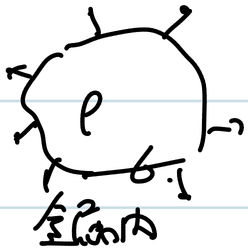


$\vec{j}=\vec{E}\sigma$

$\frac{dQ}{dt}=-\oint \vec{j}\cdot d\vec{s}=-\sigma\oint\vec{E}\cdot d\vec{s}$

$=-\frac{\sigma}{\varepsilon_0}Q.\quad Q=Q_0e^{-\frac{\sigma}{\varepsilon_0}t}$

$t\sim\frac{\varepsilon_0}{\sigma}\quad Q\to Q/e.\quad T_{电}=\frac{\varepsilon_0}{\sigma}$ 特征时间.

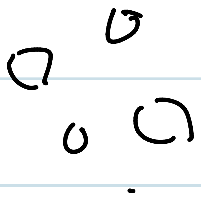
$T_K=2\pi\sqrt{\frac{m}{k}}$.

① $T_K\gg T_{电}$. 随时静电平衡 感应电荷一起动

$W\to$ 相互作用能.

② $T_K\ll T_{电}$. 感应电荷不动. 取电势能. 外场不动

格林互易

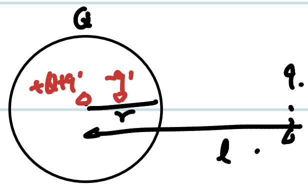


N个导体. 每个导体上带电 Q_i

电势 φ_i: 线性叠加 考虑感应

$\varphi_i(Q_1,Q_2\cdots Q_n)=\sum_{j=1}^{N}a_{ij}\varphi_j\quad i=1\cdots N$

$a_{ij}=a_{ji}$ (积点电荷对自己).



$U_Q=\frac{KQ}{r}+\boxed{\frac{Kq}{l}}\qquad q'=q\frac{r}{l}$

$U_q=\frac{K(Q+q')}{l}+\frac{K(-q')}{l-r^2/l}$

$=\boxed{\frac{KQ}{l}}+\frac{K\cdot qr}{l^2}-\frac{Kqr}{l^2-r^2}$

证明. 用唯一性定理.

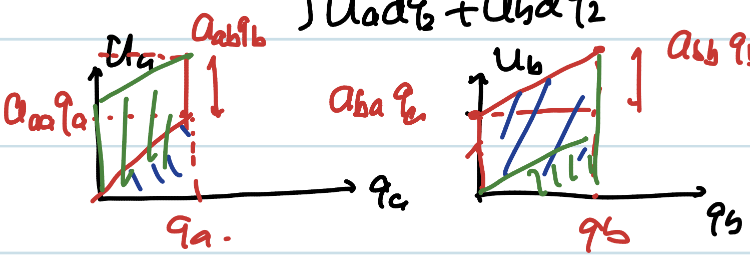
从两个为例. 加电池 慢加电压

q_a a U_a ; q_b b U_b

①. $q_a\ 0\to q_1\to q_1$

$q_b\ 0\to 0\to q_2$ 电源作功.

$\int U_adq_a+U_bdq_2$



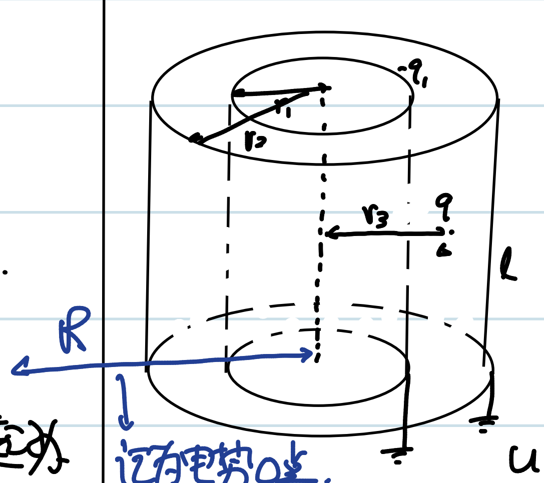
② $q_a\ 0\to 0\to q_1$

$q_b\ 0\to q_2\to q_2$

$W=\frac{1}{2}a_{aa}q_a^2+\frac{1}{2}a_{bb}q_b^2+a_{ba}q_bq_a\ /\ a_{ab}q_bq_a$

$\Rightarrow q_{ab}=q_{ba}$.

例 132. $l\gg r_1,r_2$.



电势→矩阵

$U_1=a_{11}(-q_1)+a_{12}(-q_2)+\boxed{a_{13}}q$

$U_2=a_{21}(-q_1)+a_{22}(-q_2)+\boxed{a_{23}}q$

$U_3=a_{31}(-q_1)+a_{32}(-q_2)+a_{33}q$

不算.

$U_1=0,\ U_2=0$

a_{31}: $\lambda=\frac{q_0}{l}.\quad E_x=\frac{q_0}{2\pi x\varepsilon_0 l}$.

设内筒有q_0电荷, $U_3=\frac{q_0}{2\pi\varepsilon_0 l}(-\ln r_3+\ln R)$

$a_{31}=a_{13}=\frac{1}{2\pi\varepsilon_0 l}(-\ln r_3+\ln R)$

同理 $a_{32}=a_{23}=\frac{1}{2\pi\varepsilon_0 l}(-\ln r_3+\ln R)$

$a_{11}=\frac{1}{2\pi\varepsilon_0 l}(-\ln r_1+\ln R)$

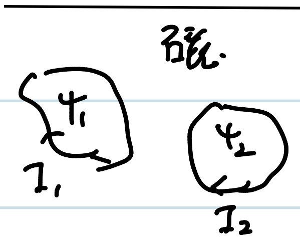
$a_{22}=\frac{1}{2\pi\varepsilon_0 l}(-\ln r_2+\ln R)$

$a_{12}=a_{21}=\frac{1}{2\pi\varepsilon_0}(-\ln r_2+\ln R)$

$0=(-\ln r_1+\ln R)(-q_1)+(-\ln r_2+\ln R)(-q_2)$
$+(-\ln r_3+\ln R)(q)$

$\Rightarrow$ 不发散. $q=q_1+q_2$; $\frac{q_1}{q_2}=\frac{+\ln r_2-\ln r_3}{-\ln r_1+\ln r_3}$

磁.

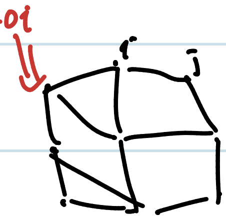


$\psi_i(I_1\cdots I_n)$
$=\sum_j a_{ij}I_j$

$a_{ij}=a_{ji}$

外电流 I_{0i} 电容.

电压 U_i, i,j之间电容 $C_{ij}=C_{ji}$



$i\to j\quad I_{ij}=(U_i-U_j)C_{ij}$ 不求和.

$I_{0i}=\sum_{j\neq i}I_{ij}=\sum_{j\neq i}C_{ij}(U_i-U_j)$

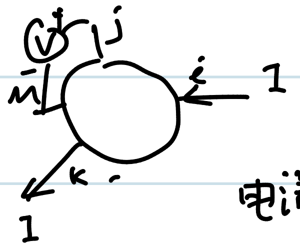
$=\sum_j C_{ij}(U_i-U_j)$

$\sum_j L_{ij}U_j=I_{0i}$ 令 $L_{ij}=\begin{cases}\sum_{j\neq i}C_{ij} & i=j\\ -C_{ij} & i\neq j\end{cases}$

$[\,][\,]=[\,]$.

2017暑1- HZ 决赛 055

电流入 i 入. k 流出;

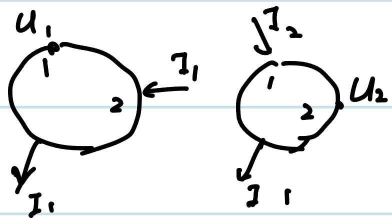


j.m 之间电压 $U_{jm}=\boxed{\ }I$ (i 入 k 出)

电流从 j 流入, m 流出. 相同.

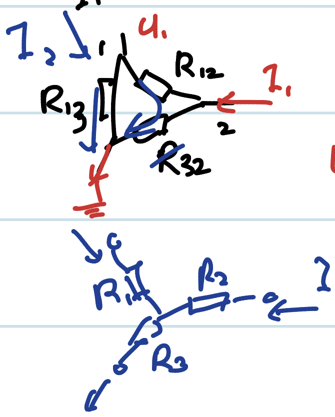
i.k 之间电压 $U_{ik}=\boxed{\ }I$ (j 入 m 出)

m.k 同一位.



$\frac{U_1}{I_1}=\frac{U_2}{I_2}$.

不要求.



$U_1=I_1\frac{R_{32}\cdot R_{13}}{R_{12}+R_{13}+R_{32}}$

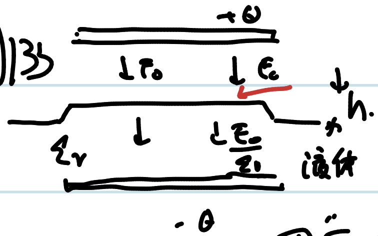
$U_2=I_2\cdot\frac{R_{13}\cdot R_{32}}{R_{12}+R_{13}+R_{32}}$

$U_1=I_1 R_3$ $\qquad R_3=\frac{R_{13}R_{32}}{R_{12}+R_{32}+R_{13}}$

$U_2=I_2 R_3$

7.4 有介质时静场能量

例13



$\rho gh=?$

① 受力. $\sigma=(E_0-\frac{E_0}{\varepsilon_r})\cdot\varepsilon_0$

用平均电场

$\rho gh=\sigma(E_0+E_0/\varepsilon_r)\frac{1}{2}=\frac{1}{2}\varepsilon_0(E_0^2-\frac{E_0^2}{\varepsilon_r^2})$ ✓

② 用虚功.

