT}}.$$

$$\int_{-\infty}^{\infty} \frac{1}{2}mv_x^2 A e^{-\frac{1}{2}mv_x^2/kT} dv_x = \frac{1}{2}m\cdot\sqrt{\frac{m}{2\pi kT}}\cdot\frac{1}{2}\sqrt{\frac{\pi}{(\frac{1}{2}m\frac{1}{kT})^3}}$$

$$= \frac{1}{2}kT.$$

$$\overline{\frac{1}{2}mv^2} = \frac{3}{2}kT.$$

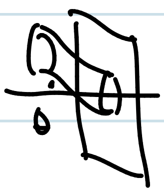
$\varepsilon \sim v^2$ $T\uparrow$

$\varepsilon \sim \omega^2$ $\Delta\varepsilon \ll kT$

$\varepsilon \sim |v|$ $T\downarrow$

光子气. $M=e$. $dp = \int_0^\infty f(\nu)d\nu$ 频率分布.

$\varepsilon = h\nu$. $P_{光} = \frac{h\nu}{c}$. 数密度 n_0



$\theta, d\Omega, dS, dt. \; c. \; \nu\sim\nu+d\nu.$

$$dn = c\cos\theta dt dS \cdot n_0 \cdot \frac{d\Omega}{4\pi} \cdot f(\nu)d\nu$$

$$P_{压} = \frac{\int dn \cdot \frac{2h\nu}{c}\cos\theta}{dS dt} \qquad \int_0^1 x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cancel{2}h\nu}{\cancel{c}} \cos\theta \cdot \cos\theta \cdot n_0 \cdot \frac{\cancel{2}\pi\sin\theta d\theta}{\cancel{4\pi}} f(\nu)d\nu$$

$$= \frac{1}{3}\cdot n_0 \cdot \int_0^\infty h\nu f(\nu)d\nu = \frac{1}{3}n_0\bar{\varepsilon} = \frac{U}{3V}$$

$$\frac{2}{3}n_0\bar{\varepsilon}$$

泄流 $J_1 = \frac{\int dn}{dS dt}$ $\int_0^1 x dx$

$$= \int n_0 dS dt\, v\cos\theta \cdot \frac{2\pi\sin\theta d\theta}{4\pi} \cdot f(v)dv$$

$$= \frac{1}{4}n_0 \int_0^\infty f(v)dv = \frac{1}{4}n_0\bar{v}$$

光子: $J = \frac{1}{4}n_0 c$

$$\bar{v} = \int_0^\infty dv\, v \cdot 4\pi v^2\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{\frac{1}{2}mv^2}{kT}} = \sqrt{\frac{8kT}{\pi m}}$$

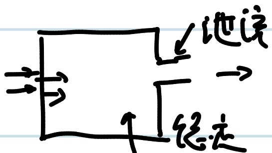
令 $\frac{1}{2}\frac{mv^2}{kT} = x^2$. $v = \sqrt{\frac{2kT}{m}}x$.

$$v = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\sqrt{\frac{2kT}{m}}\right)^4 \boxed{\int_0^\infty x^3 e^{-x^2}dx} \to \frac{1}{2}.$$

$G_3(1)$

$$= \sqrt{\frac{kT}{m}}\cdot\square.$$

DHO. THO. HDO $\;{}^1H \; {}^2H \; {}^3H$



泄流 $\frac{n_{DHO}}{n_水} = ?$

H^+ D^+

$(H_3O)^+$ $(H_2DO)^+$

$v = \square\sqrt{\frac{kT}{m}}$. $(OH)^-$ $(OD)^-$

H^+ D^+

稳 DHO. H_2O

$n_0\lambda.$ $n_0(1-\lambda)$

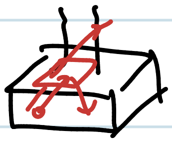
T 一样 $\frac{1}{2}mv^2 \sim \frac{3}{2}kT$

$v \sim \sqrt{\frac{3kT}{m}}$

$$出: \frac{n_{DHO}}{n_{H_2O}} = \frac{J_{DHO}}{J_{H_2O}} = \frac{\frac{1}{4}n_{内DHO}\bar{v}_{内DHO}}{\frac{1}{4}n_{内H_2O}\bar{v}_{内H_2O}}$$

D. T.

$$= \frac{\lambda}{1-\lambda}\cdot\sqrt{\frac{m_{H_2O}}{m_{DHO}}} = \frac{\lambda}{1-\lambda}\sqrt{\frac{18}{19}}$$



蒸发 $\propto$ 泄流

$$\frac{\Delta n}{\Delta S\Delta t} = J_{泄}\cdot e^{-\varepsilon/kT}$$

逸出.

5.6. 统计基础.

More is different.

时间平均

① 多次取平均值. (1个)

多个 取一次 平均值 一样!

