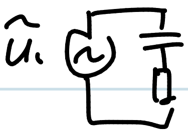
7.3. 交流电

C.L., $C\frac{dU}{dt}=\frac{dQ}{dt}=I$. $\lambda \gg l$.

$\Rightarrow$ 令 $U.I \propto e^{i\omega t}$ $\frac{d}{dt}\to i\omega$ 稳态/暂态



$\tilde{I}_c=\frac{\tilde{U}_0}{\frac{1}{i\omega C}+R}$ 由Q定

$U_0\cos\omega t=\frac{\int_0^t I\,dt+Q(0)}{C}+IR$.

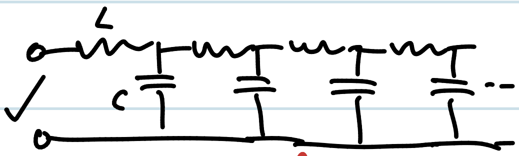
$-\omega U_0\sin\omega t=\frac{I}{C}+\dot{I}R$

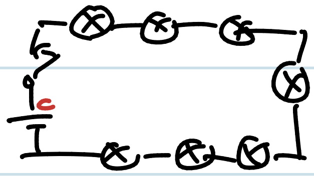
$I(t)=\frac{U_0\cos(\omega t+\varphi)}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}+I_t(t)$

$\frac{I_t}{C}+\dot{I}_tR=0$. $I_t=Ae^{-\frac{t}{RC}}$

设稳态. $C\to\frac{1}{i\omega C}$, $L\to i\omega L$ 直流电路都可以用

有以下结论不同





尺度.

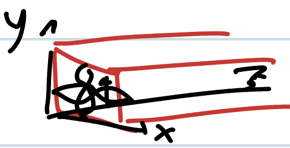
$U_0\sim U_0\cos\omega t$ $\lambda=c\cdot\frac{2\pi}{\omega}$.   (1) $l\ll\lambda$. 稳恒近似

(2) $a\ll\lambda\ll l$. 每一小段上几乎同相位.

在局域稳恒 $C\to\frac{1}{i\omega C}$. $L\to i\omega L$

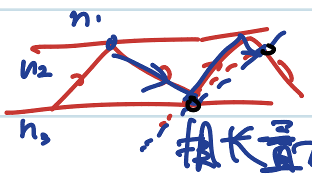
整体看,不是,形成传播.

(3) $\lambda\sim a$. $\lambda\ll l$ 波导



驻波 传导

$E\propto \sin k_x x\,\sin k_y y\; e^{ik_z z-i\omega t}$ TE TM. $e^{i\varphi}$

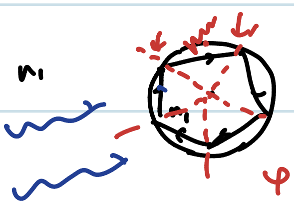


相长干涉

$r=\frac{n_1\cos\theta_1-n_2\cos\theta_2}{n_1\cos\theta_1+n_2\cos\theta_2}=1\cdot e^{i\varphi}$

全反射. $\cos\theta_2=\sqrt{1-\sin^2\theta_2}$

$=\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1}$. 虚



恰好是整数

$\frac{2\pi}{n}$角.

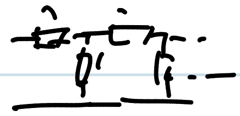
$\varphi=n_2\sin(\frac{\pi}{2}-\frac{\pi}{n})\cdot k_2+e^{i\varphi}$

$m\cdot 2\pi=n\cdot\varphi$. 共振

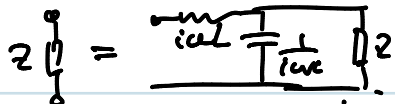
频率相同,共振吸收

加入杂质 改变频率

$n.m\to\omega_{mn}$



$R=\frac{-1+\sqrt{3}}{2}$. 取正.



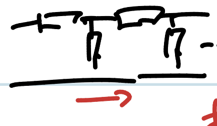
$Z=i\omega L+\frac{Z\frac{1}{i\omega C}}{Z+\frac{1}{i\omega C}}$

$Z^2+\cancel{\frac{Z}{i\omega C}}=i\omega LZ+\frac{L}{C}+\cancel{\frac{i\omega L}{i\omega C}Z}$

$Z^2-i\omega LZ-\frac{L}{C}=0 \Rightarrow Z=\frac{i\omega L\pm\sqrt{-\omega^2L^2+4\frac{L}{C}}}{2}$.