$\frac{1}{2}\varepsilon_0 E^2$

$\frac{1}{2}\vec{D}\cdot\vec{E}$

$p\cdot\Delta V=\Delta V(\frac{1}{2}\varepsilon_0 E^2-\frac{1}{2}\vec{D}\cdot\vec{E})$

$\Rightarrow p=\frac{1}{2}\varepsilon_0(E^2-\frac{E_0^2}{\varepsilon_r})$ 不同 ✗

解释以下能量:

① $\frac{1}{2}\vec{D}\cdot\vec{E}$ $\qquad \vec{E}$ $\quad$ − k +

相对弹簧能量. 介质自能

② $-\vec{p}\cdot\vec{E}$. 固定电偶极放在恒定外场中能量.

$(\Leftrightarrow)\ \Sigma U_i q_i$

③ $-\frac{1}{2}\vec{p}\cdot\vec{E}$. $=$ ①+② 全极化的电偶极放在恒定外场中能量

又: $\Leftrightarrow \Sigma\frac{1}{2}U_i q_i$

④ ⑤. $\frac{1}{2}\varepsilon_0 E^2$. $\frac{1}{2}\vec{D}\cdot\vec{E}$

出发点: $-\rho(\vec{E}+\vec{u}\times\vec{B})\cdot\vec{u}=\frac{\partial w}{\partial t}+\nabla\cdot\vec{S}$

功密度. 能量密度 能流

$-\rho\vec{E}\cdot\vec{u}=-\vec{j}\cdot\vec{E}=-(\frac{\nabla\times\vec{B}}{\mu_0}-\varepsilon_0\frac{\partial\vec{E}}{\partial t})\cdot\vec{E}$ $\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

$=\frac{1}{2}\varepsilon_0\frac{\partial E^2}{\partial t}-\frac{1}{\mu_0}(\nabla\times\vec{B})\cdot\vec{E}-\frac{1}{\mu_0}(\nabla\times\vec{E})\cdot\vec{B}+\frac{1}{\mu_0}(\nabla\times\vec{E})\cdot\vec{B}$

全微分

$=\frac{1}{2}\varepsilon_0\frac{\partial E^2}{\partial t}+\frac{1}{2\mu_0}\frac{\partial B^2}{\partial t}+\nabla\cdot(\vec{E}\times\frac{\vec{B}}{\mu_0})$

$\Rightarrow w=\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2\mu_0}B^2$. $\quad \vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$

取 j 为全部电流: $w=\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2\mu_0}B^2$.

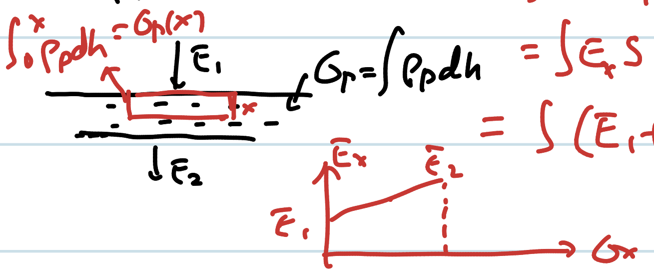
全电荷做功 + 储能 + 其它 $=0$

取 j 为自由电流. $w=\frac{1}{2}\vec{D}\cdot\vec{E}+\frac{1}{2\mu_0}\vec{H}\cdot\vec{B}$

自由电荷作功 + 储能′ + 其它 $=0$

两个储能相差, 自能项: $\frac{1}{2}\vec{p}\cdot\vec{E}$

受力:



$F=\int E S\cdot\rho_p dh$.

$\int_0^x\rho_p dh=\sigma_p(x)$ $\qquad \sigma_p=\int\rho_p dh$ $=\int E_x S\ d\sigma_x$

$=\int(E_1+\frac{\sigma_x}{\varepsilon_0})S\,d\sigma_x$

$\Rightarrow$ 取平均可行.

能量.

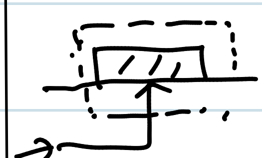
① 新建体系. 电荷是自由电荷. 大小与原全电荷

一样. 两边受力一样.

新体系 用 $\frac{1}{2}\varepsilon_0 E^2$ 虚功 ✓

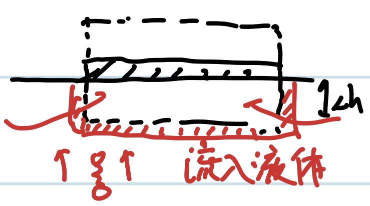
② 原体系. 用 $\frac{1}{2}\vec{D}\cdot\vec{E}$.

$\frac{1}{2}\vec{p}\cdot\vec{E}$ 极化能.



Δ 储能 $=$ 外力 + 消耗 ?

③ 用功方法.



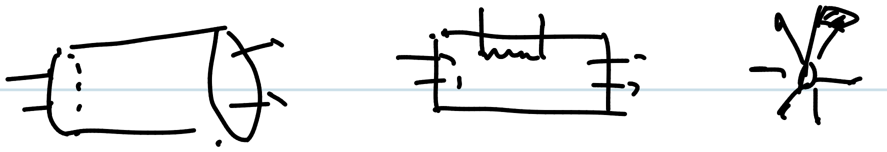
$\frac{1}{2}\vec{D}\cdot\vec{E}$

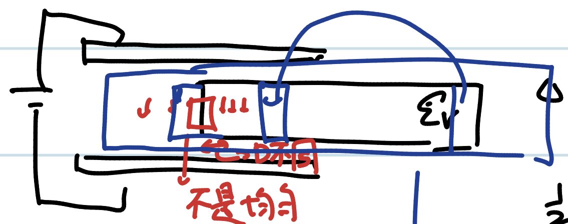
Δ贮能 = 流动 + 外力作功.

流动 → $\vec{E}\times\vec{H}$ 慢 H→0.

流动 → 液体流动. 液体自能 $\rho gh \to 0$ + $\frac{1}{2}\vec{D}\cdot\vec{E}$

外力作功 $= \Delta(\frac{1}{2}\vec{D}\cdot\vec{E})\Delta V - \frac{1}{2}\vec{P}\cdot\vec{E}\cdot\Delta V$ $\approx \Delta(\frac{1}{2}\varepsilon\vec{E}^2)\Delta V$

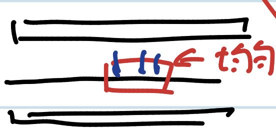




Δ(贮能) = Δ外力作功 + Δ电源.

$\frac{1}{2}\vec{D}\cdot\vec{E}$

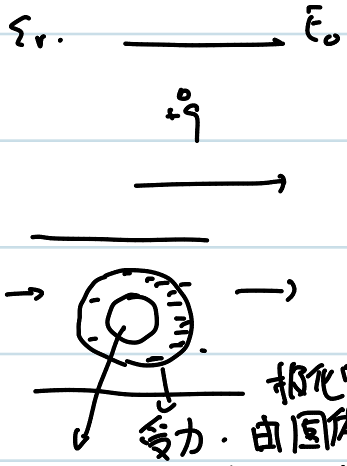
大圈不包含流入.



要算流出. D不均匀不能算.

例: 134.

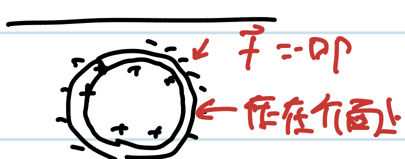
自由电荷 放在ε_r介质中. 给外场E_0 求受力.



极化电荷受力. 由固体承担.

外加金属上受力. 计算

液体



极化电荷受力最终出金属承担.

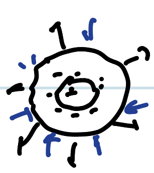
退极化场不同 ⇒ 极化电荷受力不同.

⇒ 金属上电荷分布不同.

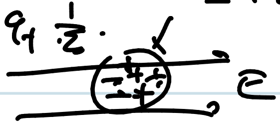
液体. $F = \frac{q}{\varepsilon_r}E_0$

$\vec{E} = \vec{D}/\varepsilon_0\varepsilon_r$ 总量 q/ε_r

$\oint\vec{D}\cdot d\vec{S} = q_f$. $\oint\vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_r}\cdot q_f\frac{1}{\varepsilon_0}$.

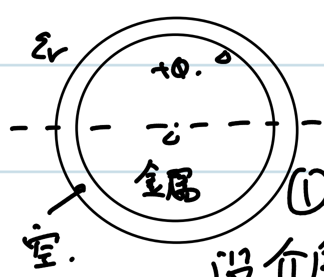


$q_总 = \frac{q}{\varepsilon_r}$



流体 电荷分布与原相同 自由电荷. $\Sigma F = E\cdot q/\varepsilon_r$

固体. 取成正球. r.



$\vec{E}(\vec{E}_0, Q) = \vec{E}(0,Q) + \vec{E}(\vec{E}_0, 0)$

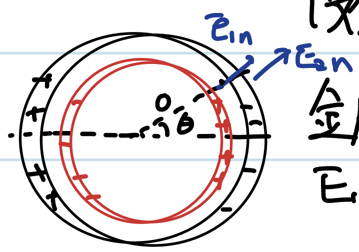
$\Delta(\frac{1}{2}\varepsilon_0 E^2)$: p. 平方.

$P(\vec{E}_0, Q) \neq \vec{P}(0,Q) + \vec{P}(\vec{E}_0, 0)$

① 只有外场E_0 无Q

设介质极化电荷为$\sigma_p(\theta) = -\sigma_{p0}\cos\theta$

设金属球表面电荷为$\sigma_f = \sigma_{f0}\cos\theta$



金属内 $E=0$.

$E_{1n} = \sigma_{f0}\cos\theta\frac{1}{\varepsilon_0}$

高斯.