系综平均

等概率. 满足约束每一个微观状态 概率一样.

$$\left\{\begin{array}{l} 1.1 \quad 1.1.1.1 \quad (\frac{1}{2})^6 \leftarrow 一个微观态 \\ 1.0 \quad 1.0.1.0 \quad (\frac{1}{2})^6 \end{array}\right.$$

概率大? $\leftarrow$ 宏观态

$\{$ 6:1.

3:1 3:0. 有 C_6^3 个微观态.

N 次. 有 $\frac{N}{2}(1+x)$ 次向上 概率 $=?$ P_x.

$$P_x = \frac{1}{2^N}\cdot C_N^{\frac{N}{2}(1+x)} = A\, C_N^{\frac{N}{2}(1+x)}$$

N 大 x 小.

$$C_N^{\frac{N}{2}(1+x)} = \frac{N!}{[\frac{N}{2}(1+x)]![\frac{N}{2}(1-x)]!}$$

$$\ln N! = N\ln N - N + \ln\sqrt{2\pi N} + O(\frac{1}{N})$$

$$\ln C_N^{\frac{N}{2}(1+x)} = \ln N! - \ln\frac{N(1+x)}{2}! - \ln\frac{N(1-x)}{2}!$$

$$= N\ln N - N - \frac{N(1+x)}{2}\ln\frac{N(1+x)}{2} + \frac{N(1+x)}{2}$$

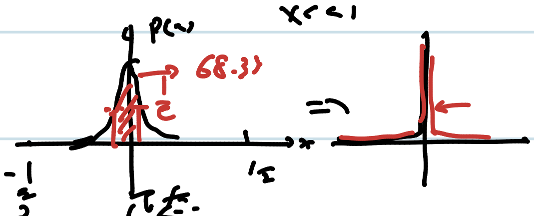
$$-\frac{N(1-x)}{2}\ln\frac{N(1-x)}{2} + \frac{N(1-x)}{2}$$

$\ln(1+x) \simeq x.$

$x \ll 1$

$$= -Nx^2$$

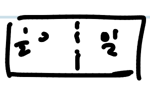
$$P(x) = Ae^{-Nx^2}$$



$Nx^2 = 1$

$x = \frac{1}{\sqrt{N}}$. 相对偏差.

~46. 3:6 $\int_{-3\delta}^{3\delta} P(x) = 99.7\%$ 10^{24}



$$e^{-N(\frac{1}{10^6})^2} \sim e^{-10^{16}} = \frac{1}{10^{10^{16}}}$$

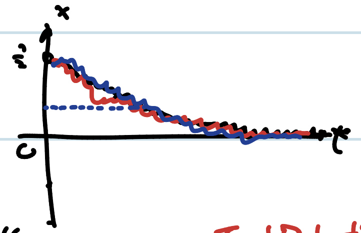
2017暑1- HZ 决赛 043

一个宏观态的熵 $S = k\cdot\ln\Omega$ ← 微观态数.

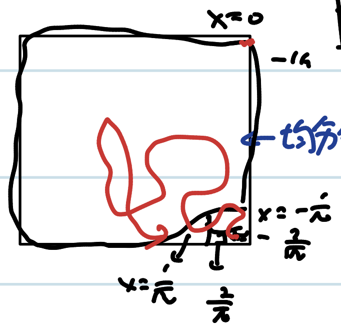
$\Omega = C_N^{\frac{N}{2}(1+x)}$

N个分子. 向上.

熵变 $N/_2$



有初始大概率转



M-B分布.

E_0 ≠ E_r, T → 微观态是等概率

$E_0 + E_r = E$; $E_0 = E - E_r$

令总体的一个态等概率.

交换能量. $dN=0$ $dV=0$

→在不同能量微观态上. $p(E_0) = A\cdot\Omega_r(E_r)$

$\Omega_r(E_r)$. 在.能量为 E_r 的宏观态对应的微观态数.

$p(E_0) = A\,\Omega_r(E-E_0)$.

最概然条件

$\Omega_0(E_0)\cdot\Omega_r(E_r) = Max$ n个系统所有可能状态

$\frac{d}{dE_0}(\Omega_0(E_0)\Omega_r(E-E_0)) = 0$

平衡条件.

$\Rightarrow \frac{d\Omega_0(E_0)}{\Omega_0\, dE_0} = \frac{d\Omega_r}{\Omega_r dE_r}$. $\Rightarrow \frac{d\ln\Omega_0(E_0)}{dE_0} = \frac{d\ln\Omega_r}{dE_r} = \frac{1}{kT}$.

$S = k\ln\Omega$

$p(E_0) = A e^{\ln\Omega_r(E-E_r)} = A e^{\ln\Omega_r(E) - \frac{1}{kT}\cdot E_0}$

$p(E_0) = A_1 e^{-\frac{E_0}{kT}}$ 一个能量为E的微观态概率

I Maxwell-Boltzmann 分布. —量子态.

取 $E = \frac{1}{2}mv^2 + E_p$.