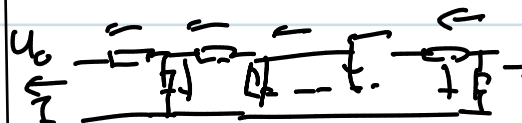
(1) $\omega^2>\frac{4}{LC}$. Z虚. $Z_\pm=\frac{i\omega L\pm i\sqrt{\omega^2L^2-4\frac{L}{C}}}{2}$. 取哪个?

(2) $\omega^2<\frac{4}{LC}$. $Z=\frac{i\omega L\pm\sqrt{4\frac{L}{C}-\omega^2L^2}}{2}$. 正或负的实部



负根 $R=\frac{-1-\sqrt{3}}{2}$.   ?代表指数变大另一个根

指数放大的根.

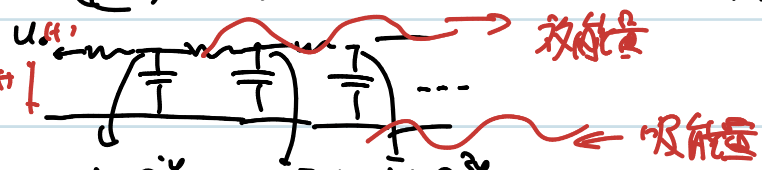


大电池

(1) $Z_\pm$ 代表振幅 指数变大或指数变小的解

U_0 λU_0 λ^2U_0 λ^3U_0 $\lambda>1$. 或 $\lambda<1$ 对应 $Z_\pm$.   取 $\lambda<1$

(2) $\omega^2<\frac{4}{LC}$; Z有>0, <0实部. 吞/吐能量



$U(t)$ $I(t)$ 放能量 吸能量. $U_0e^{i\psi}$ $U_0e^{i2\psi}$ $U_0e^{i3\psi}$

$P(t)=\overline{U(t)I(t)}=\overline{I_0\cos(\omega t)\cdot|Z|I_0\cos(\omega t+\varphi_Z)}$.

$=\frac{1}{2}|Z|I_0^2\cos(\varphi_Z)$. $=\frac{1}{2}\mathrm{Re}[\tilde{U}(t)\tilde{I}^*(t)]$.

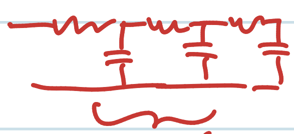
Z有实部,实部>0 $P(t)>0$ 放能量

实部<0 $P(t)<0$ 吸收.

R_n $\lim_{n\to\infty}R_n=R_\infty$

n个格子

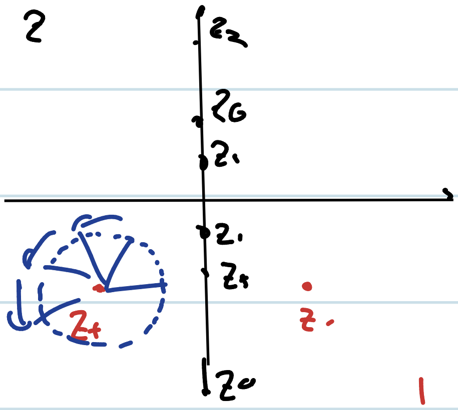
$\downarrow$



Z_n. $\lim_{n\to\infty}Z_n\neq Z$.

n个

$\omega^2<\frac{4}{LC}$ 时 Z有实部 Z_n虚数.



先解得 z_n

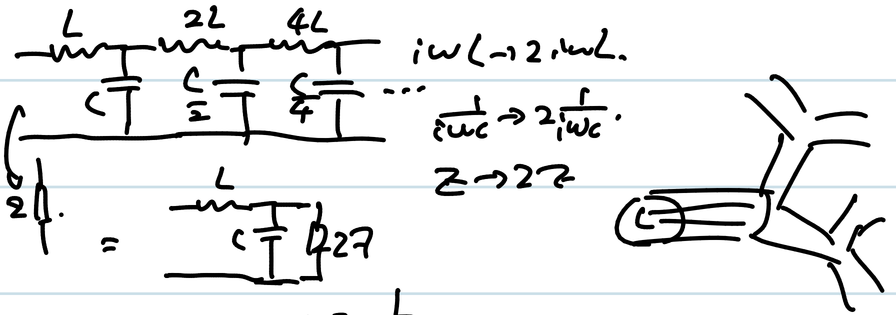
$$z_{n+1}=i\omega L+\frac{\frac{1}{i\omega c}z_n}{z_n+\frac{1}{i\omega c}}$$

令 $z_{n+1}=\frac{b_{n+1}}{b_n}$ 代入...

b_n 二阶常系数递推....