$E_{2n} = E_{1n} + \frac{\sigma_p(\theta)}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\sigma_{f0} - \sigma_{p0})\cos\theta$.

边界: $E_{1n} = E_{2n}\varepsilon_r$.

叠加. E_{1n}由σ_{f0}. σ_{p0} E_0产生.

$$E_{1n} = \frac{2\sigma_{f0}\cos\theta}{3\varepsilon_0} + \frac{\sigma_{p0}\cos\theta}{3\varepsilon_0} + E_0\cos\theta.$$

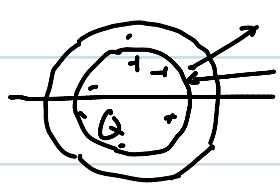
$$\begin{cases} \frac{2}{3}\sigma_{f0} + \frac{1}{3}\sigma_{p0} + E_0\varepsilon_0 = \sigma_{f0} \\ \sigma_{f0} = \varepsilon_r\cdot[\sigma_{f0} - \sigma_{p0}] \end{cases};\quad \sigma_{f0} = \frac{\varepsilon_r}{\varepsilon_r+1}\sigma_{p0}$$

$$E_0\varepsilon_0 = \frac{1}{3}\sigma_{f0} - \frac{1}{3}\sigma_{p0} = \frac{1}{3}\cdot\frac{\sigma_{f0}}{\varepsilon_r}.$$

$$\sigma_{f0} = \frac{3}{\cancel{8}}\varepsilon_r E_0\varepsilon_0,\quad \sigma_{p0} = \frac{3}{\varepsilon_r+1}\cdot E_0\varepsilon_0$$

$E_{1n} = 3\varepsilon_r E_0\cos\theta$.

只有Q. $E_{1n} = \frac{Q}{4\pi\varepsilon_0 R^2}$.



$p = \frac{1}{2}\varepsilon_0 E^2$

$$\Sigma F = \int\cos\theta\, p\, ds$$

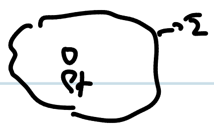
$\int_{-1}^{1}x^3dx \Rightarrow 0$ $\int_{-1}^{1}xdx = 0$

$$= \int_{-1}^{1}\cos\theta\cdot\frac{1}{2}\varepsilon_0\left[3\varepsilon_r E_0\cos\theta + \frac{Q}{4\pi\varepsilon_0 R^2}\right]^2\cdot R^2 2\pi d\cos\theta$$

$$= \int_{-1}^{1}\cos\theta\,\cancel{\tfrac{1}{2}}\varepsilon_0\cdot 3\varepsilon_r E_0\cos\theta\cdot\frac{Q}{4\pi\varepsilon_0 R^2}R^2 2\pi\, d\cos\theta$$

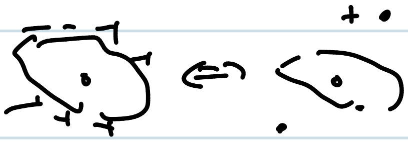
$\int x^2 dx$

7.5 电像法.



$\vec r\in\Sigma$. $\begin{cases}\varphi(\vec r)\ ✓ & 第一类\\ E_n(\vec r)\ ✓ & 第二类\end{cases}$

ρ 给定 ⇒ $\vec E(\vec r)$ $\vec r\in V$ 唯一解.

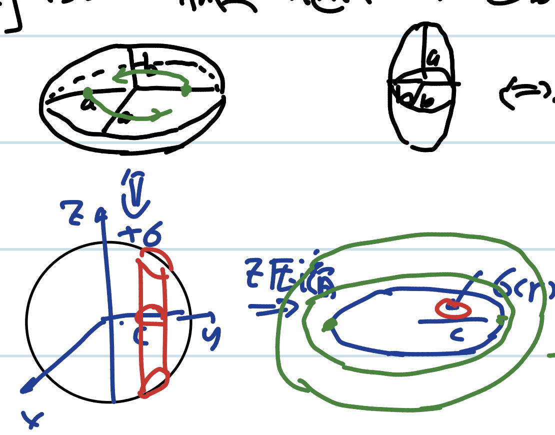


边界一样. E一样.

① $\varphi(\vec r)=C$. 边界等势面.

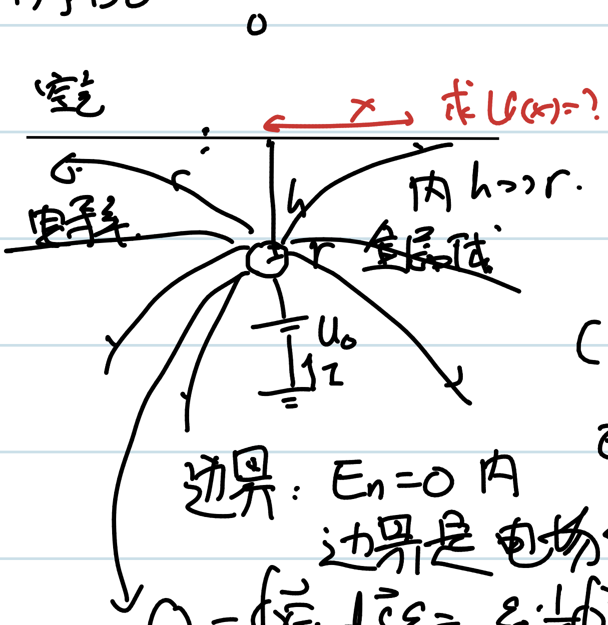
② $E_n(\vec r)=0$. 边界是电场线

例135. 孤立椭球. 求电容. (金属).



→ ±λ C 为焦点的椭球 等势.

例136.



稳. 恒.

j_n连续.

电荷受力 $=0$.

电荷分布与一个金属电荷半径分布相同.

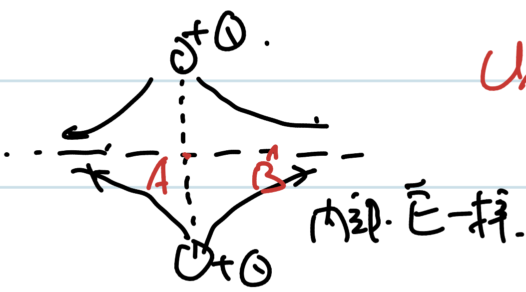
(除在地上)

边界: $E_n=0$ 内

$E_{n内}\sigma=E_{n外}\cdot 0$.

边界是电场线. 第二类.

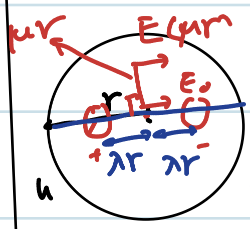
$Q=\oint\vec E\cdot d\vec S\,\varepsilon_0=\varepsilon\frac{1}{\sigma}\oint\vec j\cdot d\vec S=\frac{I}{\sigma}\varepsilon$



$U_A-U_B=\frac{2KQ}{h}-\frac{2KQ}{\sqrt{h^2+x^2}}$

内部 E一样.

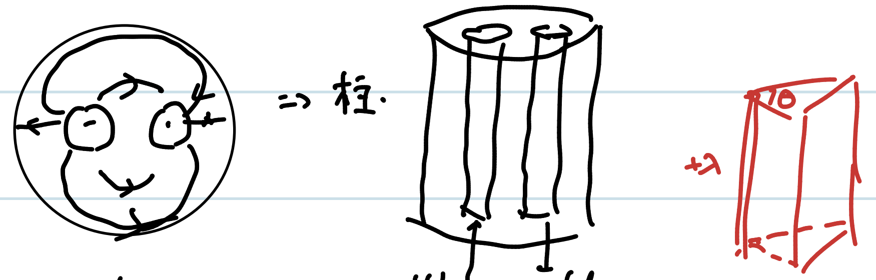
例137 圆. 半径 $r>>\lambda$. σ 电导率.



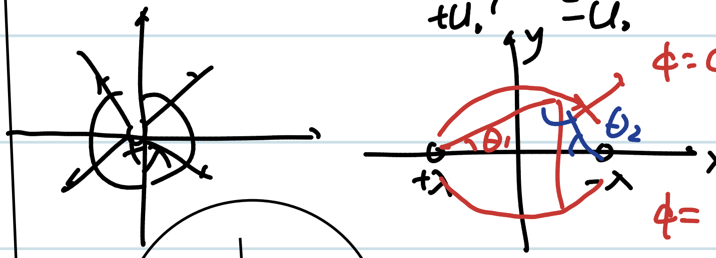
圆区域 ρ. 接入金属电极

$\rho<<r$. 位于 $\pm\lambda r$ 处.

接入 $+U_0$, $-U_0$, 求 $\frac{E(\mu r)}{E(0)}=?$



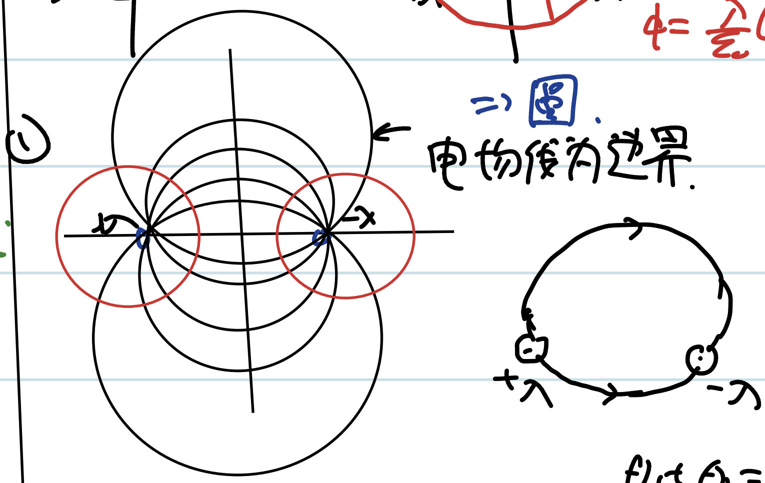
⇒ 柱.



