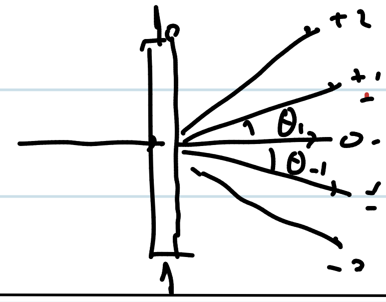
$$n(y)=n_0\left(1-Ak_1\cos(k_1y)\cos(\omega t+\varphi)\right)$$

$$\tilde{u}_R=u_0\,e^{ih\,n_0(1-Ak_1\cos k_1y\cos(\omega t+\varphi))}$$

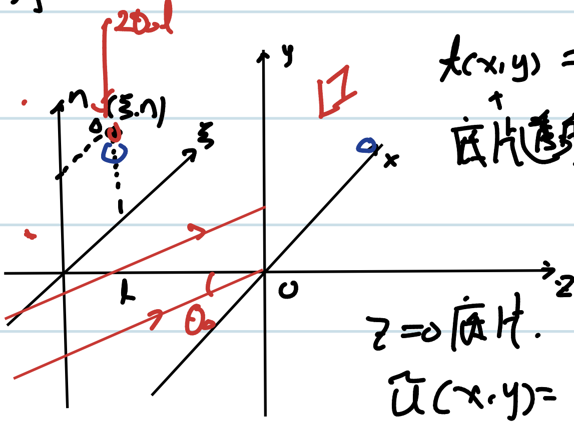
$\propto A^4\cos^4$



强度 $\propto \cos(\omega t+\varphi)|A|^2$. $\cos^5$

$$\theta_1=\frac{k_1}{k}=\frac{2\pi/\lambda_1}{2\pi/\lambda}=\frac{\lambda\ \text{光波}}{\lambda_1\ \text{声波}}.$$

例120 全息摄像. 相位+振幅.



$t(x,y)=1-\alpha I(x,y)$

底片透射率　曝光强度.

物光区:

$z=0$ 底片.

$$\tilde{u}(x,y)=u_0e^{ik\frac{-(\xi-x)^2-(\eta-y)^2}{2l}}$$

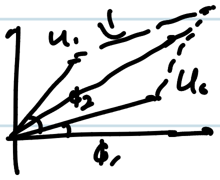
$$I(x,y)=|\tilde{u}|^2=u_0^2. \quad \times$$

参照光束 $\tilde{u}_1(x,y)=u_1e^{ik\theta_0y}$

$$\tilde{u}_{总}(x,y)=u_0e^{ik\frac{(\xi-x)^2+(\eta-y)^2}{2l}}+u_1e^{ik\theta y}$$

$$I(x,y)=|\tilde{u}_{总}(x,y)|^2 \qquad u_0e^{i\varphi_1}+u_1e^{i\varphi_2}$$

$$=u_0^2+u_1^2+2\cos(\varphi_2-\varphi_1)u_0u_1$$



$$=u_0^2+u_1^2+2\cos\left(k\theta y+\frac{(\xi-x)^2+(\eta-y)^2}{2l}\right)u_0u_1$$

$$\Rightarrow t(x,y)=1-\alpha u_0^2-\alpha u_1^2-\alpha\cdot 2u_0u_1\cos\left(k\theta y+k\frac{(\xi-x)^2+(\eta-y)^2}{2l}\right),$$

还原: 再打入参考光 (支撑物体).

$$\tilde{u}_1(x,y)=u_1e^{ik\theta y}. \qquad e^{-ik\theta y}\quad e^{ik\theta' y}.$$

$$\tilde{u}_t(x,y)=u_1e^{i\theta y}t(x,y) \to \text{平行光}$$

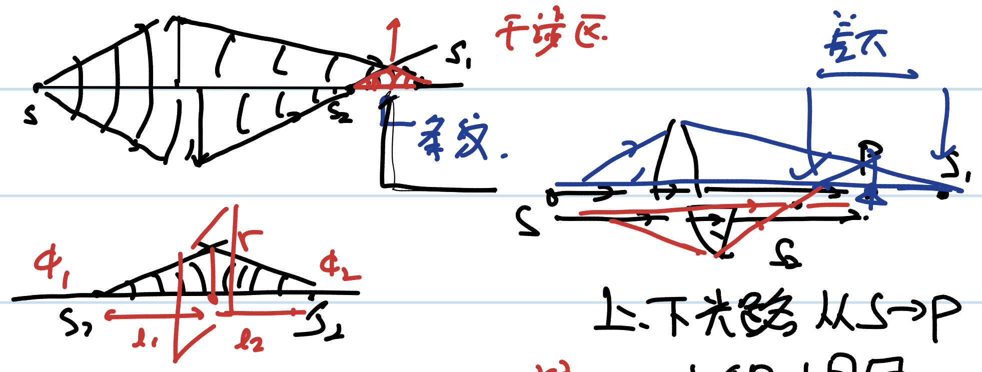
$$=u_1e^{ik\theta y}\Big[1-\alpha u_0^2-\alpha u_1^2$$

$$-\alpha u_0u_1\,e^{ik\theta y+ik\frac{(\xi-x)^2+(\eta-y)^2}{2l}}-\alpha u_0u_1\,e^{-ik\theta y-ik\frac{(\xi-x)^2+(\eta-y)^2}{2l}}\Big]$$

$$e^{2ik\theta y+ik\frac{(\xi-x)^2+(\eta-y)^2}{2l}} \qquad e^{-ik\frac{(\xi-x)^2+(\eta-y)^2}{2l}}+□$$

原物体.

例121.



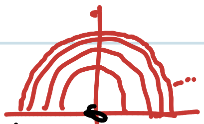
上下光路从 S→P 光程相同. 同相位

$$\varphi_1+k\frac{r^2}{2l_1}\ ;\ \varphi_2-k\frac{r^2}{2l_2}\quad \text{干涉}.$$

$$k\frac{r^2}{2l_1}+k\frac{r^2}{2l_2}=n\cdot2\pi+\varphi_0\ \text{时}\Rightarrow \text{相长}.$$

亮. ×

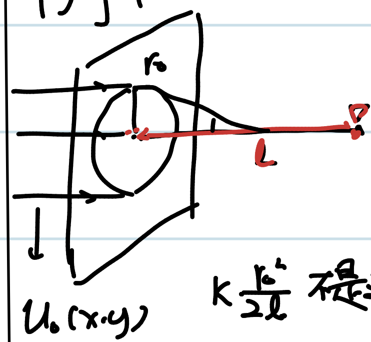
$$r_n=\sqrt{\frac{n\cdot2\pi+\varphi_0}{\frac{1}{2l_1}+\frac{1}{2l_2}}\cdot\frac{\lambda}{2\pi}}.$$



中心是亮 vs 暗?

成像前后有个相位差??