$\frac{1}{z_n-z_-}$ → 落在一个圆上, 等角度转动

Abel 极限下全落在圆上



$i\omega L\to 2i\omega L$

$\frac{1}{i\omega c}\to 2\frac{1}{i\omega c}$

$z\to 2z$

$$z=i\omega L+\frac{2z\frac{1}{i\omega c}}{2z+\frac{1}{i\omega c}}$$

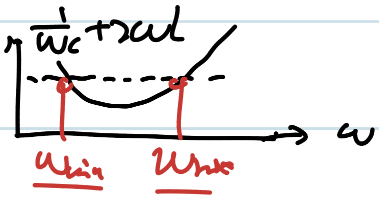
$$2z^2+z\cdot\frac{1}{i\omega c}=i\omega L\cdot 2z+\frac{L}{c}+2z\frac{1}{i\omega c}$$

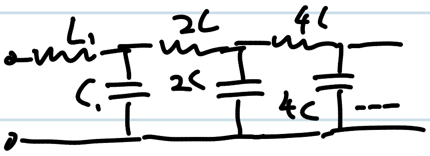
$$2z^2+\left(-\frac{1}{i\omega c}-2i\omega L\right)z-\frac{L}{c}=0$$

$$z=\frac{2i\omega L+\frac{1}{i\omega c}\pm\sqrt{-\left(\frac{1}{\omega c}+2\omega L\right)^2+4\frac{L}{c}}}{2},$$

z 有实部 有能量去∞. 传播波. $\frac{4L}{c}>\left(\frac{1}{\omega c}+2\omega L\right)^2$

变成带通.



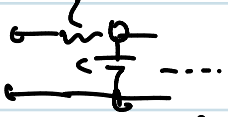


→ $Z(\omega)$?

$Z(C_i,L_i)=Z\left(\frac{1}{\omega C_i},\omega L_i\right)$ 比值.

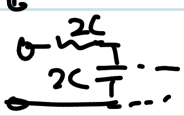
$=\omega L_1\cdot Z\left(\frac{L_i}{L_1},\frac{1}{\omega^2 C_i L_1}\right).$

第一层



$Z=i\omega L\cdot Z\left(\frac{L_i}{L_1},\frac{1}{\omega^2 C_i L_1}\right)$

第二层



$Z=2\omega L_1 Z\left(\frac{1}{\omega^2 2C 2L_1}\right)$

$\frac{L_i}{L_1}$ 不变, $\frac{C_i}{C_1}$ 不变

$f(x)=Z\left(\frac{1}{\omega C_1 L_1}\right).$

令 $Z=i\omega L_1 f(x)$ 第二层: $Z'=2\omega L_1 f\left(\frac{x}{4}\right)$

$$\Rightarrow i\omega L_1 f(x)=i2\omega L_1+\frac{\frac{1}{i\omega C_1}\cdot 2i\omega L_1 f\left(\frac{x}{4}\right)}{\frac{1}{i\omega C_1}+2i\omega L_1 f\left(\frac{x}{4}\right)}.$$

$x=\frac{1}{\omega^2 L C}.$

$$\cancel{\Rightarrow i\omega L_1 f(x)}=i2\cancel{\omega L_1}+\frac{\frac{1}{i\omega C_1}\cdot 2\cancel{i\omega L_1} f\left(\frac{x}{4}\right)}{\frac{1}{i\omega C_1}+2i\omega L_1 f\left(\frac{x}{4}\right)}.$$

$x=\frac{1}{\omega^2 L C}.$

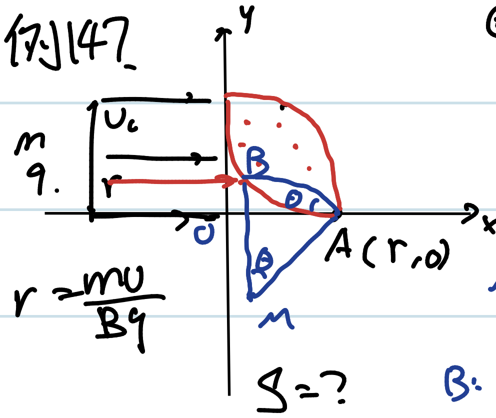
$$f(x)=2+\frac{2f\left(\frac{x}{4}\right)}{1-2\frac{1}{x}\cdot f\left(\frac{x}{4}\right)}$$

令 $f(x)=a_0+a_1x+a_2x^2+a_3x^3+\dots$ 高斯展开.