$\phi=C$

$\frac{\phi}{l}=\frac{\lambda}{\varepsilon_0}\cdot\frac{\theta}{2\pi}$

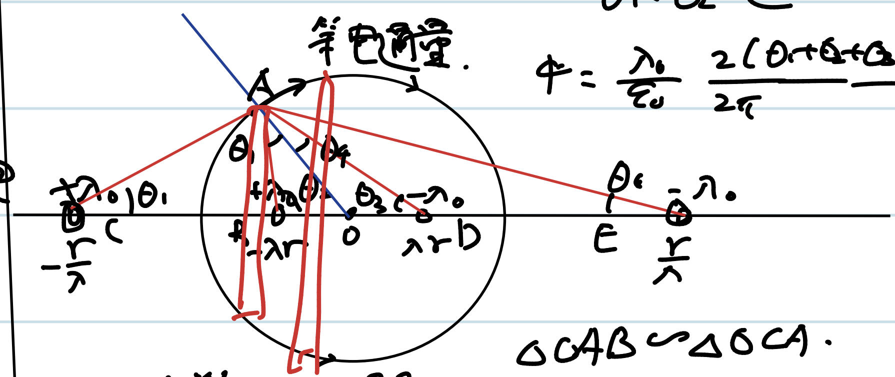
$\phi=\frac{\lambda}{\varepsilon_0}(2\theta_1+2\theta_2)$



⇒ 圆

电场线为边界.

$\theta_1+\theta_2=C$ ×



等电荷量.

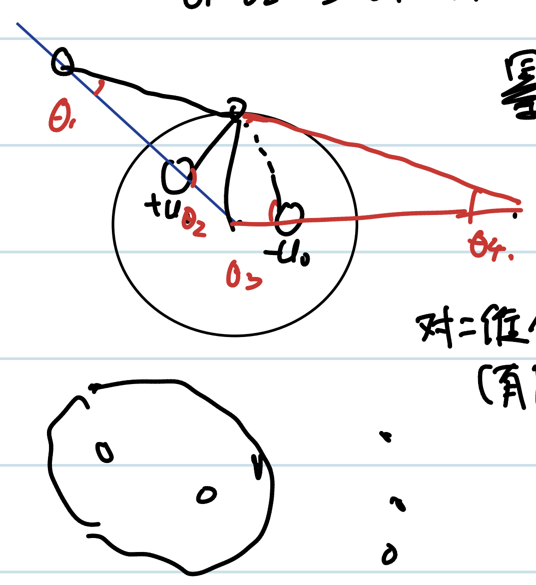
$\phi=\frac{\lambda_0}{\varepsilon_0}\frac{2(\theta_1+\theta_2+\theta_3+\theta_4)}{2\pi}$

入数 λ_0电荷密度

$\triangle CAB\backsim\triangle OCA$.

$\angle OCA=\angle BAO$

$\theta_1+\theta_2+\theta_3+\theta_4=\pi$.

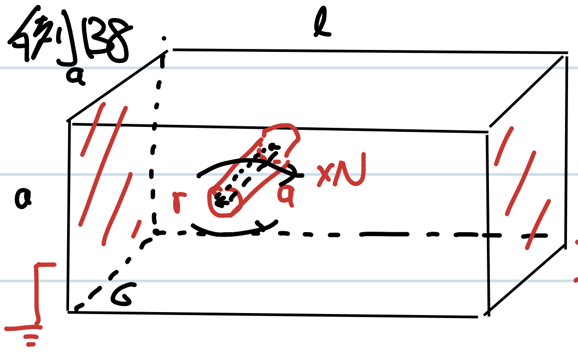


受力求稳.

对二维体系只存在电像.

(有限多个线电像).

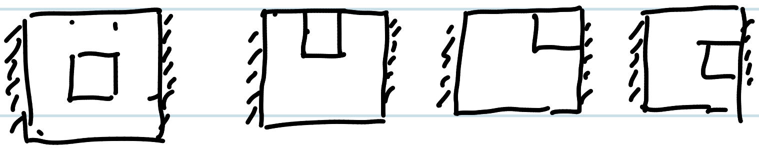
例138



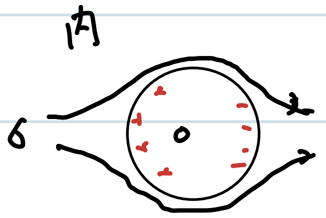
$R=\frac{l}{\sigma a^2}$

$\Delta R \ll R$

挖一个柱形洞，远离边界，r. a. 求 $\Delta R=?$



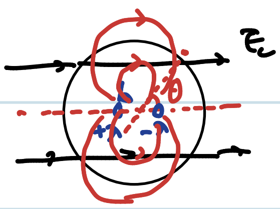
有几个模?



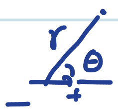
电场线是圆. / $E_n=0$

$E_{n外}=-E_0\cos\theta$

$\ln(1+x)\simeq x$

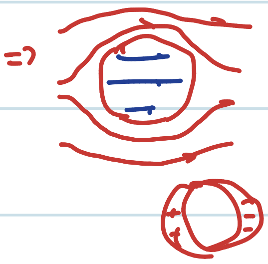


$\varphi=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r+b\sin\theta}{r}$



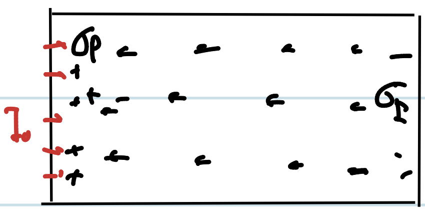
$=\frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{b}{r}\sin\theta$.

$E_n=\frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{b\cos\theta}{r^2}$.



$\Rightarrow \frac{\lambda b}{2\pi\varepsilon_0 r^2}\cos\theta=E_0\cos\theta.\quad \Rightarrow \lambda b=2\pi\varepsilon_0 r^2 E_0$

一个电偶极子会算 ×N 相加.



$\vec P$ 极化强度 $=\frac{\lambda a\cdot b\,N(\hat x)}{a^2\cdot L}=\frac{\lambda b N}{a l}\cdot(-\hat x)$

I_0 不变. 对每个柱子. 外场不变.

$\Rightarrow \sigma_P=P=\frac{\lambda b N}{a l}$.

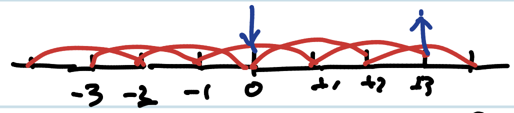
$\Rightarrow \Delta U=\frac{\sigma_P}{\varepsilon_0}\cdot l=\frac{\lambda b N}{a}=N\cdot\frac{1}{a}\cdot 2\pi r^2 E_0$

$I_0=E_0\cdot\sigma\cdot a^2$

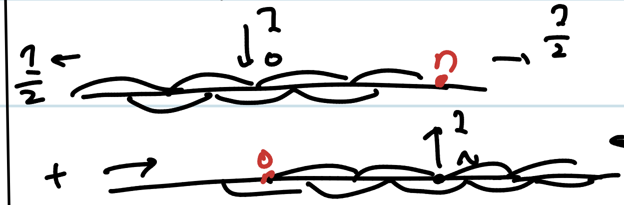
$\Rightarrow \Delta R=\frac{\Delta U}{I_0}=\frac{\frac{1}{a}\cdot 2\pi r^2}{\sigma a^2}$. $(N\to 1)$

8. 电路.

8.1 基尔霍夫. [mathematica].



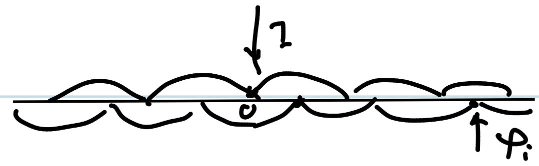
求 $R_{0n}=?$



$U_{0n}=\square I$

$U_{0n}=\square I$

+ $U_{总,0n}=2\square I.\quad R_{0n}=2\square.$



令 φ_i 对 i $4\varphi_i-\varphi_{i-1}-\varphi_{i+1}-\varphi_{i-2}-\varphi_{i-2}=0$

对 $i=1$ ....

对 $i=0$ ....

令 $\varphi_i=A\lambda^n$ 代入.

$\lambda^4+\lambda^3-4\lambda^2+\lambda+1=0$

解得: $\lambda_1=\lambda_2=1\quad \lambda_\pm=\frac{-3\pm\sqrt5}{2}$.

$\varphi=\sum A_i\lambda_i^n$ 重根

$=A_1 1^n+A_2 n 1^n+A_3\lambda_+^n+A_4\lambda_-^n$

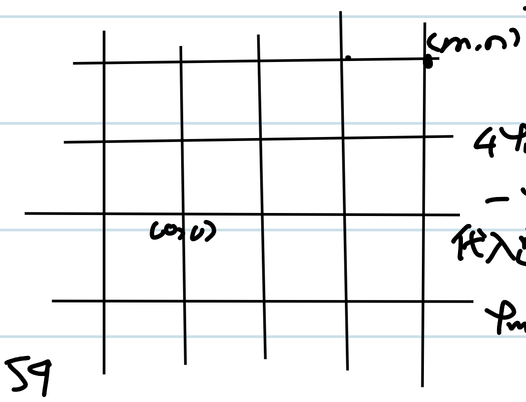
两个边界. $2(\varphi_0-\varphi_1)+2(\varphi_0-\varphi_2)=I$

$\varphi_1-\varphi_2+\varphi_1-\varphi_3+\varphi_1-\varphi_0+0=0$

A_1 任意. , $A_3=0$ φ 不会趋于 $+\infty$

$\varphi=C+A_2 n+A_4\lambda_-^n$ 代入.