$p(\vec{v}, \vec{r}) = A\, e^{-\frac{\frac{1}{2}mv^2 + E_p(\vec{r})}{kT}}$

$f(\vec{v})\, dv_x dv_y dv_z = dp$

$\psi(\vec{r}, t) = A e^{i\frac{\vec{p}\cdot\vec{x}}{\hbar} - i\frac{Et}{\hbar}}$

限在L长箱子中. 驻波. $\frac{P\cdot L}{\hbar} = n\pi$.

$P_n = \frac{nh}{2L}$; 每个 $\Delta p = \frac{h}{2L}$ 对应一个态.

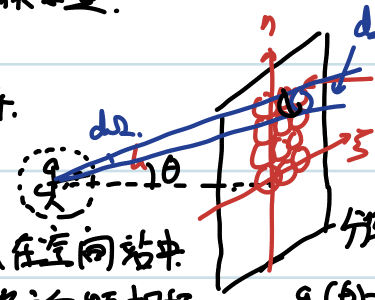
P上 微态观等分. $f(\vec{p})dp_x dp_y dp_z$.

$f(p_x, p_y, p_z)\, dp_x dp_y dp_z = dp$

$f(p_x, p_y, p_z) = A\, e^{-\frac{\sqrt{p^2c^2 + m^2c^4}}{kT}}$

核定量.

例1.14.



在(ξ, η) 有多大概率打到

$dp = f(\xi, \eta)\cdot d\xi\, d\eta$.

$g(\theta) = A$

放在空间各中 向各方向均匀打射.

分布概率 $\int g(\theta)\, d\Omega = 1$.

$g(\theta) = \frac{1}{4\pi}$.

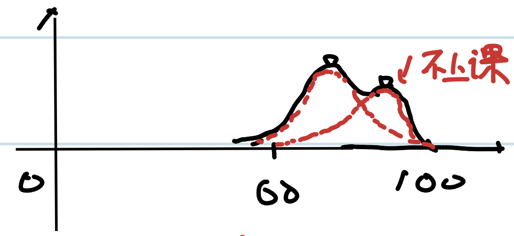
概率: $g(\theta)\cdot d\Omega = f(\xi, \eta)\, ds$. $ds = \frac{r^2 d\Omega}{\cos\theta}$

$f(\xi,\eta) = g(\theta)\cdot\frac{d\Omega}{ds}$

$= \frac{1}{4\pi}\cdot\frac{d\Omega\,\cos\theta}{(h^2+\xi^2+\eta^2)\, d\Omega}$

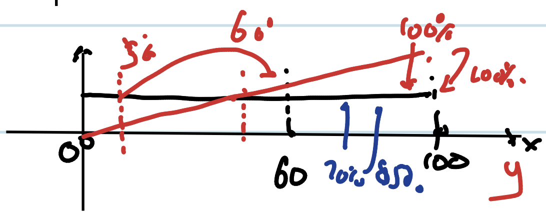
$= \frac{1}{4\pi}\cdot\frac{h}{(h^2+\xi^2+\eta^2)^{3/2}}$

$\iint_{-\infty}^{\infty} d\xi\, d\eta\, \frac{1}{4\pi}\,\frac{h}{(h^2+\xi^2+\eta^2)^{3/2}} = \frac{1}{2}$.



$x \to \sqrt{x} \times 10$

$f(x) = \frac{1}{100}\cdot dx$.



$g(y)$

$y = 10\sqrt{x}$.

$x \sim x+dx \to y+dy$.

$x = \left(\frac{y}{10}\right)^2$

$g(y)dy = f(x)\, dx$

$g(y) = f(x)\cdot\frac{dx}{dy}$

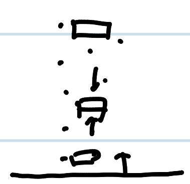
$= \frac{1}{100}\cdot\frac{d\left(\frac{y}{10}\right)^2}{dy} = \frac{1}{100}\cdot\frac{2y}{100}\cdot dy$.

$f(0.05) = 0.6$ 二次函数.

$f(0.70) = 0.85$

$f(1.00) = 1$

例1.15. 大气. 气温.



分布h~h+dh 的概率.

$p(h) = A\, e^{-E/kT}$

$p(h) = A e^{-mgh/kT}$

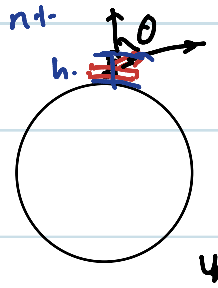
$p(h) = n(h) \propto P_压$

$P_压(h) = P_0\, e^{-mgh/kT}$

地: $\frac{P_{O_2}(0)}{P_{N_2}(0)}=\frac{21}{78}$ 求 $\frac{P_{O_2}(20km)}{P_{N_2}(20km)}=?$

$$\frac{P_{O_2}(20km)}{P_{O_2}(0)} = e^{-\frac{m_{O_2} g h N_A}{kT\cdot N_A}} = e^{-\frac{\mu_{O_2} g h}{RT}}$$

$$\left(\frac{P_{O_2}(20km)}{P_{O_2}(0)}\right)\Big/\left(\frac{P_{N_2}(20km)}{P_{N_2}(0)}\right) = e^{-\frac{(\mu_{O_2}-\mu_{N_2})gh}{RT}}$$



$\frac12 mv^2 > \frac{GMm}{R}$ $v^2 > gR$.