8. 磁场中带电粒子运动

圆周.

例147



$r=\frac{mv}{Bq}$

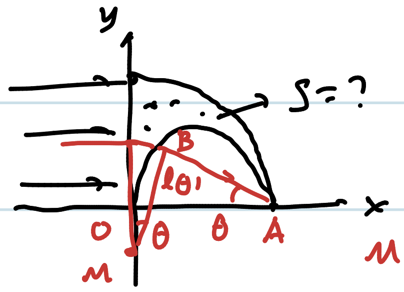
①均匀磁场在一定范围内有.

一束平行粒子汇聚在 A 点

求 B 边界

$M:\begin{cases}x=r-r\sin\theta\\ y=-r\cos\theta.\end{cases}$

$B:\begin{cases}x=r-r\sin\theta\\ y=-r\cos\theta+r\end{cases}$ → 圆.



令 AB → $l(\theta)$

$B:(r-l\cos\theta,\ l\sin\theta).$

$M:(r-l\cos\theta-r\sin\theta,\ l\sin\theta-r\cos\theta)$

$\quad\ \ \|$

$\quad\ \ 0$

$r-l\cos\theta-r\sin\theta=0$; $l=\frac{r(1-\sin\theta)}{\cos\theta}.$

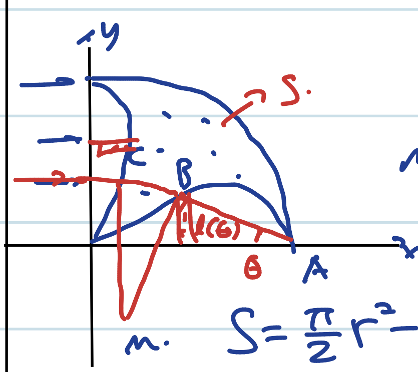
$B:\left(r\sin\theta,\ \frac{r(1-\sin\theta)\sin\theta}{\cos\theta}\right).$

$$S=\frac{\pi}{2}r^2-\int_{\theta=0}^{\theta=\pi/2}y_B\,dx_B$$

$$=\frac{\pi}{2}r^2-\int_0^{\theta=\pi/2}\frac{r(1-\sin\theta)\sin\theta}{\cancel{\cos\theta}}\cdot d\,r\cancel{\sin}\theta$$

$$r^2\int_0^{\pi/2}(\sin\theta-\sin^2\theta)\,d\theta$$

$=\frac{\pi}{2}r^2-r^2\cdot\left(1-\frac{1}{2}\cdot\frac{\pi}{2}\right)$ 同一个面积.

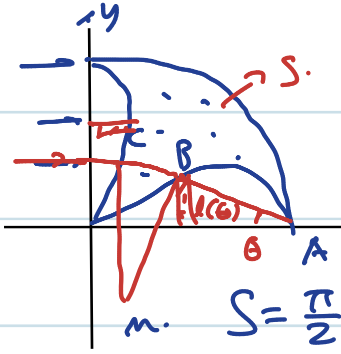


$B:(r-l\cos\theta,\ l\sin\theta).$

$m:(r-l\cos\theta-r\sin\theta,\ l\sin\theta-r\cos\theta)$

$C:(r-l\cos\theta-r\sin\theta,\ l\sin\theta-r\cos\theta+r)$

$S=\frac{\pi}{2}r^2-\int y_B\,dx_B-\int x_C\,dy_C$



$B: (r-l\cos\theta,\ l\sin\theta)$.

$m: (r-l\cos\theta-r\sin\theta,\ l\sin\theta-r\cos\theta)$

$C: (r-l\cos\theta-r\sin\theta,\ l\sin\theta-r\cos\theta+r)$

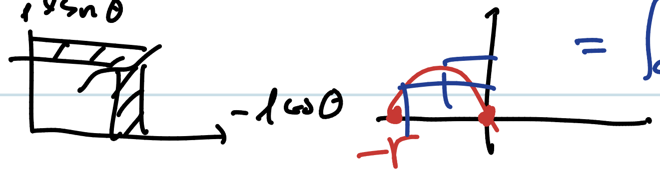
$S=\frac{\pi}{2}r^2-\int y_B dx_B-\int x_C dy_C$

$\int y_B dx_B+\int x_C dy_C$

$=\int_{\theta=0}^{\pi/2} l\sin\theta\ d(-l\cos\theta)+\int_0^{\pi/2}[r(1-\sin\theta)-l\cos\theta]\,d[l\sin\theta-r\cos\theta]$.