例122. 圆孔. 近.



$$u=?\quad \tilde{u}=\frac{e^{ikl}}{i\lambda l}\int_0^{r_0}\tilde{u}_0(x,y)e^{ik\frac{r^2}{2l}}2\pi r\,dr$$

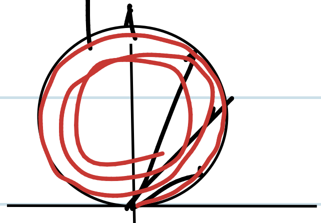
$$=\frac{e^{ikl}}{i\cdot2\cdot}\int_0^{r_0}\tilde{u}_0(x,y)e^{ik\frac{r^2}{2l}}\,d\,k\frac{r^2}{2l}$$

$\varphi(r)$

$k\frac{r^2}{2l}$ 相位增加.

$$\left|\int_0^{\varphi_r}e^{ik\frac{r^2}{2l}}\,d\,k\frac{r^2}{2l}\right|=2\left|\sin\frac{\varphi_r}{2}\right|.$$

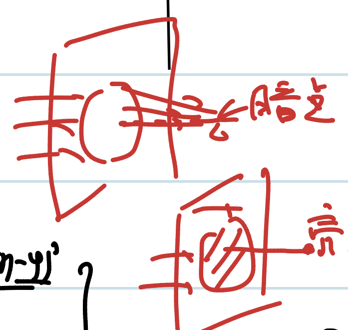
$k\frac{r_0^2}{2l}$ 对 r_0 变化

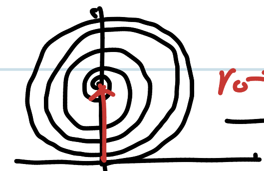


r_0 足够大

$$k\frac{r_n^2}{2l}=2\pi n. \qquad \frac{\cos\theta_1+\cos\theta_2}{2}$$

r_0 大

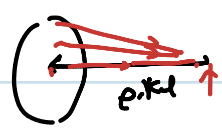




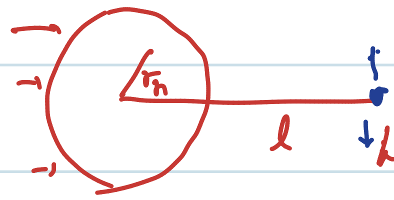
$r_0\to\infty$

$$u=\tilde{u}_0\,e^{ikl}$$

平面波



总振幅. $r_0\to\infty$ $e^{ikl}\cdot e^{i\frac{\pi}{2}}$



令 r_n, $k\frac{r_n^2}{2l} = 2\pi n$.

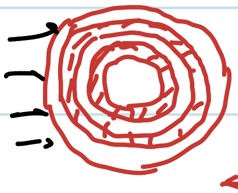
从 $r_0 \sim r_{1/2}$, $t=1$.

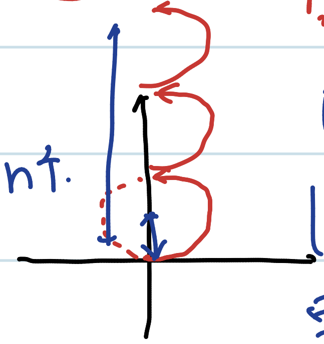
$r_{1/2} \sim r_1$ $t=0$.

$r_1 \sim r_{3/2}$ $t=1$

$r_{3/2} \sim r_2$ $t=0$

$r_{n-1/2} \sim r_n$ $t=1$





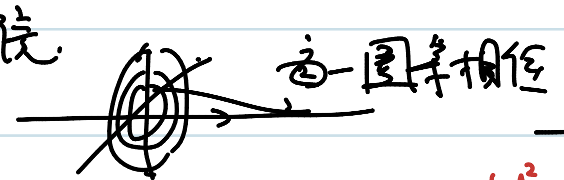
$\tilde{u} = 2n \cdot 1 \cdot \frac{u_0}{i} \cdot e^{ikl}$.

$|\tilde{u}|^2 = 4n^2 u_0^2$ 亮.

实际: $\frac{k r_n^2}{2l} = 2\pi n$; l 决定

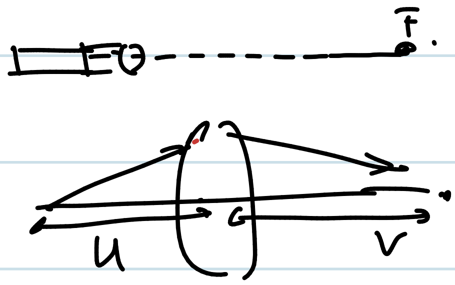
"焦点"

透镜.



每一圈等相位

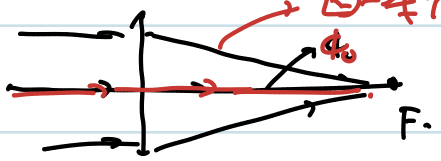
$u^2 = (n \cdot 2\pi)^2 u_0^2$

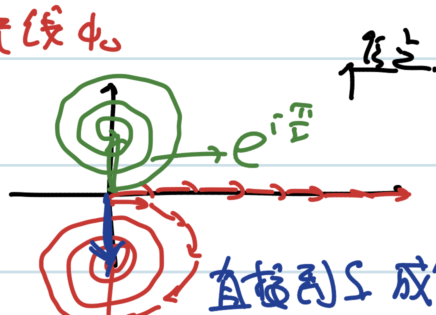


$\frac{kr^2}{2u} + \frac{kr^2}{2v} = \frac{kr^2}{2f}$ 时.

$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

每一条光线 ϕ_0

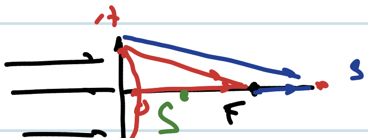




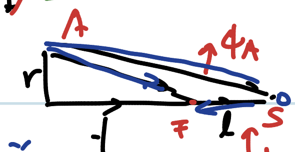
$e^{i\frac{\pi}{2}}$

直接看S 成像看S

相差 $e^{i\frac{\pi}{2}}$.

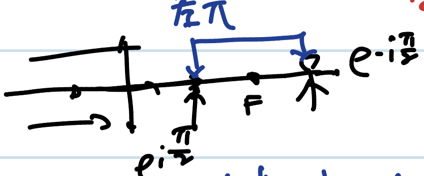


$|FS| = l$.



$|AS| - |AF| - |FS|$.

$\phi_A - \phi_0 - k|AS| + k|FS| + k|AF| = 0$ (+k|AS|. (−k|FS| (−k|AF|)



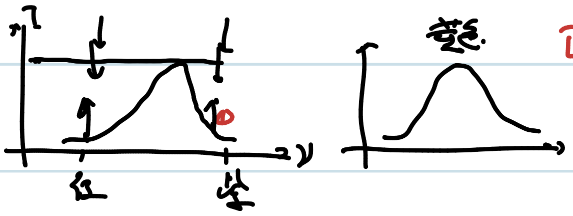
差π $e^{-i\frac{\pi}{2}}$ $e^{i\frac{\pi}{2}}$

$\therefore \phi_A - \phi_0 = k(|AS| - |AF| - |FS|)$

$= k\left[\frac{r^2}{2(l+f)} + l + f - f - \frac{r^2}{2f} - l\right]$

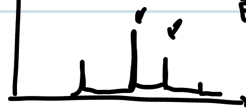
$= k\left(-\frac{r^2 \cdot l}{2f(l+f)}\right)$. $r\uparrow$ $\Delta\phi \propto r^2$

6.3. 光学干涉器件.