$v_h^2 > \frac{GMm}{R+h}$; $v_h^2 > g\frac{R^2}{R+h}$ 高h. $h \ll R$.

$$n(h)=n_0(h)\, e^{-mgh/kT}.$$

$v_h > \sqrt{g\frac{R^2}{R+h}}$ 才脱离.

$$P = \frac12\int_{v_h}^{\infty} f(v)\,dv = \int_{v_h}^{\infty} 4\pi v^2\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{\frac12 mv^2}{kT}}\,dv$$

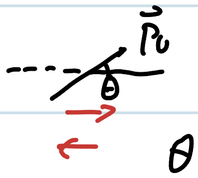
$\frac12 mv^2 \gg kT$.

$$\frac{\Delta N}{\Delta t} = n(h)\cdot 4\pi(R+h)^2\cdot\frac14\bar{v}\times P.$$

$$\frac{\Delta N}{\Delta t} = n(h)\cdot 4\pi(R+h)^2\left[\int_{v_h}^{\infty} f(v)\,dv\int\frac{d\Omega}{4\pi}\cdot v\cos\theta\right]$$

$$N = \int_0^\infty n_0 e^{-\frac{mgh}{kT}} S\, dh\ ;\ \frac{\Delta N}{\Delta t} = -\Omega N$$

例115. 偶极子极化



→E 方向取值 $\vec{P} = \frac{\Sigma\vec{p}}{V} = \varepsilon_0\chi E$. 极化率

θ: $\varepsilon = -\vec{p}_0\cdot\vec{E}$

$$\overline{p_0} = \int p_0\cos\theta\, A\cdot e^{-\varepsilon/kT}\,d\Omega = p_0 A 2\pi\int\cos\theta\, e^{-\Omega}\,d\cos\theta$$

$$= p_0 A 2\pi\cdot\left(\frac{kT}{pE}\right)^2\cdot\int_{-\frac{pE}{kT}}^{\frac{pE}{kT}} x e^x\,dx$$

$$1 = \int A e^{-\varepsilon/kT}\,d\Omega$$

$$= \int_{-1}^{1} A\, e^{+\frac{p_0 E_0\cos\theta}{kT}}\cdot 2\pi\, d\cos\theta$$

$$= 2\pi A\cdot\frac{kT}{p_0 E_0}\int_{-\frac{pE}{kT}}^{\frac{pE}{kT}} e^x\,dx = 2\pi A\cdot\frac{kT}{pE_0}\, 2\sinh\frac{pE_0}{kT}$$

$$\frac{d}{d\beta}\int_{-a}^{a} e^{\beta x}\,dx = \frac{d}{d\beta}\left[\frac1\beta\left(e^{\beta x}-e^{-\beta x}\right)\right]$$

$$\int_{-a}^{a} x e^{+\beta x}\,dx = -\frac{e^{\beta x}-e^{-\beta x}}{\beta^2} + (ae^{\beta x}+e^{-\beta x}a)\Big|_{-a}^{a}$$

取高温 $\frac{pE}{kT} \ll 1$. $e^{\frac{pE}{kT}} \simeq 1+\frac{pE}{kT}$.

$1 = 2\pi A\cdot 2\cdot\frac{kT}{pE}\cdot\frac{pE}{kT}$ $A \simeq \frac{1}{4\pi}$.

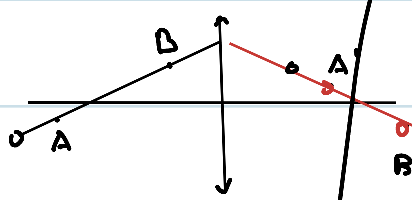
$$\overline{p} = \frac{1}{4\pi}\cdot\int_{-1}^{1} p\cos\theta\left(1+\frac{pE}{kT}\cos\theta\right)2\pi\, d\cos\theta \to \varepsilon\chi.$$

$$= \frac13\frac{p^2}{kT}E \qquad P = \boxed{n_0\frac{p^2}{3kT}}E \quad \text{2017暑I-H2 决赛0.45}$$

6. 光学.

6.1 几何光学.

近轴成像



直线成像? 直线:

$\frac1u+\frac1v=\frac1f$.

过A光线 ⇒ 过A' 过B光线 ⇒ 过B'

过AB光线 ⇒ 过A'B'.