$A_2=..\ A_4=...\quad \frac{2(\varphi_n-\varphi_0)}{I}=R_{0n}$



φ_{mn}.

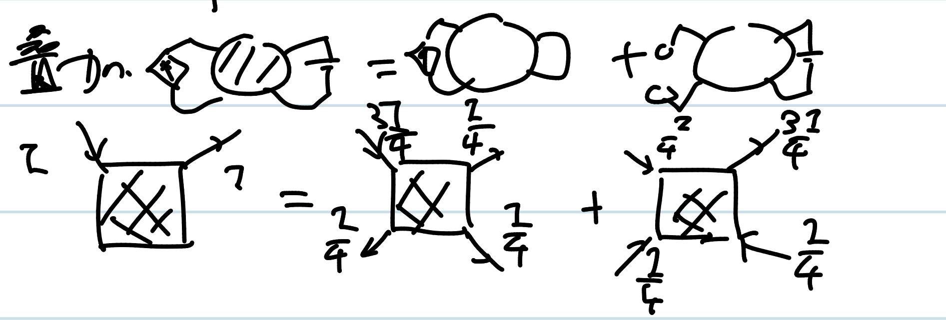
$4\varphi_{mn}-\varphi_{m,n-1}-\varphi_{m,n+1}-\varphi_{m-1,n}-\varphi_{m+1,n}=0$

代入边界.

$\varphi_{mn}=\sum A e^{am}e^{bn}$

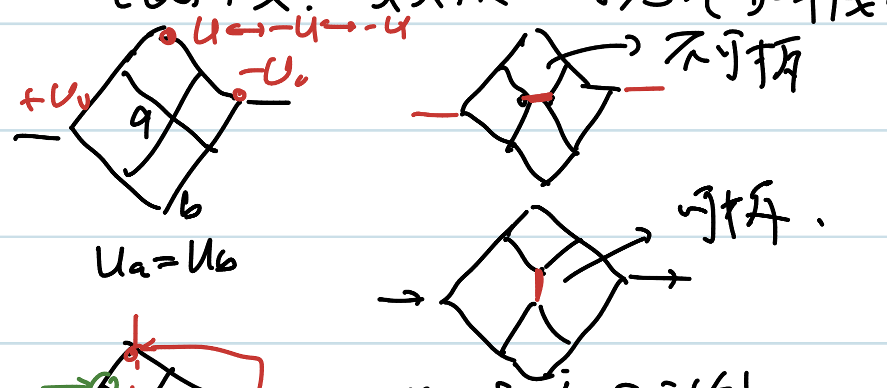
8.2 化简

叠加:

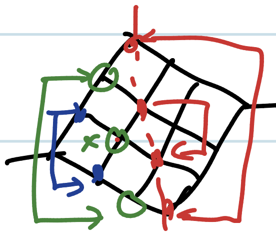


对称性: ① 保持电路不动上的变换.

电路不变. 被变换点等势. (加导线)



$U_a = U_b$



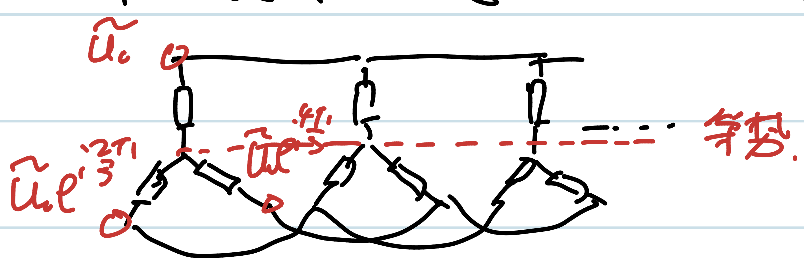
② 作C变换 再变换点.

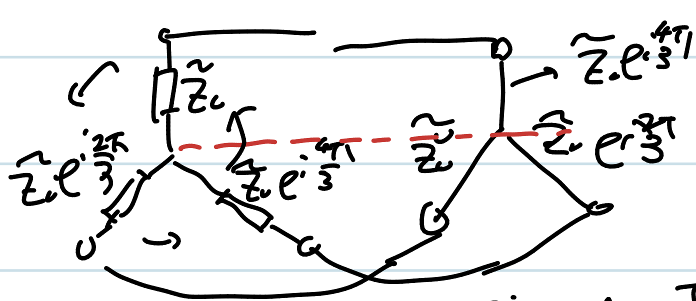
$q \leftrightarrow -q$. $U \leftrightarrow -U$. $I \leftrightarrow -I$.

保持电路不变 (正负交换)

不被换点等势 (中点等势)

对交流电. $\varphi \to \varphi + \frac{2\pi}{3}$ …… 保持电路不变 ……

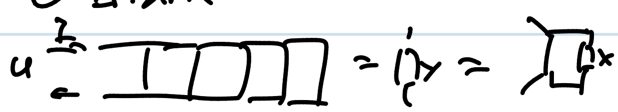




相当于 $\tilde{\varepsilon}_0$ 或 $\tilde{I}$

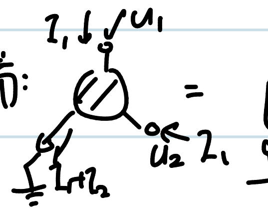
转120° 电压不变. $t \to \frac{T}{3} + t$ 相移.

③ 自相似



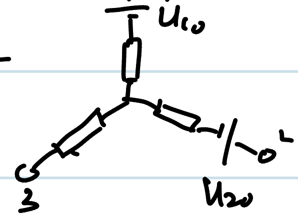
端口响应函数相同. $U(I) = U_0 - I r_0$

三端:



$$\begin{cases} U_1(I_1, I_2) = U_{10} - a_{11} I_1 - a_{12} I_2 \\ U_2(I_1, I_2) = U_{20} - a_{21} I_1 - a_{22} I_2 \end{cases}$$

= 三个独立电阻

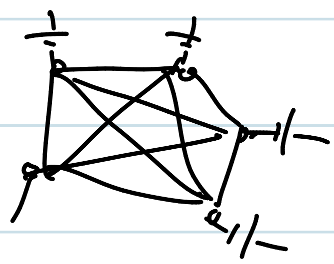


N端: $U_i(I_j) = U_{i0} - \sum_j a_{ij} I_j$

N-1个电压

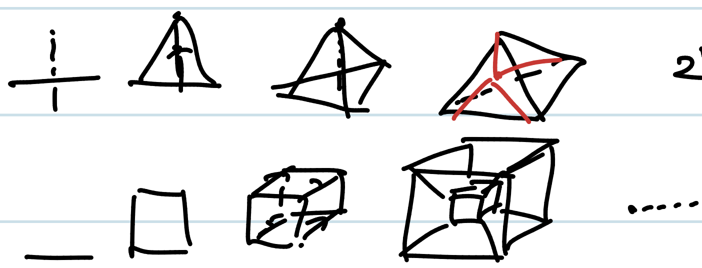
N×N 对称.

有 $\frac{N(N-1)}{2}$ 个电阻

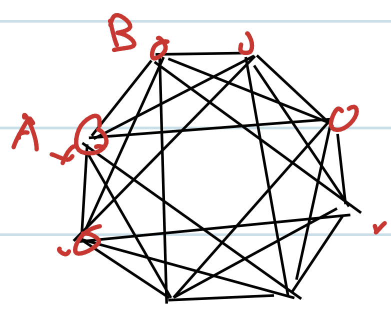


N维空间第二类多面体.

N	2	3	4	5	6……
	∞	5	6	3	3



2^n个点.

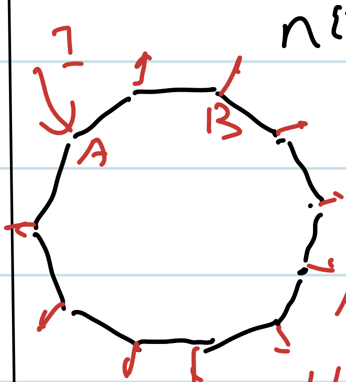


有2N个点. 两两之间连线. 除了对角线上点.

$\frac{8\times6}{2} = 24$边.

一.由2个四面体 $\frac{24\times2}{6} = 8$

N维: 2N个点.

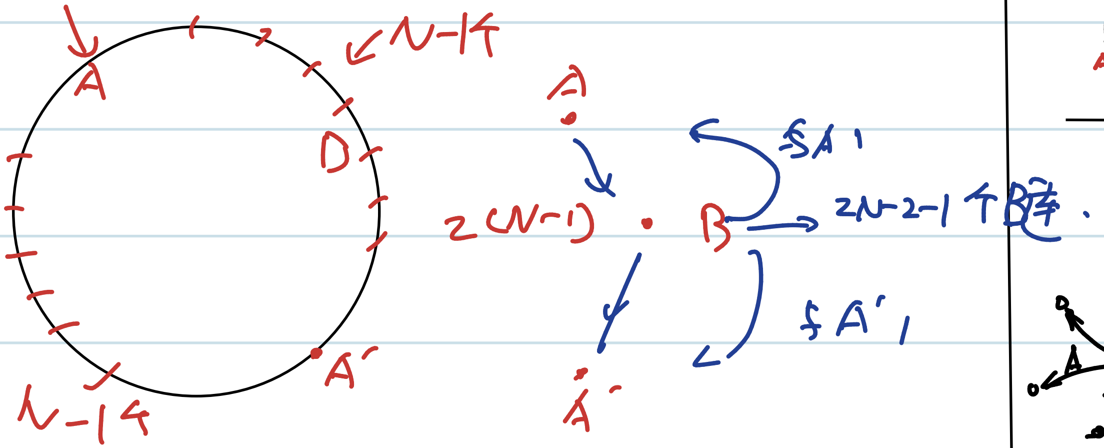


有 $\frac{2N\cdot(2N-2)}{2}$ 边.