能补 卫带吸收分方法. 放补大. ⇒ 颜色 少:

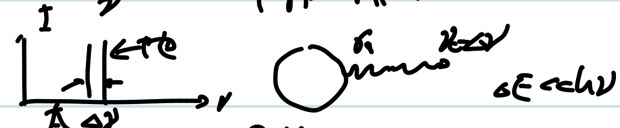
分辨率 - 门



精度量 ⇒ 性质.

$r \cdot \left(\frac{1}{n^2} - \frac{1}{m^2}\right) \times 4$

Helium.

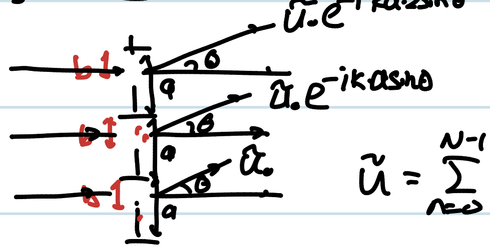


$\Delta E \ll h\nu$

$\frac{GMm}{r} = \Delta E$.

Na: 589.3 nm | 589.0 nm, 589.6 nm

例. 透射光栅.



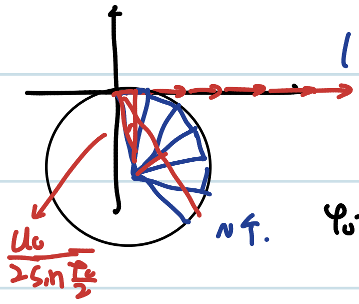
$\tilde{u}_0 e^{-ik2a\sin\theta}$, $\tilde{u}_0 e^{-ika\sin\theta}$, $\tilde{u}_0$

$\tilde{u} = \sum_{n=0}^{N-1} \tilde{u}_0 e^{-inka\sin\theta}$

$ka\sin\theta = \varphi_0$

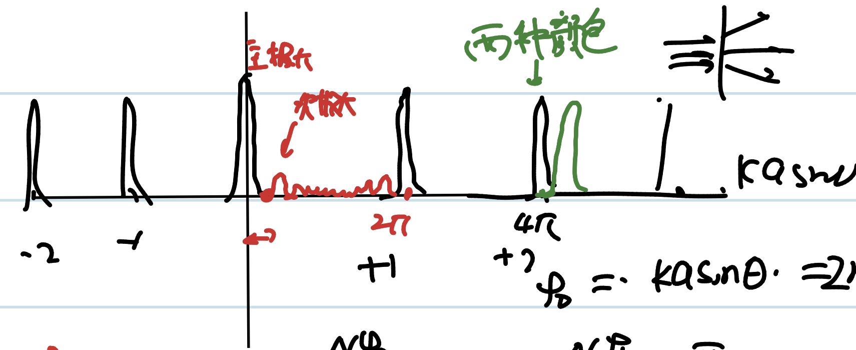
$|\tilde{u}| = \frac{u_0}{2\sin\frac{\varphi_0}{2}} \cdot 2\sin\frac{N\varphi_0}{2}$

$= \frac{u_0 \sin\frac{N\varphi_0}{2}}{\sin\frac{\varphi_0}{2}}$



$\frac{u_0}{2\sin\frac{\varphi_0}{2}}$ $N\varphi_0$.

$\varphi_0 \to 0$ $\frac{\sin\frac{N\varphi_0}{2}}{\sin\frac{\varphi_0}{2}} \to N$. $|\tilde{u}| = Nu_0$



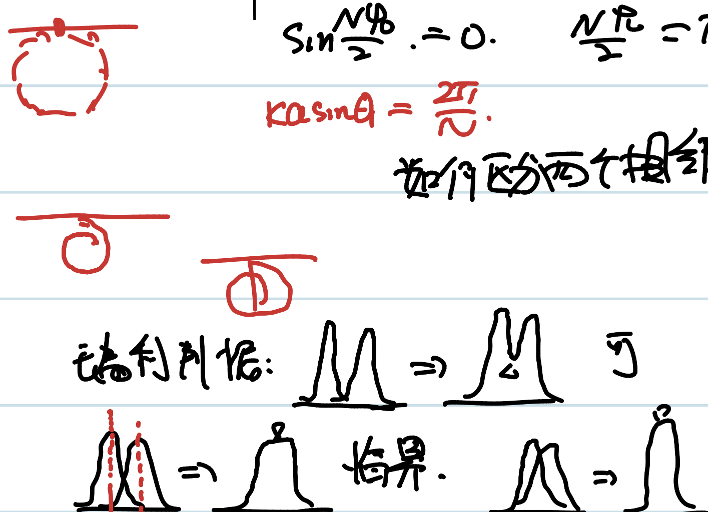
主极大 次极大 两种颜色 $ka\sin\theta$ −2 −1 2π 4π +1 +2

$\varphi_0 = ka\sin\theta = 2n\pi$ 时

$\sin\frac{N\varphi_0}{2} = 0$. $\frac{N\varphi_0}{2} = \pi$. $\varphi_0 = \frac{2\pi}{N}$

$ka\sin\theta = \frac{2\pi}{N}$.

如何区分两个相邻频率



瑞利判据: 可

临界. 不可.

一个峰值在另一个零点 $\frac{2\pi a\sin\theta}{\lambda} = m2\pi$

$\lambda+\Delta\lambda$ 的零点 $\Rightarrow \frac{2\pi a\sin\theta}{\lambda+\Delta\lambda} = m2\pi + \frac{2\pi}{N}$

2017暑I-HZ 决赛049

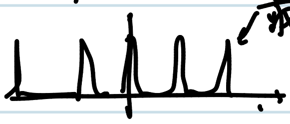
$$\frac{2\pi a\sin\theta}{\lambda}=m\,2\pi$$

$$\frac{2\pi a\sin\theta}{\lambda+\Delta\lambda}=m2\pi+\frac{2\pi}{N}$$ → 分辨.

$$1+\frac{\Delta\lambda}{\lambda}=\frac{m}{m+\frac{1}{N}}.\qquad \frac{\Delta\lambda}{\lambda}=\frac{1}{mN}$$. 分辨本领.

级数 ↑ 周期数 ↑

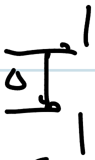
$m\uparrow.\ N\uparrow$



最高级数. ①

$\{ka\sin\theta\}_{max}=ka.$

$m\le ka.$ $\quad a\uparrow$ 光源极限.