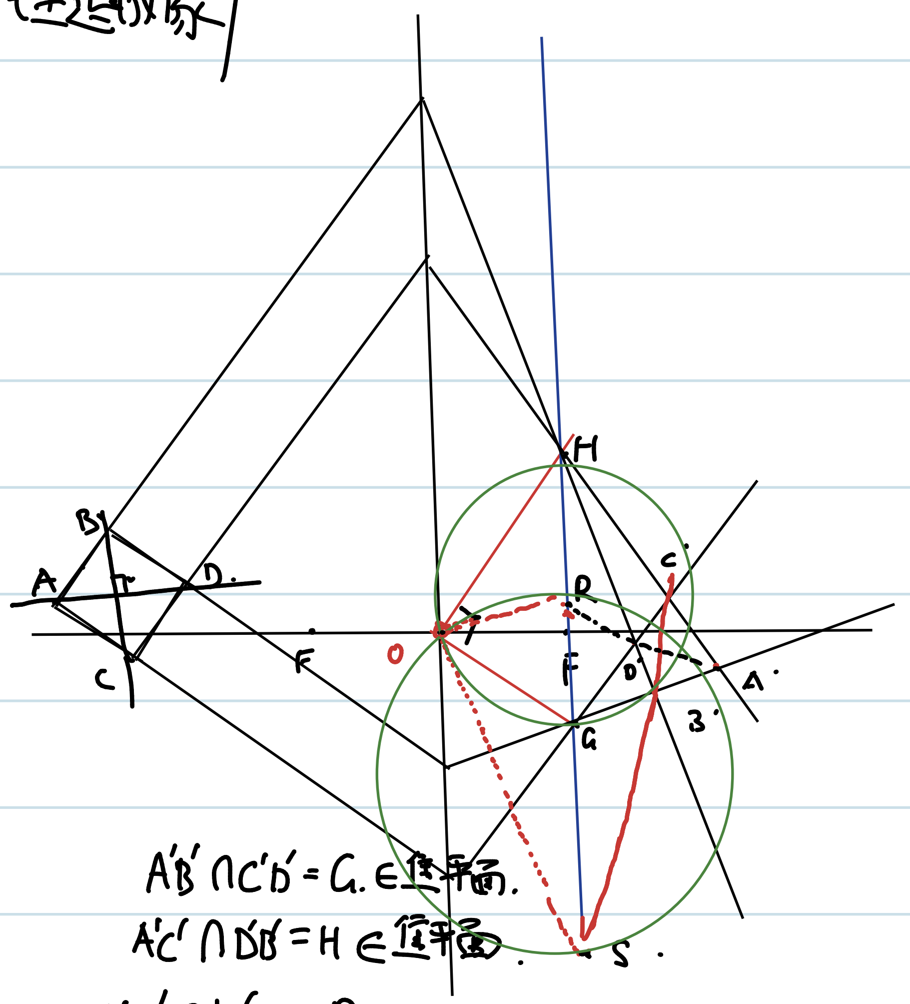
圆成像? 正方形成像?

例) 116

已知一个四边形. 由正方形成像得到.

求尺规作图. 画光心 焦点.

理想成像



$A'B' \cap C'D' = G \in$ 焦平面.

$A'C' \cap D'B' = H \in$ 焦平面.

$A'D' \cap HG = R$.

$B'C' \cap HG = S$

$OR \perp OS \Leftarrow$ AD与A'D'光线在焦平面上相交

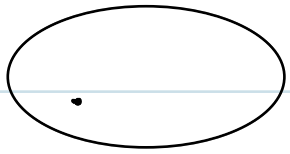
$OR \parallel AD$

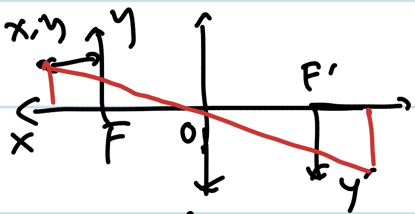
以HG为直径作圆. 同理 $OS \parallel AC$.

以RS为直径作圆 交于光心

例117. 一个圆成像为了椭圆.
圆心的像和椭圆已知.

尺规作图还原光心,焦点.





$$\frac{1}{f+x}+\frac{1}{f+x'}=\frac{1}{f}$$

$$f+x'+f+x=\frac{f^2+fx+fx'+xx'}{f}$$

$$\Rightarrow f^2=xx'$$

$$\begin{cases} x=\frac{f^2}{x'} \\ y=\frac{y'f}{x'}\end{cases}$$

$$\frac{y}{y'}=\frac{x+f}{x'+f'}\ ;\ y=y'\cdot\frac{f+\frac{f^2}{x'}}{x'+f}=\frac{y'f}{x'}$$