以 l 展开. l^2. lr. r^2

l^2: $\int_0^{\pi/2} l\sin\theta\ d(-l\cos\theta)+\int_0^{\pi/2}(-l\cos\theta)\,d\,l\sin\theta$



$=\int_0^{\pi/2} d[-l\cos\theta\ l\sin\theta]$

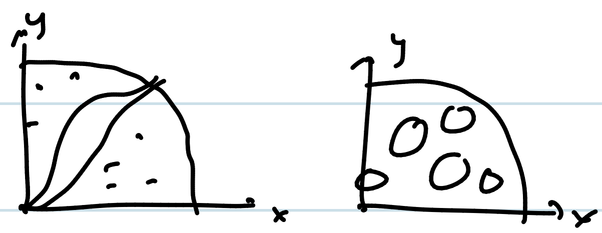
$\to 0$

lr: $\int r(1-\sin\theta)\cdot d\,l\sin\theta+\int(-l\cos\theta)\,d(-r\cos\theta)$

$=\int r(1-\sin\theta)\,d\,l\sin\theta+\int +l\cdot\sin\theta\ d[r\sin\theta+r]$

$=\int_0^{\pi/2} d[l\sin\theta\ r(1-\sin\theta)]\to 0$ $\theta=0,\ \theta=\frac{\pi}{2}$

r^2: $\int_0^{\pi/2} r^2(1-\sin\theta)\sin\theta\,d\theta$ 与① 相同.



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7.2 其它运动

$\vec F=q\vec v\times\vec B+$

① 线性 齐性/非齐性 $-k\vec r$, $-\beta\vec v$, $+m\vec g$.

$-\beta\vec v+m\vec g$.

猜指数解.

② 自然坐标 $-f(v)\hat v$.

拆成 切/法.

③ 多个守恒量, 正则动量 正则角动量. 组合

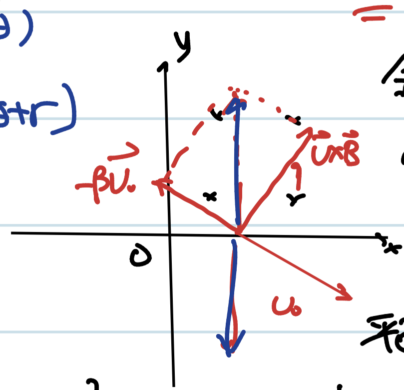
摩擦对称.

① 线性.

例148: $\vec F=m\vec a=q\vec v\times\vec B-\beta\vec v+m\vec g$ 不吃.

令 $\vec v=\vec v_1(t)+\vec v_0$

$q\vec v_0\times\vec B-\beta\vec v_0+m\vec g=0$



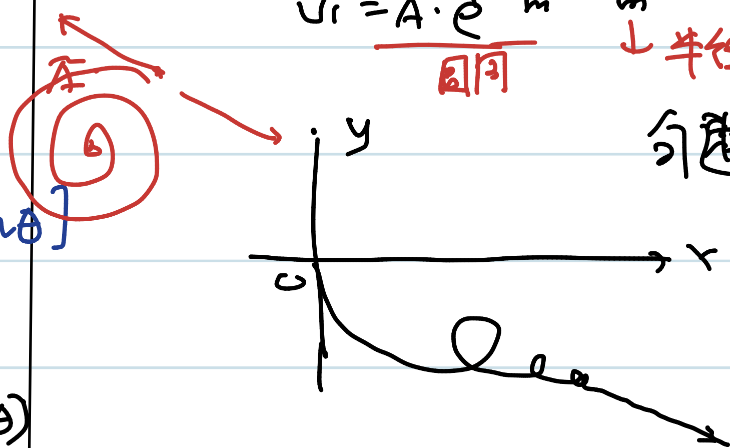
$m\dot{\vec v}_1(t)=q\vec v_1\times\vec B-\beta\vec v_1$

平面→复数 $\tilde v=v_x+iv_y$.