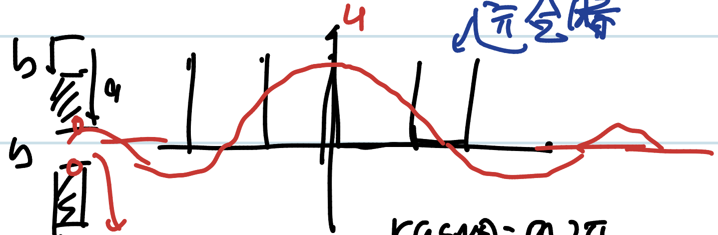
$Na\le\Delta.$

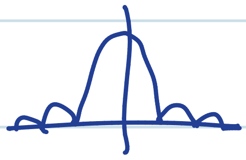
光源宽度Δ

$$\frac{\Delta\lambda}{\lambda}=\frac{1}{\frac{ka}{2\pi}\cdot\frac{\Delta}{a}}=\boxed{\frac{\lambda}{\Delta}}$$

m_{max} $\qquad N_{max}$

② m 大. 光强弱

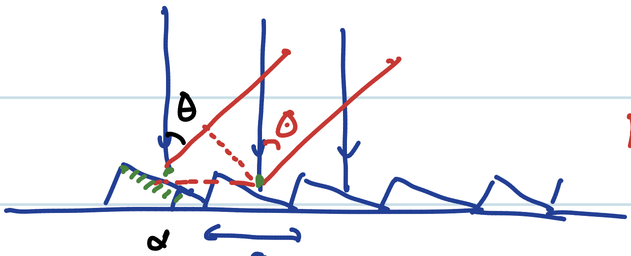




闪合膝

$ka\sin\theta=m\,2\pi.$

光强问题

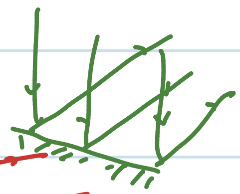


相邻两缝之间.

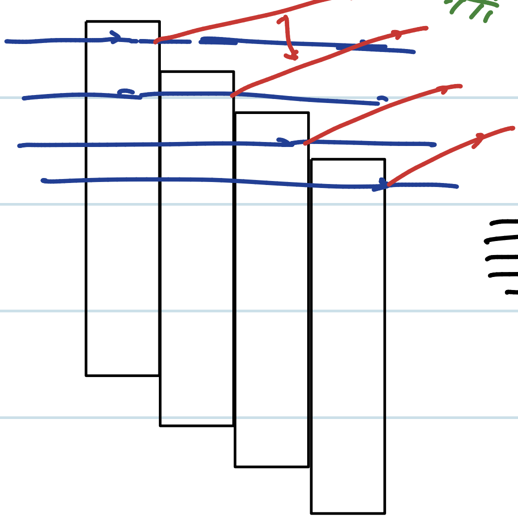
$ka\sin\theta=m\cdot2\pi$

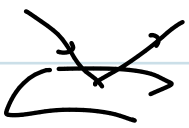
第 m 级.

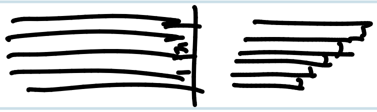
反射镜面



$\theta=2\alpha$ 时最亮.





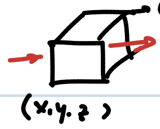


7. 电学.

7.1 静场属性.

$$\begin{cases}\nabla\cdot\vec E=\rho/\varepsilon_0 & (c)\\ \nabla\cdot\vec B=0\end{cases}\qquad\begin{cases}\nabla\times\vec E=-\frac{\partial\vec B}{\partial t} & (c)\\ \nabla\times\vec B=\mu_0\vec j\end{cases}$$

$$\int\nabla\cdot\vec A\,dV=\oint\vec A\cdot d\vec s$$



$$dxdydz\left[\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right].$$

$$RHS=E_x(x+dx,y,z)dydz-E_x(x,y,z)dxdy+c.c.$$

$$=\left(\frac{\partial E_x}{\partial x}\cdot dx\right)dydz+c.c.=LHS.$$

电场分量之间满足约束. $\partial_iE_i=0$

$$(\nabla\times\vec A)_z\cdot dxdy.$$

$$=(\partial_xA_y-\partial_yA_x)dxdy$$

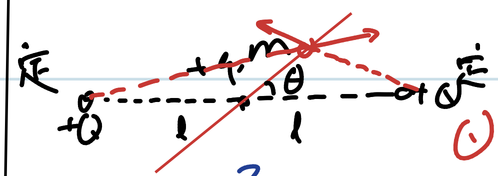
$$=[A_y(x+dx,y)-A_y(x,y)]dx\qquad=\oint\vec A\cdot d\vec l$$

$$-[A_x(x,y+dy)-A_x(x,y)]dy\qquad dxdy$$

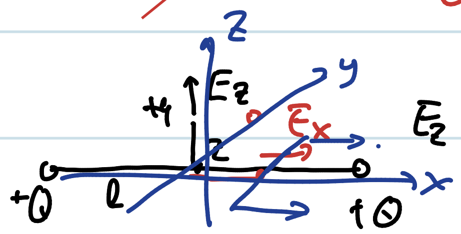
$\partial_iE_j-\partial_jE_i=0$ 还有一个约束.

求振动周期.

稳定条件.. $\tan\theta<\sqrt{2}.$



① 写 ΣF. ② 小展到第一阶



$$E_z=\frac{Q}{4\pi\varepsilon_0l^2}\cdot\frac{z}{l}\cdot2.$$

$z\ll l.$

$$E_x=\frac{Q}{4\pi\varepsilon_0l^2}\left[\frac{1}{(1+\frac{x}{l})^2}-\frac{1}{(1-\frac{x}{l})^2}\right]$$

$$=\frac{Q}{4\pi\varepsilon_0l^2}\cdot\frac{-4x}{l}.$$

E 展开. $\vec r=0$ $\vec E=0$

$y\leftrightarrow-y.$ E_x 不变.

$z\leftrightarrow-z.$ E_x 不变. E_y 不变

$$\begin{cases}E_x=\square x+\cancel{\square y}+\cancel{\square z}\\ E_y=\cancel{\square x}+\square y+\cancel{\square z}\\ E_z=\cancel{\square x}+\cancel{\square y}+\square z\end{cases}$$

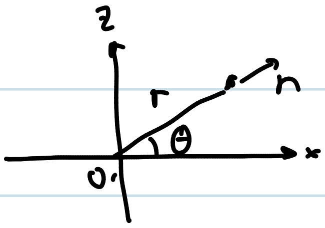
$\partial_jE_i=\partial_iE_j$

$y\leftrightarrow z.$

$E_x=ax$, $E_y=by$. $E_z=bz$

$\sum\partial_i E_i=0$ $\quad a+b+b=0$.

$E_z=\frac{Q}{4\pi\varepsilon_0 R}\cdot\frac{2z}{R}$. $\quad b=\frac{2Q}{4\pi\varepsilon_0 R^3}$. $\quad a=-\frac{4Q}{4\pi\varepsilon_0 R^3}$



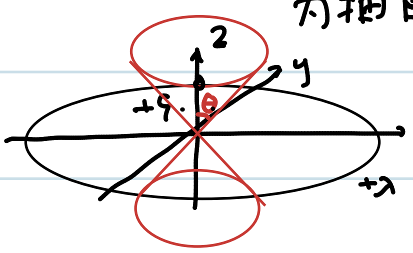
(r,θ) $\quad r\ll l$

$$\begin{cases}E_x=-2b\cdot r\cos\theta\\ E_z=b\, r\sin\theta\end{cases}$$

$E_n=E_x\cos\theta+E_z\sin\theta=b\,r\cdot(-2\cos^2\theta+\sin^2\theta)<0$.