$ax+by=1$ 直线

$$\Rightarrow a\frac{f^2}{x'}+b\frac{y'f}{x'}=1 \Leftrightarrow af^2+by'f=x'$$

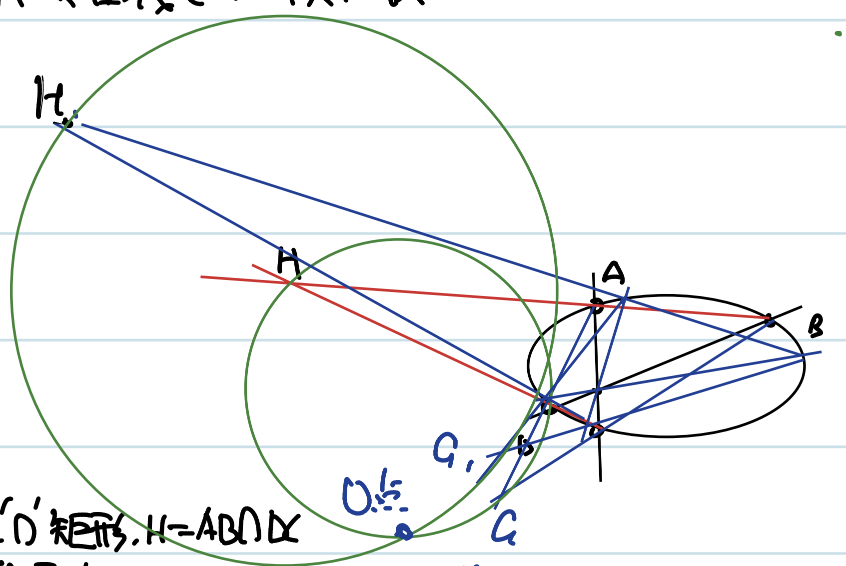
$$ax^2+bxy+cy^2+dx+ey=1.$$

$$a\cdot\frac{f^4}{x'^2}+b\frac{f^2}{x'}\cdot\frac{y'f}{x'}+c\frac{y'^2f^2}{x'^2}+d\frac{f^2}{x'}+e\frac{y'f}{x'}=1.$$

$$af^4+bf^3y'+cy'^2f^2+df^2x'+fex'y'=x'^2$$

仍二次.

n次曲线 ⇔ n次曲线



A'B'C'D'矩形, H=AB∩DC

H在焦面上.

G=AD∩BC. H在焦面.

光心O. OH // A'B'.

OG // A'D'

O在HGH为直径圆上.

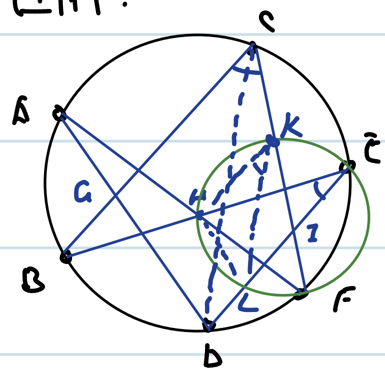
再作A'B'C'D', ⇒ H_1G_1

以H_1G_1为直径.

HG 焦面.

应用.

I ∈ GH.



过HEF圆交CF, K

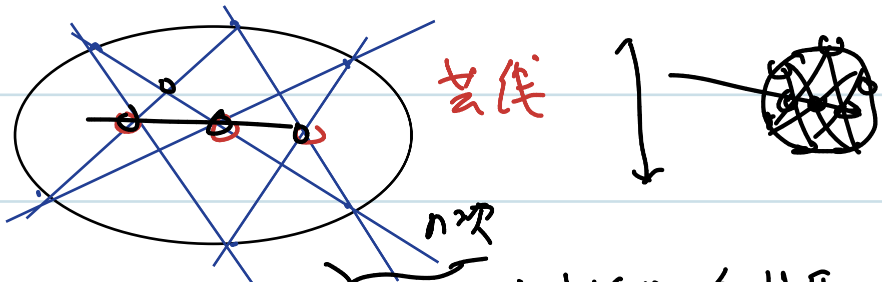
交BE于L

∠HKL=∠HEL=∠BCD

⇒ BC//HK

同理. HL//AD. CD//KL

位似 ⇒ CK.GH.DL交于一点 为I



由二次=次曲线的交点构成n个曲度.

保持成像.不变

6.2 用波动光学表述成像.

$\vec E(x,y,z,t)=\vec E_0 e^{i(\vec k\cdot\vec r-\omega t)}$ 平面波.

$\vec E_0\cdot\vec k=0$ 横波. $\vec B=\vec B_0 e^{i(\vec k\cdot\vec r-\omega t)}$

$\nabla\times\vec E=-\frac{\partial\vec B}{\partial t}$: $i\vec k\times\vec E=+i\omega\vec B$: $\frac{\vec k}{\omega}\times\vec E=-\vec B$

同一个偏振方向 $U_0 e^{i(\vec k\cdot\vec r-\omega t)}=\tilde U(t)$

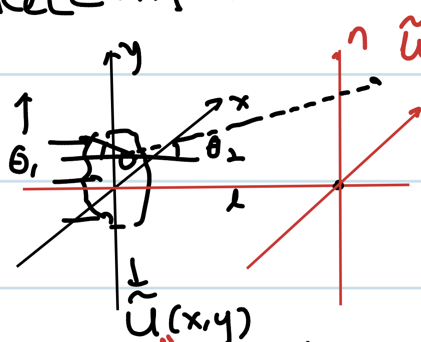
电场振幅.