$m\dot{\tilde v}=(iqB-\beta)\tilde v_1$

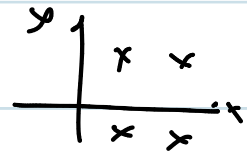
$\tilde v_1=\tilde A\cdot e^{\frac{iqB}{m}-\frac{\beta}{m}t}$.

圆周 半径指数缩小



匀速叠加. 缩小圆周.

例149 $m\vec a=q\vec v\times\vec B+F_0\hat n(t)$



$\hat n(t)=(\cos\omega t,\ \sin\omega t)$

$m\vec a=q\vec v\times\vec B\Rightarrow\tilde v=\tilde v_0 e^{i\frac{qB}{m}t}$

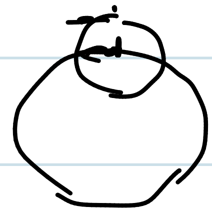
猜 $\tilde v=\tilde v_1 e^{i\omega t}$ 代入

$mi\omega\tilde v_1 e^{i\omega t}=iqB\tilde v_1 e^{i\omega t}+F_0 e^{i\omega t}$

$\tilde v_1=\frac{F_0}{im\omega+qB}$.

$\tilde v=\tilde v_0 e^{i\frac{qB}{m}t}+\frac{F_0}{im\omega+qB}e^{i\omega t}$

两圆周运动叠加



行星运动

例150 $m\vec a=q\vec v\times\vec B-f(v)\hat v$.

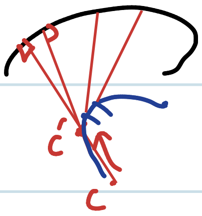
$f(v)=-\beta v,\ -a_0,\ -bv^2$

$\vec a=\dot v\hat v+\omega v\hat n$.

$\Rightarrow m\dot v\hat v+\omega v\hat n m=qB\hat n-f(v)\hat v$

$\omega=\frac{qB}{m}$. $\dot v=-\frac{f(v)}{m}$; $\int\frac{dv}{f(v)}=-\int\frac{dt}{m}$

$v(t)\Rightarrow r=\frac{mv(t)}{Bq}$.



曲率中心只能沿法线运动.

$v_c = \frac{dr_c}{dt} = \frac{m\dot{u}}{Bq} = \frac{mf(u)}{Bq}$.

法向与切向一样, 均匀随时变化

$\vec{v}_c = \frac{mf(u)}{Bq}[\cos(\omega t+\varphi), \sin(\omega t+\varphi)]$

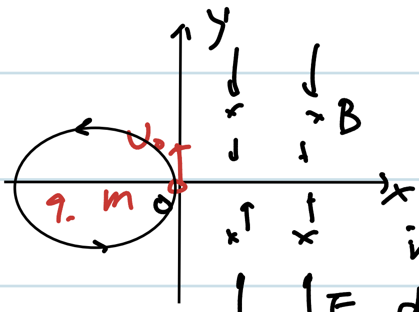
定轨迹. 物体轨迹是曲率中心轨迹渐开线

例151. 加一个对称性.

$[\frac{\partial L}{\partial q}=0. \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}=0. \quad \frac{\partial L}{\partial \dot{q}}=p \Rightarrow$ 守恒$]$

找一个方向平移/转动对称, 写动量/角动量定理

变成全微分 ⇒ 守恒量



$\vec{B}=B(-\hat{z}) \quad \vec{E}=\hat{y}\cdot(-\alpha y)$

$\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2=\frac{1}{2}mv_0^2-\frac{1}{2}q\alpha y^2$ ← 电势能

沿X轴平移.

$\frac{dP_x}{dt}=-qv_yB$; $dP_x=-qBdy$

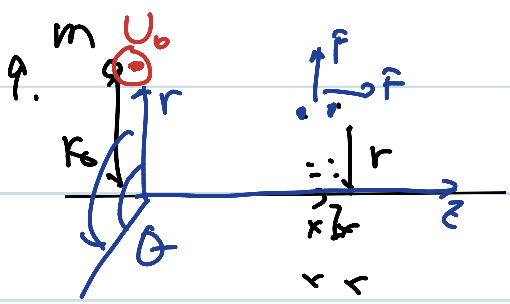
$\Rightarrow d(P_x+qBy)=0$

$P_x+qBy=0 \Rightarrow P_x=-qBy$ 代

$\Rightarrow \frac{1}{2}mv_y^2+\underbrace{\frac{1}{2m}q^2B^2y^2+\frac{1}{2}\alpha q y^2}_{V_{eff}}=\frac{1}{2}mv_0^2$

y方向简振.