稳定: $-2\cos^2\theta+\sin^2\theta<0$ $\quad \tan\theta<\sqrt2$.

例124. 均匀带电圆环. +q在过中心的以z为轴圆锥面上. 求在小振动下稳定条件



$$\begin{cases}E_x=\square x+\cancel{\square y}+\cancel{\square z}\\ E_y=\cancel{\square x}+\square y+\cancel{\square z}\\ E_z=\cancel{\square x}+\cancel{\square y}+\square z\end{cases}$$

$y\leftrightarrow -y$ 镜面对称. $E_y\leftrightarrow -E_y$. $E_x\leftrightarrow E_x$. $E_z\leftrightarrow E_z$

$z\leftrightarrow -z$ 镜面对称. $E_z\leftrightarrow -E_z$. $E_x\leftrightarrow E_x$ $E_y\leftrightarrow E_y$

$y\leftrightarrow x$ 对称 $\quad E_x=ax$. $E_y=ay$. $E_z=bz$.

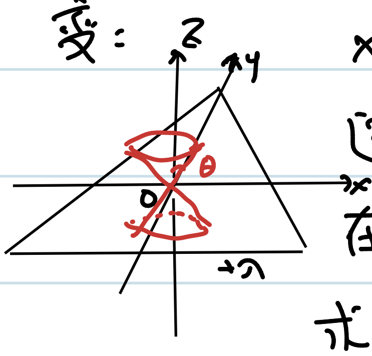
$\partial_i E_i=0$. $\quad a+a+b=0$. $\quad a=-\frac{b}{2}$. $\quad b>0$

$\begin{cases}x=r\sin\theta\\ z=r\cos\theta\end{cases}$ $\quad E_n=E_z\cos\theta+E_x\sin\theta$

$=r\cdot b(\cos^2\theta-\frac12\sin^2\theta)<0$

$\tan\theta>\sqrt2$.

变: x-y平面正三角形边上带电+Q. 过中心O垂线为z轴. 在O上+q. 在一个以z轴的锥面上. 求稳定条件.



展开 $\varphi(x,y,z)=\varphi_0+\underbrace{\square x+\square y+\square z}_{=0}+\square x^2+\square y^2+\square z^2\cancel{+\square xy}\cancel{+\square yz}\cancel{+\square zx}$.

$y\leftrightarrow -y$ φ不变 $\quad z\leftrightarrow -z$, φ不变

$\varphi(x,y,z)=ax^2+by^2+cz^2$.

转120°. φ不变.

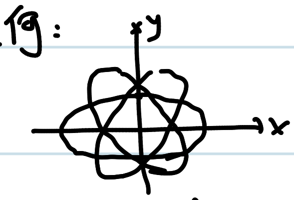
转θ: $\begin{cases}x=x'\cos\theta+y'\sin\theta\\ y=-x'\sin\theta+y'\cos\theta\end{cases}$ $\quad$ z不变.

$\varphi(x',y')=a(x'\cos\theta+y'\sin\theta)^2+b(-x'\sin\theta+y'\cos\theta)^2$

$=(a\cos^2\theta+b\sin^2\theta)x'^2+(a\sin^2\theta+b\cos^2\theta)y'^2$

$=x'^2+y'^2$ $\quad\Leftrightarrow\quad a=b$.

几何:



等φ值. 转120°不变. $\Rightarrow$ 正圆 $a=b$

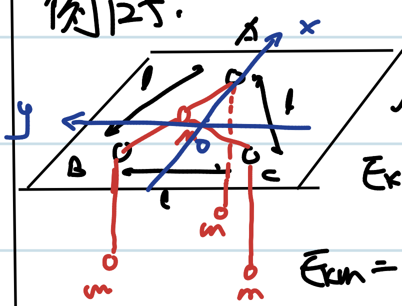
$\varphi=ax^2+ay^2+cz^2$ $\quad c<0$

$\partial_i E_i=0$. $\sum\partial_i^2\varphi_i=0$, $\quad a+a+c=0$ $\quad c=-2a$.

令 $\sqrt{x^2+y^2}=\tan\theta z$. $\quad \tan\theta>?$.

$\varphi=(a\tan^2\theta-2a)z^2>0$. 稳定.

例125. 求在扰动下. M的运动.

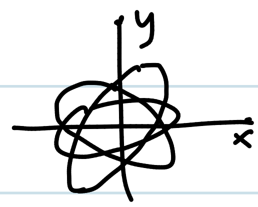


$M:(x,y)$ $\quad(\dot x,\dot y)$

$E_{kM}=\frac12 M\dot x^2+\frac12 M\dot y^2$

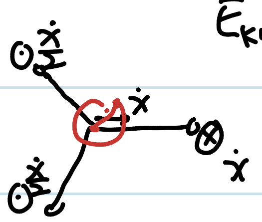
$E_{km}=\square\dot x^2+\square\dot x\dot y+\square\dot y^2$

$r\sim O(1)$ $\quad u\sim O(1)$ $\quad$ 在$\dot x,\dot y$平面上 E_{km}等值线 椭圆.



转120°不变. $\Leftrightarrow$ 正圆.

$E_{km}=a\dot x^2+a\dot y^2$

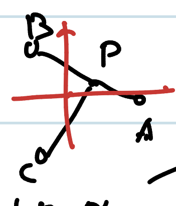


$\dot x$✓ $\dot y=0$. $\quad E_{km}=\frac12 m\dot x^2+\frac12 2m(\frac{\dot x}{2})^2$

$=\frac12\cdot\frac32 m\dot x^2$ $\quad a=\frac34 m$

$E_k=\frac12(M+\frac32 m)\cdot(\dot x^2+\dot y^2)$.

位置的变化对速度二阶. $E_k\propto$速度². 不计.

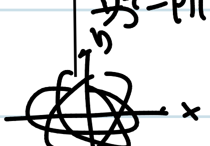


$E_p=C-mg[|AP|+|PB|+|PC|]$

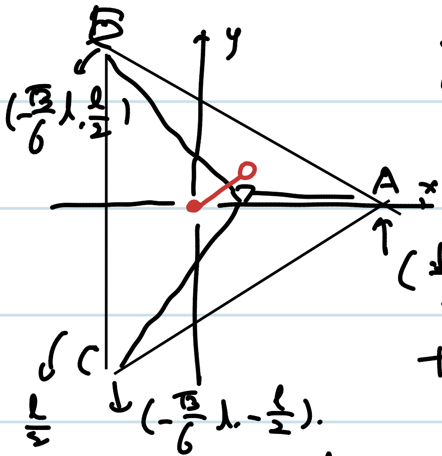
$=C\cancel{+\square x+\square y+\square z}$ ← 原点平衡

$+\square x^2+\square xy+\square y^2$

势二阶



$E_p=C+kx^2+ky^2$



取$(x,0)$

$|AP|+|PB|+|PC|$

$= (\frac{\sqrt{3}l}{3}-x)$

$+2\sqrt{(x+\frac{\sqrt{3}l}{6})^2+(\frac{l}{2})^2}$ $\qquad \frac{l^2}{4}+\frac{3}{36}l^2$