$\vec S=\vec E\times\vec H=\frac{1}{\mu}\vec E\times\vec B\Rightarrow\bar{\vec S}=\hat k\frac{1}{2\mu c}E_0^2$

记 $I=|\tilde U(t)|^2\frac12$; $[\vec S=\hat k\frac{1}{\mu c}\vec E^2]$ $=\hat k\frac{1}{\mu c}\overline{E^2(t)}$

$E(t)=E_0\cos(kx-\omega t)$ $\overline{E^2(t)}=\frac12E_0^2$.

$Re[\tilde E(t)]=E(t)$



$\tilde U(\xi,\eta)=?$

$\tilde U_0(x,y)$

$$ikl+ik\frac{x^2+y^2-2\xi x-2\eta y+\xi^2+\eta^2}{2l}$$

$$\tilde U(\xi,\eta)=\iint\frac{dxdy\,\tilde U_0(x,y)\,e^{ik\sqrt{l^2+(\xi-x)^2+(\eta-y)^2}}}{ik\sqrt{l^2+(\xi-x)^2+(\eta-y)^2}}\cdot\frac{\cos\theta_1+\cos\theta_2}{2}$$

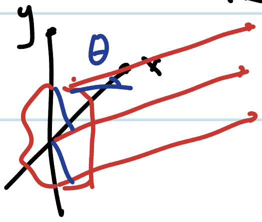
$\xi,x,\eta,y\ll l$

$$=\frac{e^{ikl}\,e^{ik\frac{\xi^2+\eta^2}{2l}}}{ikl}\iint dxdy\,\tilde U_0(x,y)\,e^{-ik\frac{\xi x}{l}-ik\frac{\eta y}{l}+ik\frac{x^2+y^2}{2l}}$$

$e^{ikl}\cdot\frac{e^{ik\frac{\xi^2+\eta^2}{2l}}}{ikl}\iint dxdy\,\tilde{u}_0(x,y)\,e^{-ik\frac{\xi x}{l}-ik\frac{\eta y}{l}+ik\frac{x^2+y^2}{2l}}$

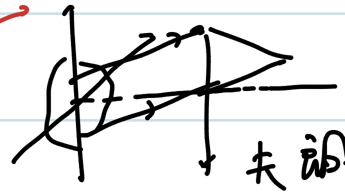
① $k\cdot\frac{x^2+y^2}{2l}\ll1$, $x,y\ll\sqrt{l\lambda}$, $\xi,\eta,x,y\ll l$.

$$u(\xi,\eta)=\frac{e^{ikl+ik\frac{\xi^2+\eta^2}{2l}}}{ikl}\cdot\iint dxdy\,\tilde{u}_0\,e^{-i\frac{k\xi x}{L}-ik\frac{\eta y}{L}}$$



$\frac{\xi}{L},\frac{\eta}{L}$ 角度　$\sin\theta_x\cdot x;\ \sin\theta_y\cdot y$

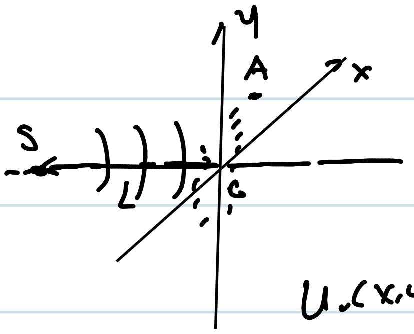
往平面上看.



夫琅禾费

② $k\frac{x^2+y^2}{2l}$ 不能忽略. 菲涅耳.

例118. 点光源. 点光源成像



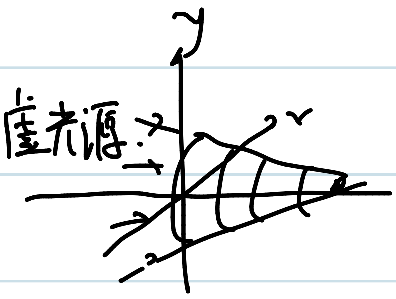
$u_0(x,y)=u_0e^{i\phi(x,y)}$

$\phi(x,y)-\phi(0,0)=(|SA|-|SO|)\cdot k$.

$=k\cdot\frac{x^2+y^2}{2L}$

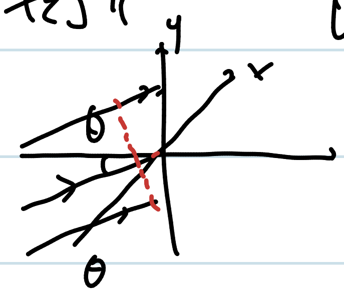
$u_0(x,y)=u_0\,e^{ik\frac{x^2+y^2}{2L}}$

面上振幅和相位信息.