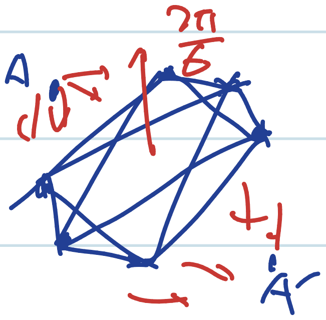
Vertex Profile

从2N-1点流出 $\frac{2}{2N-1}$.

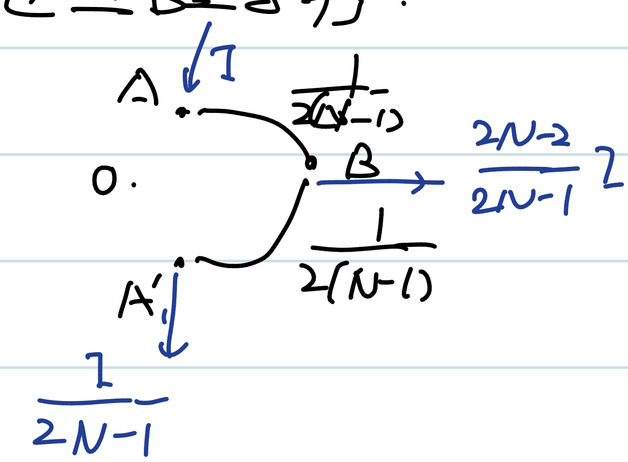
从B流出 从2N-1点流入 $\frac{1}{2N-1}$.



将B点 2(N-1)个 转 $\frac{2\pi}{2(N-1)}$ 角



这些B点等势.



$$U_{AB} = I\cdot\frac{1}{2(N-1)}$$

$$\text{总}\ U_{AB} = 2\times\frac{I}{2(N-1)} = \frac{I}{N-1}$$

$$\text{总电流}\ I = I + \frac{1}{2N-1}I = \frac{2N}{2N-1}I.$$

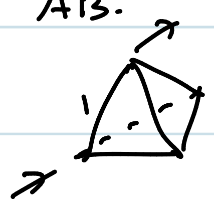
从而得电阻.

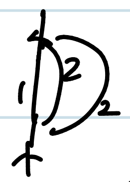
每两个点之间连R电阻或不连.

如果相连. 称为相邻. 连通.

若相邻. 则 R_{AB}, $\sum_{i<j} R_{ij} = (N-1)R$.

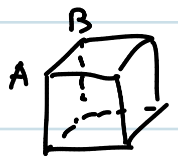
AB. N点的个数.



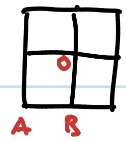


$R_{AB} = \frac{1}{2}$.

$6\times R_{AB} = 3. = 4-1$



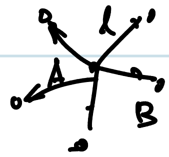
$R_{AB} = \frac{\frac{3}{5}\cdot\frac{5}{7}}{} = \frac{7}{12}$. 有12个条. $12\times R_{AB} = (8-1)$.



$4R_{OB} + R_{AB}\cdot 8 = (9-1).$

第一类 多面体. 每点一样每边一样.

一个点连 l 条边, 有N个点.



电流要加: A入 $\frac{N-1}{N}I$. 其它点 流出 $\frac{I}{N}$.

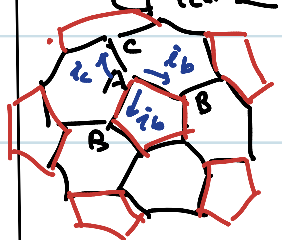
$$I_{AB} = \frac{N-1}{N}\cdot\frac{1}{l}, \quad U_{AB} = \frac{N-1}{N}\frac{1}{l}\cdot r\cdot I$$

$$\text{总值}: R_{AB} = \frac{2U_{AB}}{I} = 2\cdot\frac{N-1}{N}\frac{r}{l}$$

$$\text{有个}\ \frac{N\cdot l}{2}\ \text{条边}.\quad \sum R_{AB} = \frac{Nl}{2}\,2\frac{N-1}{N}\frac{r}{l} = (N-1)r.$$

第二类. 有N个点一样.

每个点连 l_1 个第一种边. l_2 个第二种边



$l_1 = 1,\ l_2 = 2.$

6条的边 5条的边.

A点流入电流 $\frac{I(N-1)}{N}$. 其它出 $\frac{I}{N}$.

$$l_1\cdot i_1 + l_2 i_2 = \frac{I}{N}(N-1)$$

$$U_{AB} = r\cdot i_1,\quad U_{BC} = r i_2.$$

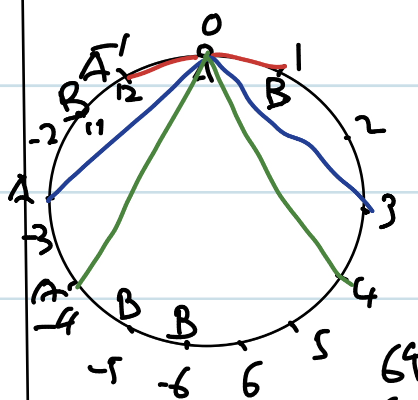
$$\Rightarrow \text{对应}\ R_{AB} = \frac{2ri_1}{I},\quad R_{AC} = \frac{2ri_2}{I}.$$

$$\sum R = \left(\frac{2ri_1}{I}\right)\cdot\frac{Nl_1}{2} + \left(\frac{2ri_2}{I}\right)\frac{Nl_2}{2}$$ 代入

$$= (l_1 i_1 + l_2 i_2)\cdot\frac{N}{I} = r(N-1).\quad \text{证毕.}$$

例) KC0 13个点. mod 13. 二次剩余

设点与 (±1, ±3, ±4) 相连.

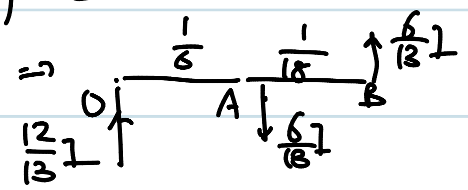


O — 每个O与 6个A相连.

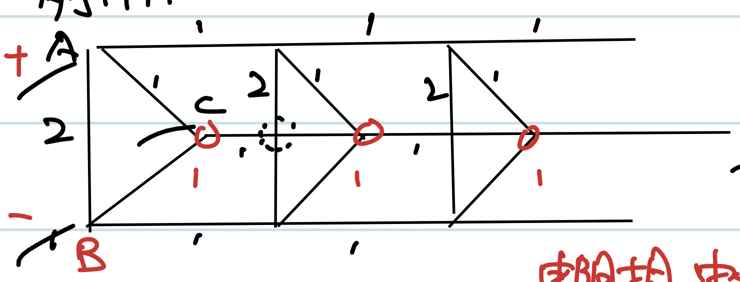
6个A

每个A与 (1个O, 2个A, 3个B) 相连

6个B 每个B与 (3个A, 3个B) 相连



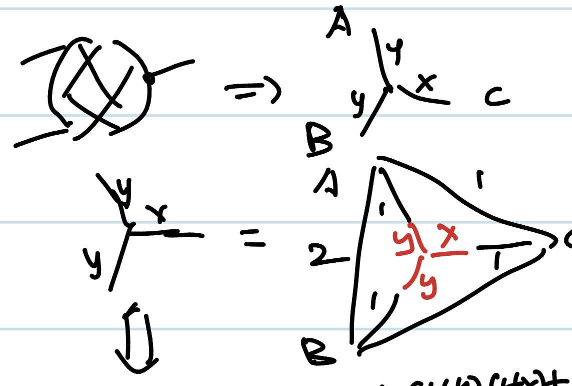
例141.



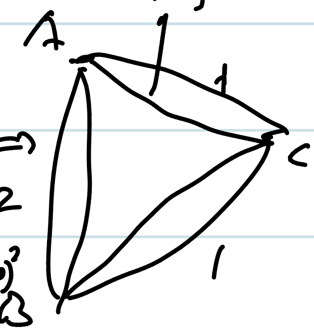
求R_{AC}

电阻相.中之 第二类对称.

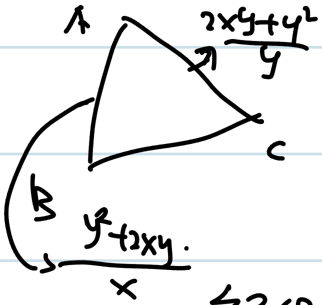
电阻为2. 不知道 下行.



$\frac{(1+y)^2+(1+x)(1+y)2}{1+y}$



$\frac{2(1+y)(1+x)+(1+y)^2}{1+x}$

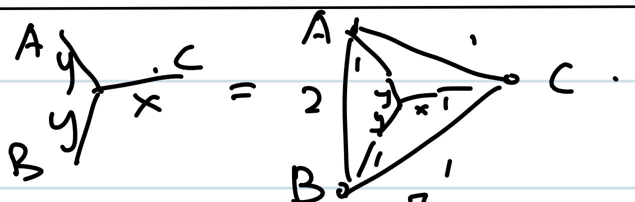


$\frac{2xy+y^2}{y}$

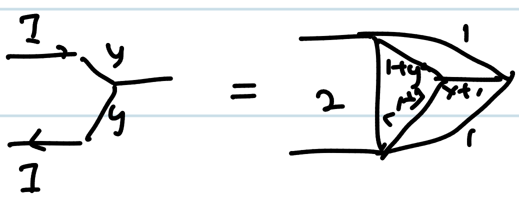
$\frac{y^2+2xy}{x}$

AB
$$\frac{x}{y^2+2xy}=\frac{1+x}{2(1+y)(1+x)+(1+y)^2}+\frac{1}{2}$$
$$\frac{y}{y^2+2xy}=\frac{1+y}{(1+y)^2+2(1+y)(1+x)}+1$$

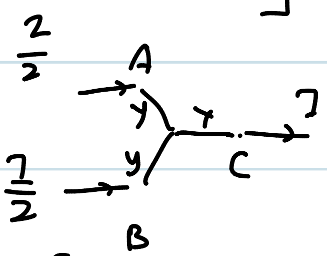
解得: ....