$= \frac{\sqrt{3}l}{3}-x+2\cdot\sqrt{(\frac{l}{\sqrt{3}})^2+2\cdot x\frac{\sqrt{3}l}{6}+x^2}$

$= \frac{\sqrt{3}l}{3}-x+2\frac{l}{\sqrt{3}}(1+\sqrt{3}\frac{x}{l}+3(\frac{x}{l})^2)^{1/2}$

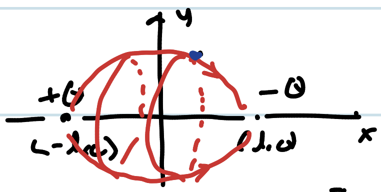
$= \frac{\sqrt{3}l}{3}-x+\frac{2l}{\sqrt{3}}(1+\frac{1}{2}\sqrt{3}\frac{x}{l}+\frac{1}{2}\cdot 3\frac{x^2}{l^2}+\frac{\frac{1}{2}(-\frac{1}{2})}{2}\cdot\frac{3x^2}{l^2})$

$= \sqrt{3}l+0\cdot x+\frac{2l}{\sqrt{3}}\cdot\frac{9}{8}\cdot\frac{x^2}{l^2}$ $\qquad \frac{3}{2}-\frac{3}{8}$

$k=\frac{3\sqrt{3}}{4\times 3}$. $\qquad E_p = mg\cdot\frac{3\sqrt{3}}{12}l\cdot(\frac{x^2}{l^2}+\frac{y^2}{l^2})$

$E_k=\frac{1}{2}(M+\frac{3m}{2})(x^2+y^2)$.

例126. 计算电场线

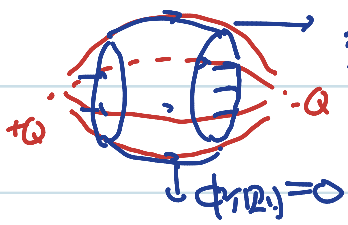


$E_x = \frac{x-l}{[(x-l)^2+y^2]^{3/2}}\cdot\frac{Q}{4\pi\varepsilon}+\frac{x+l}{[(x+l)^2+y^2]^{3/2}}\frac{Q}{4\pi\varepsilon_0}$

$E_y=\frac{Q}{4\pi\varepsilon_0}\left\{\frac{y}{[(x-l)^2+y^2]^{3/2}}+\frac{y}{[(x+l)^2+y^2]^{3/2}}\right\}$

记电场线 $y=f(x)$

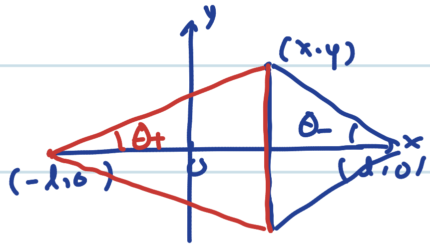
$y'=\frac{E_y}{E_x}=\frac{+}{+}$ 难.



对其高斯. 前后面电通量相同.

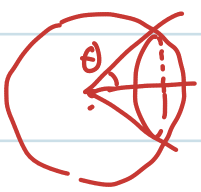
$\phi=C \iff$ 电场线

$\phi(D)=0$



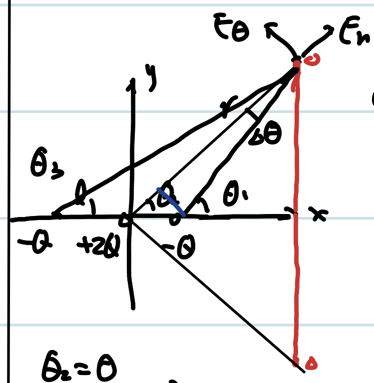
$\phi=\frac{q}{\varepsilon_0}(\frac{1-\cos\theta_+}{2}+\frac{1-\cos\theta_-}{2})$

$\frac{x-l}{[(x-l)^2+y^2]^{1/2}}+\frac{x+l}{[(x+l)^2+y^2]^{1/2}}=C$



$\phi=\frac{q}{\varepsilon_0}\cdot\frac{\Omega}{4\pi}=\frac{q}{\varepsilon_0}\frac{1-\cos\theta}{2}$

例127. 电四极子在远场电场线.



$\phi=C$

$\phi=(-\frac{1-\cos\theta_1}{2}+2\frac{1-\cos\theta_2}{2}-\frac{1-\cos\theta_3}{2})\frac{Q}{\varepsilon_0}$

$=[\frac{1}{2}\cos\theta_1-\cos\theta+\frac{1}{2}\cos\theta_2]\frac{Q}{\varepsilon_0}$

$\theta_2=\theta$ $\qquad \theta_2+\delta\theta=\theta_1,\ \Delta\theta=\frac{l\sin\theta}{r}$

$\cos\theta_1=\frac{r\cos\theta-l}{\sqrt{(r\cos\theta-l)^2+r^2\sin^2\theta}}=\frac{r\cos\theta-l}{\sqrt{r^2-2rl\cos\theta+l^2}}$

$=[\cos\theta-\frac{l}{r}]\cdot(1-2\cos\theta\frac{l}{r}+\frac{l^2}{r^2})^{-1/2}$

$=[\cos\theta-\frac{l}{r}](1+\cos\theta\frac{l}{r}-\frac{1}{2}\frac{l^2}{r^2}+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2}4\cos^2\theta\frac{l^2}{r^2})$

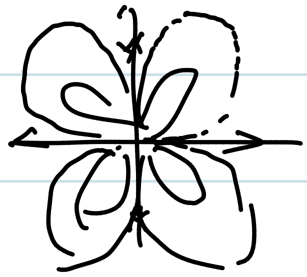
$=\cos\theta-\frac{l}{r}+\cos^2\theta\frac{l}{r}+(-\cos\theta\frac{l^2}{r^2})+\cos\theta(-\frac{1}{2}+\frac{3}{2}\cos^2\theta)\frac{l^2}{r^2}$

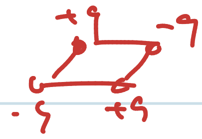
$=\underbrace{\cos\theta}_{C}+(\underbrace{-1+\cos^2\theta}_{0})\frac{l}{r}+(-\frac{3}{2}\cos\theta+\frac{3}{2}\cos^3\theta)\frac{l^2}{r^2}+\square$.

$C=\cos\theta_1-2\cos\theta+\cos\theta_3$

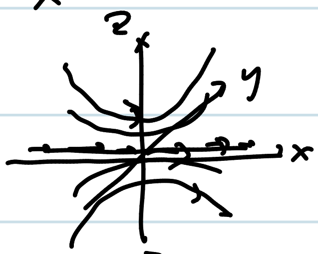
$=\frac{l^2}{r^2}(-3\cos\theta+3\cos^3\theta)$

$r^2=A\cos\theta\sin^2\theta$.