$P_x=qBy \Rightarrow$ X方向简振.



$B=\frac{\mu_0 I}{2\pi r}$

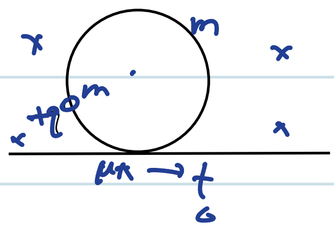
不在平面内运动

求 r_{max} 满足方程

$\Delta E=0$ 在平移对称 $\boxed{P_z-q\frac{\mu_0 I}{2\pi}\ln r=C.}$

$\Delta L_z=0$. $\frac{dP_z}{dt}=qB(r)\frac{dr}{dt}$

2017暑1-H2 决赛 067



正则动量组合

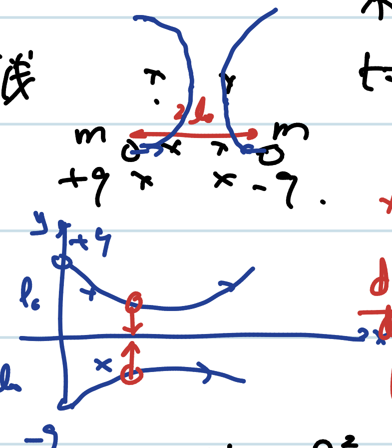
正则角动量

选择 得守恒量

例153 有相互作用. $q\vec{v}\times\vec{B}$

不算辐射. 求轨迹

$t=0. \ v=0 \quad 2l_0$



x方向无库仑力.

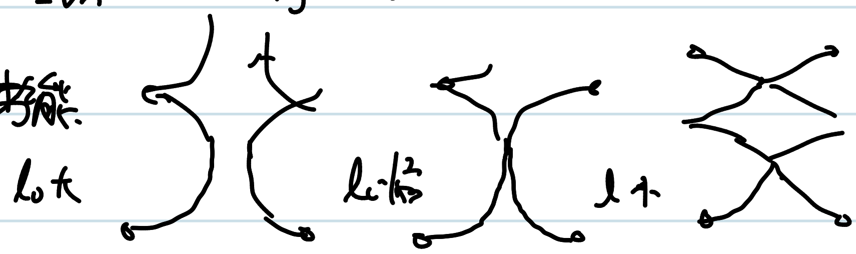
$\frac{dP_x}{dt}=-qv_yB$

$P_x+qyB=C=ql_0B.$

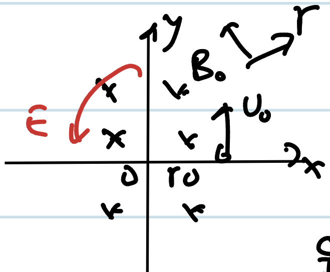
$2\times(\frac{1}{2}\frac{P_x^2}{m}+\frac{P_y^2}{2m})-\frac{Kq}{2y}=-\frac{Kq}{2l_0}$

$\frac{P_x^2}{2m}+\frac{P_y^2}{2m}-\frac{Kq}{4y}+\frac{Kq}{4l_0}=0$ 代入.

$\frac{(ql_0B-qyB)^2}{2m}-\frac{Kq}{4y}+\frac{Kq}{4l_0}+\frac{P_y^2}{2m}=0 \quad y_{min}$——



8.3 浸渐不变量



$r_0=\frac{mv}{Bq}$

B_0 慢变为 $2B$ 时. $r\to$?

$\frac{dL}{dt}=\dot{r}qB\cdot r+Eq\cdot r$

$L=mv_0r_0$

$=r\cdot rBq.$

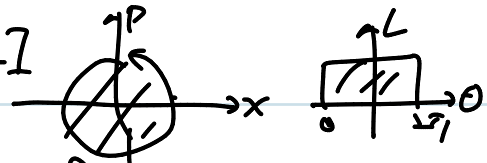
$=qB\frac{dr^2}{2dt}+qr\cdot\frac{\pi r^2}{2\pi r}\frac{dB}{dt}$

$\frac{dL}{dt}=\frac{d}{dt}[\frac{qBr^2}{2}].$

$d(\frac{1}{2}r^2Bq)=0 \quad r^2Bq\to C. \quad r\to\frac{1}{\sqrt{2}}r_0$

$I\cdot S=\pi r^2\frac{qBq}{2\pi m}\propto L$ 不变.