构造两个情景.

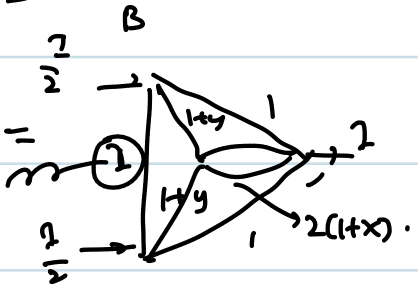


$2y=2//(2+2y)//2$. 解得y...



$U_{AC}=(\frac{1}{2}y+x)I$.

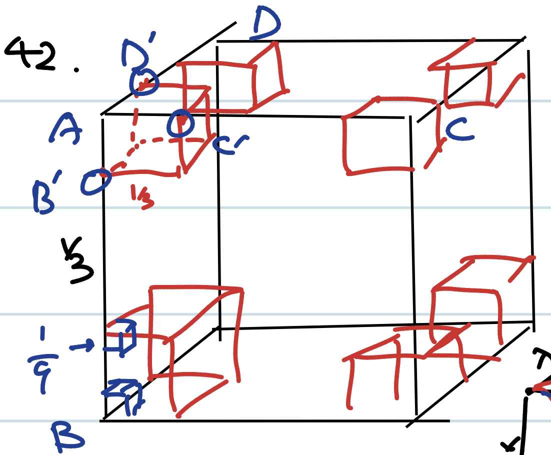
y已知



$U_{AC}=\frac{I}{2}\cdot 1//(1+\underbrace{y+2(1+x)}_{y+2x})$

∴ 一元二次 $x=..$

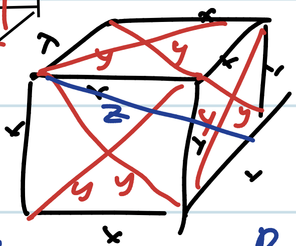
例142.



无穷分. $R_{AB}=?$

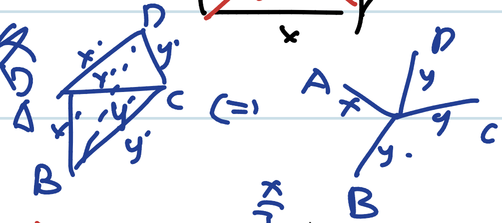
电阻单位长一样

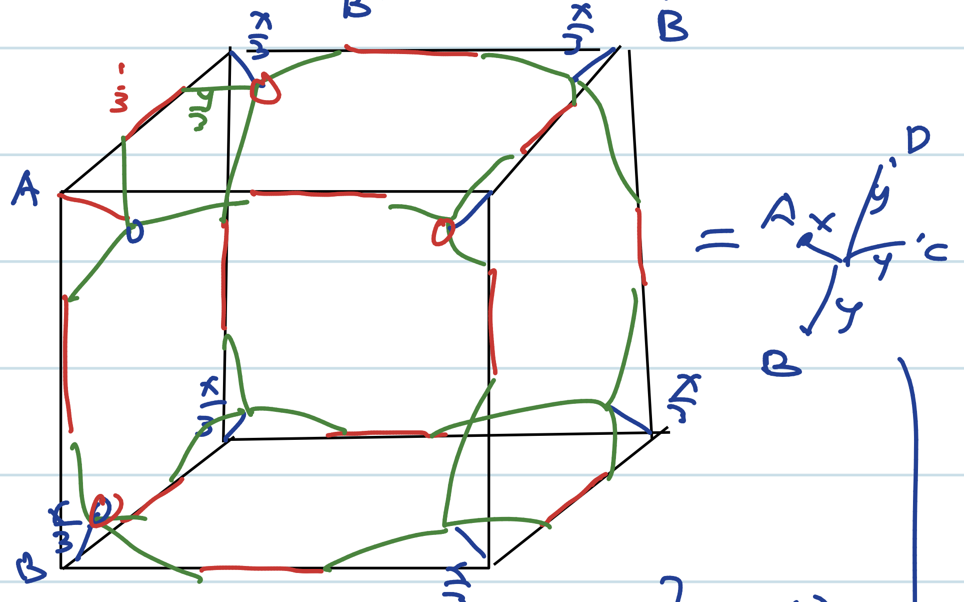
8端 $\frac{8\times7}{2}=28$.



3个变量

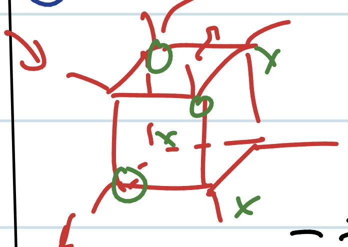
⇒四端网络





$R_{AB}=x+y=\frac{x}{3}+\frac{x}{3}+(\frac{1}{3}+\frac{y}{3})\frac{7}{12}$. ①

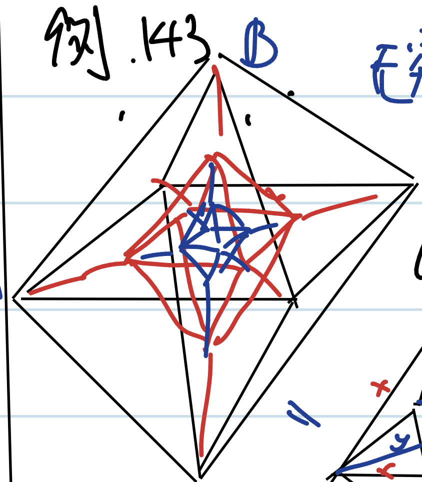
②: A入I. BCD出$\frac{I}{3}$. $U_{AB}=(x+\frac{y}{3})I$.



$U_{AB}=I\frac{x}{3}+\frac{I}{3}(\frac{1}{3}+\frac{2y}{3})+\frac{I}{3}\cdot\frac{x}{3}$

二元一次 解得——.

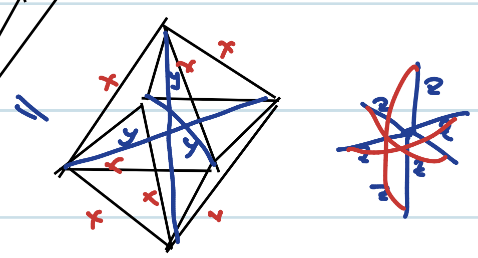
例143

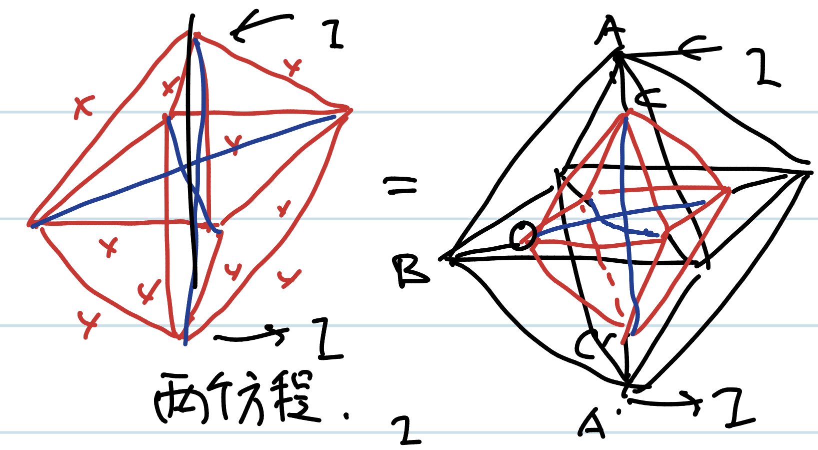


无穷层 每边电阻1.

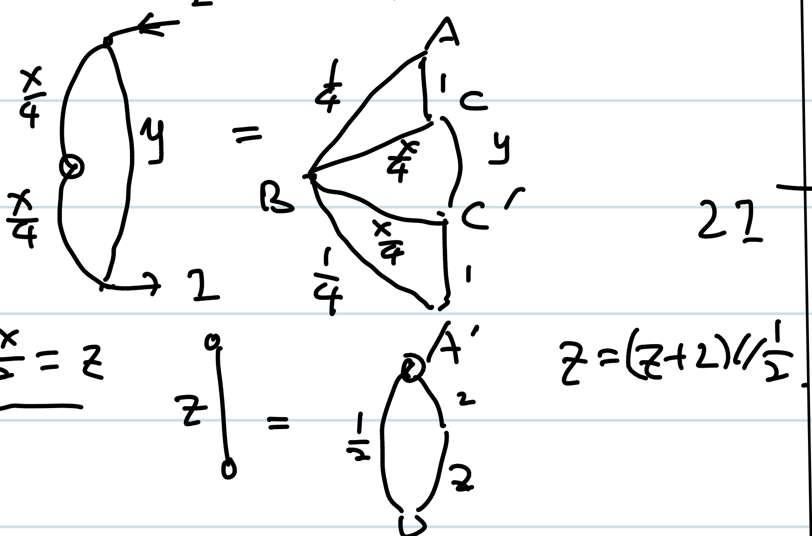
求R_{AB}

6端 $\frac{6\times5}{2}=15$.