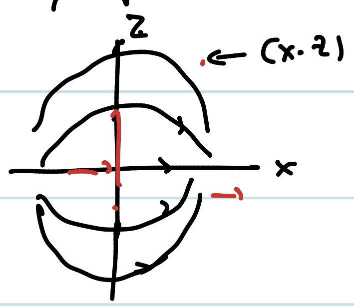


变. 平行于 y 轴, $E_x=\frac{1}{a-bx^2}$ $\quad b>0,\ a>0$



令 $x=0, y>0, z=0$. $\vec{E}=0$

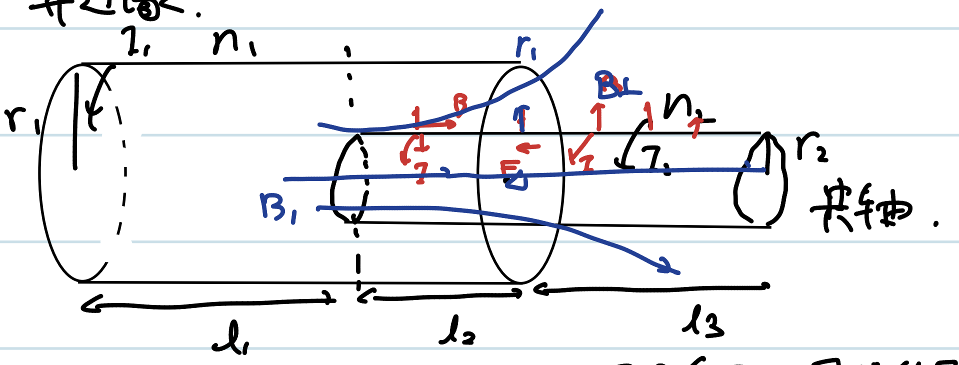
求 $|x|<\frac{a}{b}$. 电场线



$z(x)\cdot E_x=C$.

$z(x)\frac{1}{a-bx^2}=C$.

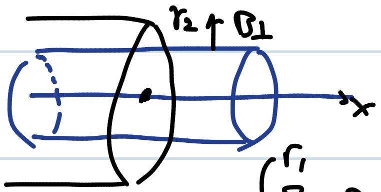
算边缘.



$l_1, l_2, l_3 \gg r_1, r_2$ 求小圆柱受到大圆柱作用力

$dW_{外力} + dW_{电源} = dW_{磁}$ 恒流源

$dW_\varepsilon = U\,dQ = U\,dt\,\frac{dQ}{dt} = d\psi \cdot I$.



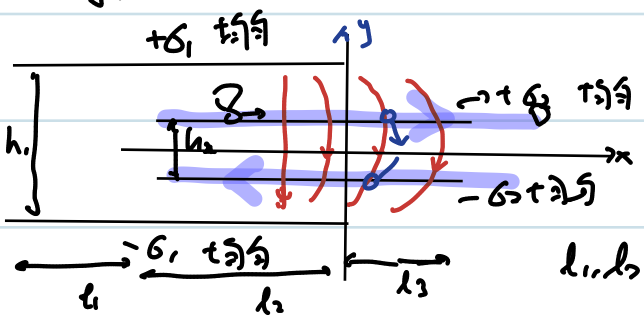
$F = \int_{x \ll -r_1}^{r_1} I_1 B_\perp 2\pi r_2 \, dx\, n_2$.

$F = I_2 n_2 \cdot \int B_\perp 2\pi r_2 \, dx$

$= I_2 n_2 \cdot \Phi_{(x)} = I_1 n_2 (\Phi_2 - \Phi_1)$

$= I_2 n_2 \cdot \pi r_2^2 \cdot n_1 \mu_0 I_1$

例128



$l_1, l_2, l_3 \gg h_1, h_2$

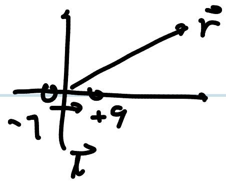
求小电容器受力. 宽 D.

$\Sigma F_x = \int E \cdot \sigma\, ds = \int_{x_L \ll h_2}^{x_R \gg h_2} E_x(x, \frac{h_2}{2}) \cdot \sigma_2 D \cdot dx + \int_{x_R \gg h_2}^{x_L \ll -h_2} -E_x(x, -\frac{h_2}{2})\, \sigma_2 D\, dx$

$= \sigma_2 D \cdot [\oint \vec E \cdot d\vec l - (-\frac{\sigma_1}{\varepsilon_0} \cdot h_2) + 0]$

$= \sigma_2 D \cdot \frac{\sigma_1}{\varepsilon_0} \cdot h_2$

7.2 偶极子.



$\varphi(\vec r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r + \hat r \cdot \vec l}\right)$

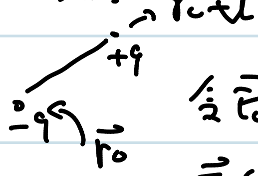
$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat r \cdot \vec l}{r^2} = \frac{\vec p \cdot \vec r}{4\pi\varepsilon_0 r^3}$

$\vec E = -\nabla \varphi(\vec r)$; $E_i = -\frac{p_j}{4\pi\varepsilon_0} \partial_i\left(\frac{r_j}{r^3}\right)$

$E_i = -\frac{p_j}{4\pi\varepsilon_0}\left[\frac{\delta_{ij}}{r^3} - 3\frac{r_j r_i}{r^5}\right]$.

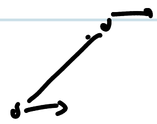
$\vec E = \frac{3(\vec p \cdot \vec r)\vec r - \vec p r^2}{4\pi\varepsilon_0 r^5}$. $E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$, $E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$

受力. ① $\vec F = \vec E(\vec r_0 + \vec l) q - \vec E(\vec r_0) q$.



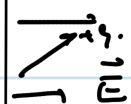
令 $\vec E_0 = \vec E(\vec r_0)$

$\vec E(\vec r_0 + \vec l) = \vec E_0 + \sum_i \frac{\partial \vec E}{\partial x_i} l_i$



$F_j = q \sum_i \frac{\partial E_j}{\partial x_i} l_i = \sum_i p_i \partial_i E_j$

$F_x = p_x \partial_x E_x + p_y \partial_y E_x + p_z \partial_z E_x$; $\vec F = \vec p \cdot \nabla \vec E$



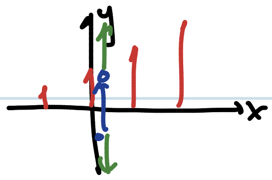
② $U = q\varphi(\vec r_0 + \vec l) - q\varphi(\vec r_0)$.

$= q \sum_i (\partial_i \varphi) l_i = \vec p \cdot \nabla \varphi = -\vec p \cdot \vec E$

p_j常数. $\vec F = -\nabla U$; $F_i = +\partial_i(\vec p \cdot \vec E) = \partial_i(p_j E_j)$

$F_j = p_i \partial_j E_i$ $p_i \partial_j E_i$; $p_i \partial_i E_j$

例.127. $\vec E = (\alpha x + \beta)\hat y$, $\vec p = p_0 \hat y$ 在[illegible]动



① $F_j = p_i \partial_i E_j$ $\Sigma \vec F = 0$ ✓

随y增   不随y变化

② $-\vec p \cdot \vec E = -p_0(\alpha x + \beta)$

$\vec F = -\nabla U = p_0 \alpha \cdot \hat x$

$U = -\vec p \cdot \vec E = \sum q_i \varphi_i$ 保守场 静电. $\nabla \times \vec E = 0$

$\partial_x E_y = \alpha$ $\partial_y E_x = 0$. $\nabla \times \vec E \neq 0$

不是静电. 不可以用 $U = -\vec p \cdot \vec E$.