$u(x,y)=u_0e^{-ik\frac{x^2+y^2}{2L}}$

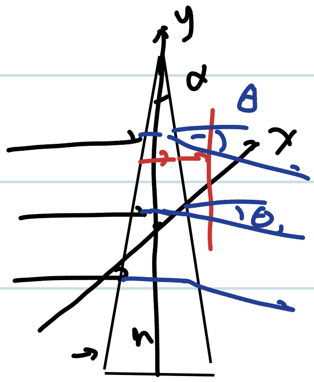
平行光



$u=u_0e^{i\sin\theta y k}$

$\sim u_0\,e^{iy\boxed{\theta k}}$

向上 θ 角向 平行光.



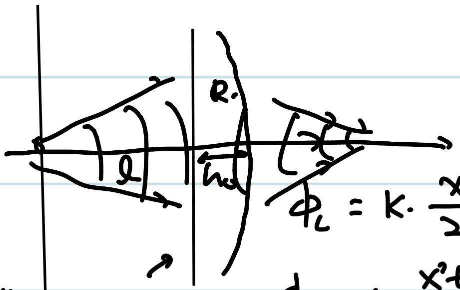
$h(y)=D-\alpha y$

$u_L=\tilde{u}_0$.

$\tilde{u}_R=\tilde{u}_0\,e^{ik[n\cdot h(y)+(1-h(y))]}$

$=\tilde{u}_0\,e^{i(k(n-1)(-\alpha y))}\cdot e^{i(\varphi_0+\varphi')}$

平行光. $\theta=-\frac{\alpha k(n-1)}{k}=-\alpha(n-1)$



$h(x,y)=h_0-\frac{x^2+y^2}{2R}$

$\phi_L=k\cdot\frac{x^2+y^2}{2l}$

$\phi_R=k\frac{x^2+y^2}{2l}+(n-1)kh(x,y)+C$.

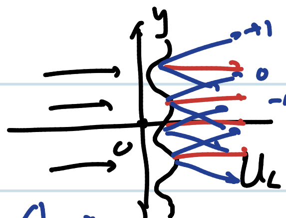
$=k\frac{x^2+y^2}{2l}+\frac{(n-1)(x^2+y^2)}{-2R}k+C'$

$=-k\frac{x^2+y^2}{2l'}+C'$　会聚 l' 处

$\Leftrightarrow\ \frac{1}{l'}+\frac{1}{l}=\frac{n-1}{R}$ 成像公式

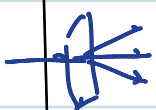
例119. 光栅.

振幅透射率.



$t(y)=a+b\cos(k_1y)$

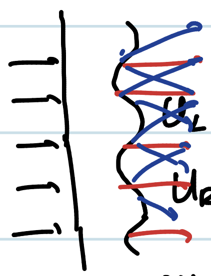
$u_L=u_0$



$\phi(x,y)=u_0(a+b\cos k_1y)$

$=au_0+\underbrace{bu_0e^{ik_1y}}_{\frac{k_1}{k}=\theta\text{角向上平行}}+\underbrace{bu_0e^{-ik_1y}}_{\text{向下}}$

au_0：平行



$n_0,\ h(y)=h_0+h_1\cos k_1y$

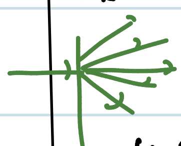
相位光栅.

u_L,u_0

$u_R,\ u_0e^{i[(n-1)kh_1\cos k_1y]}\,e^{i(\phi+\phi')}$.

$\ll1$

$u_R:\ u_0e^{i(\phi+\phi')}[1+i(n-1)kh_1\cos k_1y+\frac{1}{2!}(i(n-1)kh_1\cos k_1y)^2+\dots]$.

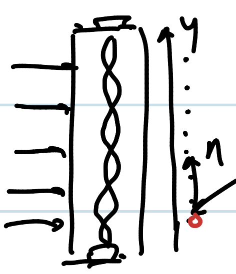


±1级　$\theta=\frac{k_1}{k}$

$=u_0e^{i(\phi+\phi')}[1+i(n-1)kh_1\frac{e^{ik_1y}+e^{-ik_1y}}{2}+\frac{1}{2!}(-1)\cdot kh_1(\frac{1}{2}+\underbrace{\frac{1}{4}e^{i2k_1y}}_{+2}+\underbrace{\frac{1}{4}e^{-i2k_1y}}_{-2})+\dots$

1：平行光

超声光栅.



$\eta(y,t)$ 振动.

$\eta(y,t)=A\sin(k_1y)\cdot\cos(\omega t+\varphi)$

$y\sim y+dy$. $\Rightarrow dy$.

$dy'=dy+\eta(y+dy,t)-\eta(y,t)$

$=dy(1+\frac{\partial\eta}{\partial y})$.　$\frac{\partial\eta}{\partial y}\ll1$

超声波 $\frac{dy'}{dy}=1+\frac{\partial\eta}{\partial y}=1+Ak_1\cos(k_1y)\cos(\omega t+\varphi)$

$\frac{\rho'}{\rho}=1-Ak_1\cos(k_1y-\omega t)$.

$\frac{\Delta n}{n}=1\cdot\frac{\Delta\rho}{\rho}$