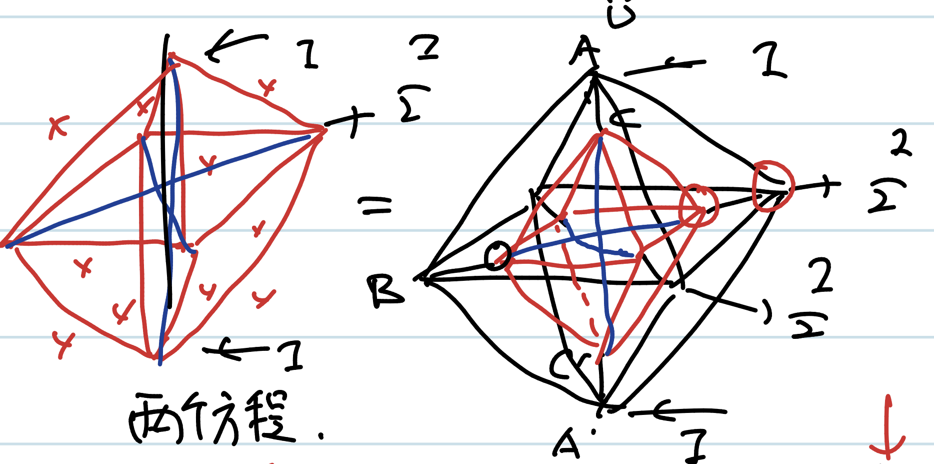


两方程.

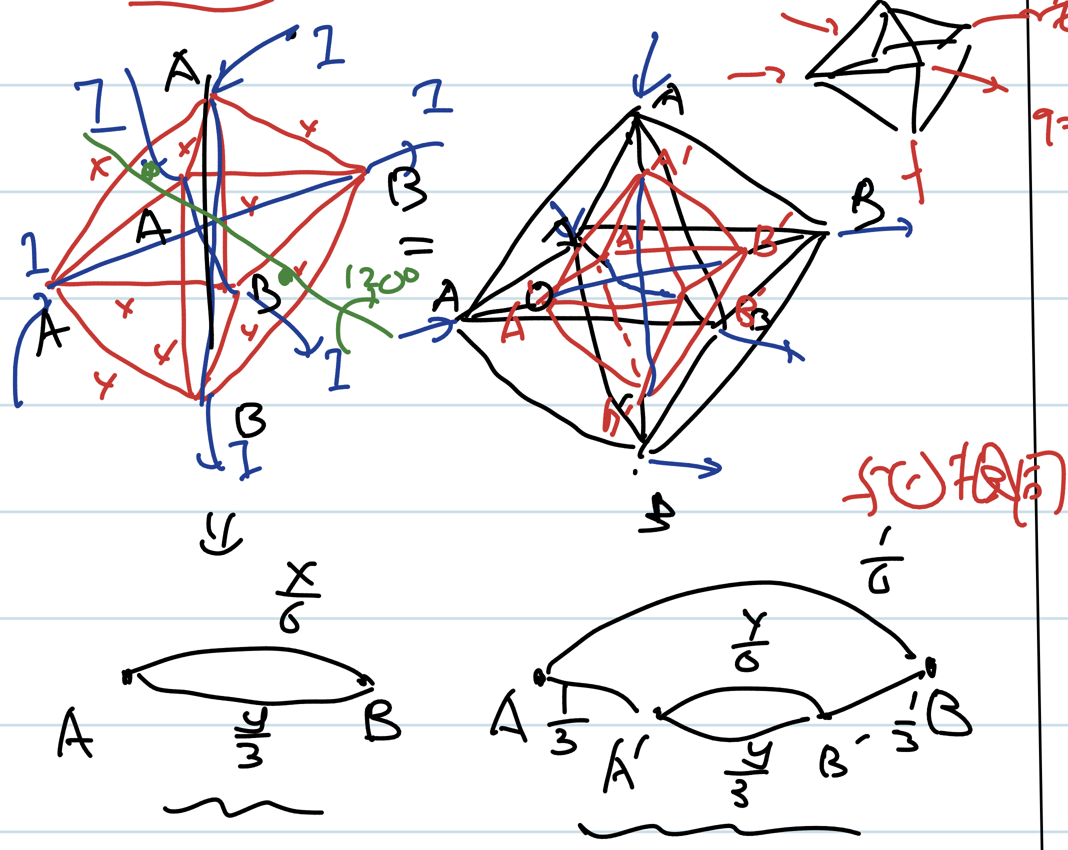


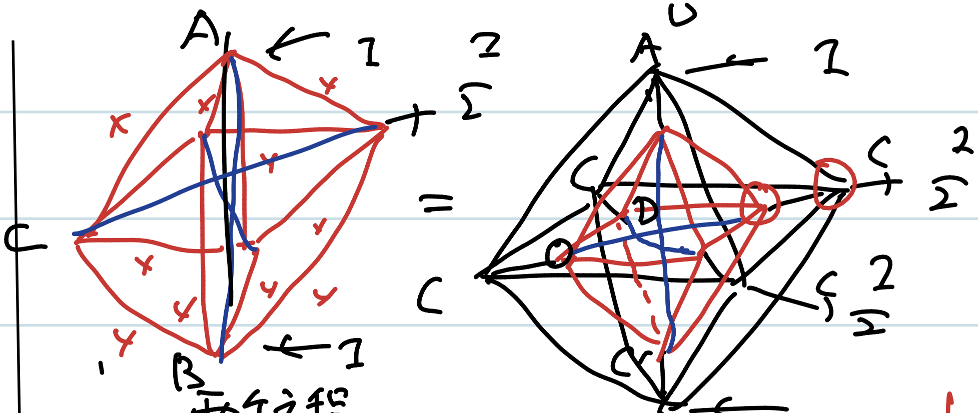
$y \,//\, \frac{x}{2} = z$

$z = (z+2) \,//\, \frac{1}{2}$

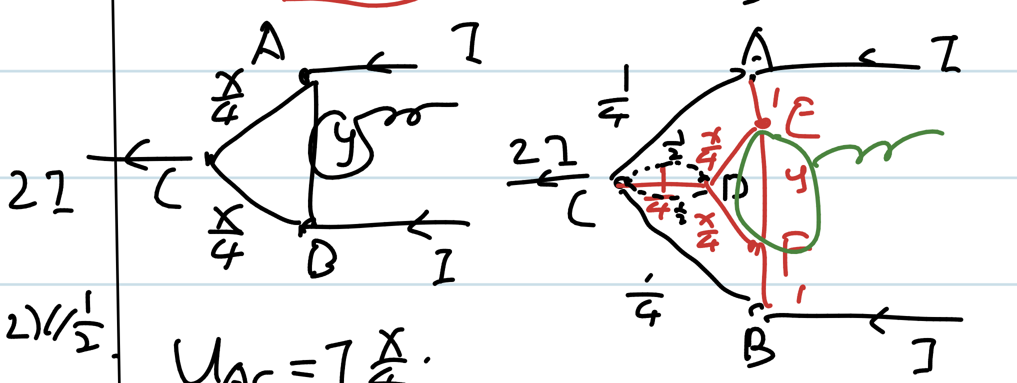


两方程.





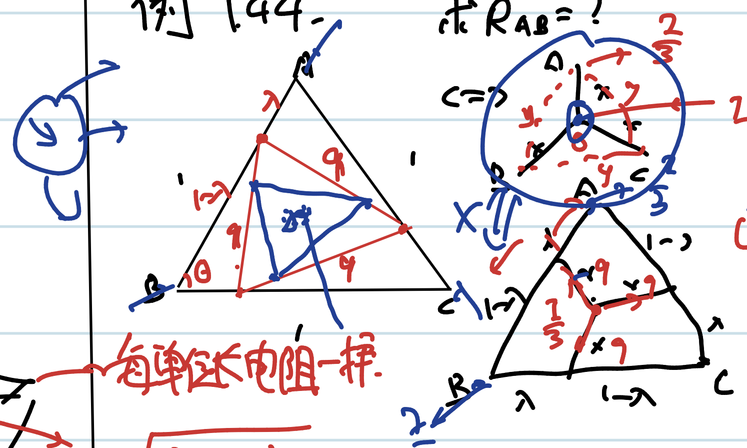
两方程.



$U_{AC} = I\,\frac{x}{4}.$

$U_{AC} = I \cdot \frac{1}{4} \,//\, \left(1 + \frac{x}{4} + \frac{1}{2}\right)$

例 144. 求 $R_{AB}=?$



ABCO 四端

$U_{AO} = I\,\frac{x}{3}.$

$U_{AO} = I\,\frac{x}{3}\,q. + \frac{2}{3}\cdot(1-\lambda)\lambda.$

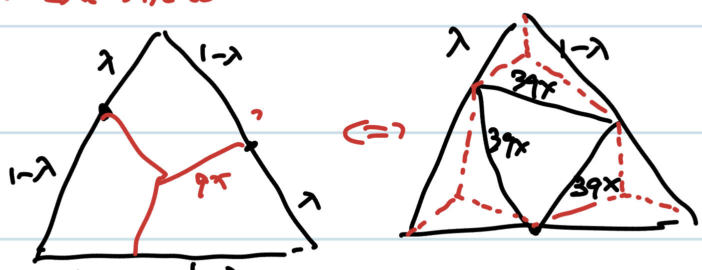
$\frac{x}{3} = \frac{x}{3}\,q + \frac{1}{3}\lambda(1-\lambda)$

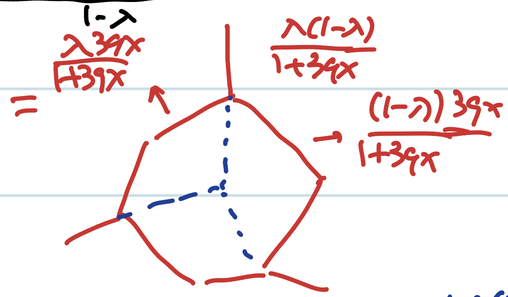
$x = \dots$

每条边电阻一样

$q = \sqrt{\lambda^2 + (1-\lambda)^2 - 2\lambda(1-\lambda)\cos\theta}$

→求 $x = \boxed{}$.





$$\frac{\lambda(1-\lambda)}{1+39x} + \frac{39x}{1+39x}\cdot\frac{1}{3} = x.$$

∴方程