$F_j = p_i \partial_j E_i = p_i \partial_i E_j \Leftrightarrow \partial_j E_i = \partial_i E_j$

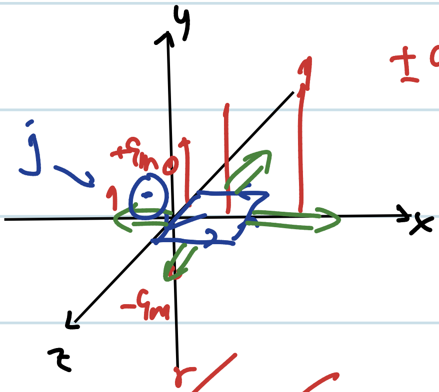
$\Leftrightarrow \nabla \times \vec E = 0$

例130. $\vec{B}=(\alpha x+\beta)\hat{y}$. $\vec{m}=m_0\hat{y}$. 求受力.

磁矩 $I\cdot S$.

$\vec{F}=\nabla(\vec{m}\cdot\vec{B})$ ✓

$\vec{F}=(\vec{m}\cdot\nabla)\vec{B}$ ✗

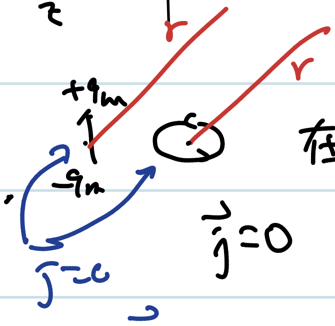


$\pm q_m$. $\vec{F}=\vec{m}\cdot\nabla\vec{B}$ $F=0$

$I\cdot B(x+\Delta x,y)\cdot\Delta y-IB(x,y)\Delta y$

$=I\cdot\frac{\partial B}{\partial x}\cdot\Delta x\Delta y=IS\frac{\partial}{\partial x}B$

$=\frac{\partial}{\partial x}(\vec{B}\cdot\vec{m})$. 磁常数.



在远场相同. 场源远离物体.

$\vec{j}=0$ $\vec{j}(\vec{r})\to 0$. $|\vec{r}|\gg l$

或 $\varepsilon_0\frac{\partial\vec{E}}{\partial t}\to 0$

$\vec{j}=0\iff\nabla\times\vec{B}=0\iff\partial_i B_j=\partial_j B_i$

$\vec{F}=\int\vec{j}\times\vec{B}\,dv$ ------ $=\nabla[\vec{B}\cdot I\vec{S}]$

$B_j=B_{0j}+\partial_i B_j r_i$ $I\vec{S}=I\frac{1}{2}\int\vec{r}\times d\vec{r}=\frac{1}{2}\int\vec{r}\times\vec{j}\,dv$

7.3. 自由电荷在场中能量.

$W=\sum q_i U_i$ 对自由电荷.

q_i在一个已知, 不受外场中移动.

外力作功 恒对应.

$W=\frac{1}{2}\sum_i q_i U_i$ 对所有电荷. U_i中不包括 q_i在r_i处电势.

相互作用. q与世界, 场源有相互作用.

外力动q_i时, 感应电荷也会动

$\int\frac{1}{2}\varepsilon_0E^2dv$ → 约束外场, 场源不动 → $\sum U_i q_i$
→ 场源也动, 考虑电势能 $\frac{1}{2}\sum U_i q_i$

$\int\frac{1}{2}\varepsilon_0E^2dv$

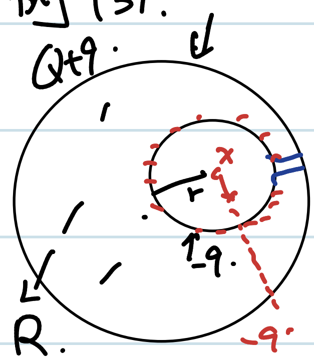
H.下不要求. $\int\frac{1}{2}\varepsilon_0E^2dv$

$=-\int\frac{1}{2}\varepsilon_0\vec{E}\cdot\nabla\varphi\,dv=-\frac{1}{2}\varepsilon_0\int\nabla\cdot(\varphi\vec{E})dv+\frac{1}{2}\varepsilon_0\int\varphi\nabla\cdot\vec{E}\,dv$

$=-\frac{1}{2}\varepsilon_0\oint\varphi\vec{E}\cdot d\vec{S}+\frac{1}{2}\int\rho\varphi\,dV$

$=\frac{1}{2}\sum_i\varphi_i q_i$ 在电荷边界上 $+\frac{1}{2}\int\rho\varphi\,dv$ 体电荷.

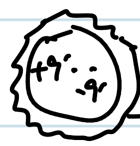
例131. ① q电势能 ② 相互作用能. ③ $q\to\infty$ 做功



① $Q+q$. $U=\frac{k(Q+q)}{R}$.

$q'=-\frac{r}{x}q$. $l=\frac{r^2}{x}$

$q'\neq q$? $-q$对中心. $U=-\frac{kq}{r}$. $-q$和$-q'$ 对内电场一样. 电势差一样.



$+q$ 高斯 $q_{感}=q_{像}$

$q=q$.

$U_{-q'}(x)-U_{q'}(0)=U_q(x)-U_q(0)$

$-\frac{kq'}{(l-x)}$ $\cancel{-\frac{kq'}{l}}$ $\cancel{-\frac{kq}{r}}$

$U_{-q}(x)=-\frac{k\cdot q\cdot\frac{r}{x}}{l-\frac{r^2}{x}}$

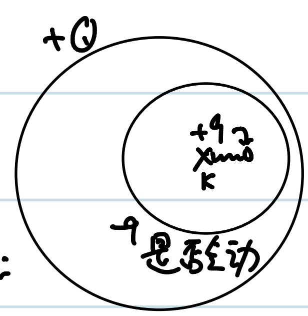
$U_{电势}=q\cdot\left[\frac{k(Q+q)}{R}-\frac{kq\frac{r}{x}}{l-\frac{r^2}{x}}\right]+$

$W=\frac{1}{2}\sum q_i U_i$

$=\frac{1}{2}q\cdot\left[\frac{k(Q+q)}{R}-\frac{kq\frac{r}{x}}{l-\frac{r^2}{x}}\right]$ $\frac{k}{2R}$

$+\frac{1}{2}Q\cdot\frac{k(Q+q)}{R}$

$W(Q,0)-W(Q,q)=$作功.



平衡位 $x=0$处, 振动. 求周期=? 小振动

E_k+E_p. $\frac{1}{2}kx^2+W_电$?

相互作用 电势能?

是否动 屏蔽→0

驰豫时间?