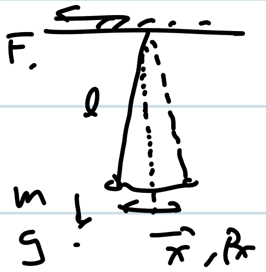
一个周期运动 $\oint p\,dq = I$



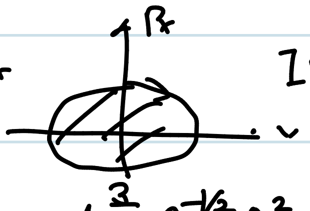
一慢参数缓变 $\frac{\Delta A}{\Delta t} \ll \frac{A}{T}$.

$\Rightarrow$ I 不变

例154. $l \to \frac{l}{2}$, $\theta_0 \to ?$



$$\begin{cases} x = l\theta_0 \cos\omega t. \\ P_x = -m\omega\, l\theta_0 \sin\omega t \end{cases}$$



$I = \pi \cdot l\theta_0 \cdot m\omega l\theta_0 = C.$

$l^2 \omega \theta_0^2 = C$

$l^{\frac{3}{2}} g^{-1/2} \theta_0^2 = C.$ $\quad l^3\theta^4 = C$

9. $mg \to Eq$.

$E \to 2E$. $\theta \to ?$ $\quad \frac{l^{\frac{3}{2}}}{(Eq)^{1/2}}\theta^2 = C$

$E^{-1/2}\cdot\theta^2 = C.$

E缓变. 无势能项. 不能写 $\Delta E = 0$

令 $E = E_0 + \alpha \frac{t}{T} E_0$ 代入. $\alpha \ll 1$. 微扰展开

算一个周期.

$m\ddot{x} = -E_0 q\left(1+\alpha\frac{t}{T}\right)\frac{x}{l}.$

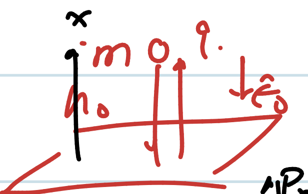
$O(0)$: $x = A\cdot\cos\omega t.$ $\quad \omega = \sqrt{\frac{E_0 q}{ml}}.$

$O(1)$ $\quad x = A\cos\omega t\cdot(1+\alpha f(t))$ $\quad \alpha \ll 1$

$-\omega^2 A\cos\omega t - A\cos\omega t\, \alpha f''(t)$ ← $O(1)$.

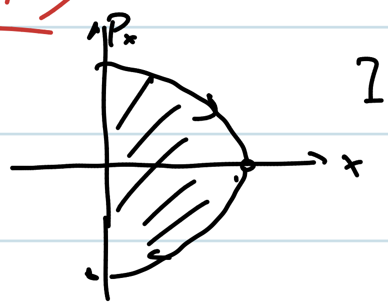
$= -\frac{E_0 q}{l} A\cos\omega t\,(1+\alpha f(t))$ ← $O(1)$

$-\frac{E_0 q}{l}\alpha\frac{t}{T} A\cos\omega t$ ← $O(1)$ $\quad$ 解 $f(t)$...



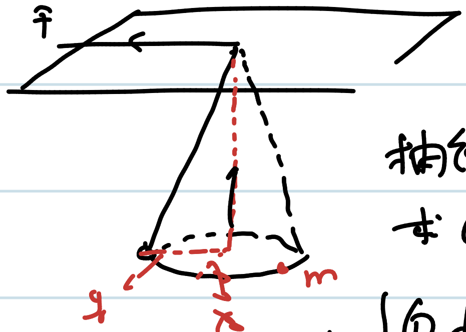
弹性

$E_0 \to 2E_0$ 慢 $\quad h_0 \to ?$



$I = \oint p\,dq.$

$\propto h^{\alpha} E^{\beta} q^{\gamma} m^{\gamma}$



$\frac{y_1}{l_1} = \theta_{y_0}$ $\quad \frac{x_1}{l} = \theta_{x_0}$

抽绳子 $l \to \frac{l}{2}$

求 $\theta_y \to ?$ $\quad \theta_x = ?$

$$\begin{cases} \int (P_x dx + P_y dy) = I \text{ 不变 ①} \\ L_z \text{ 守恒. ②} \end{cases}$$

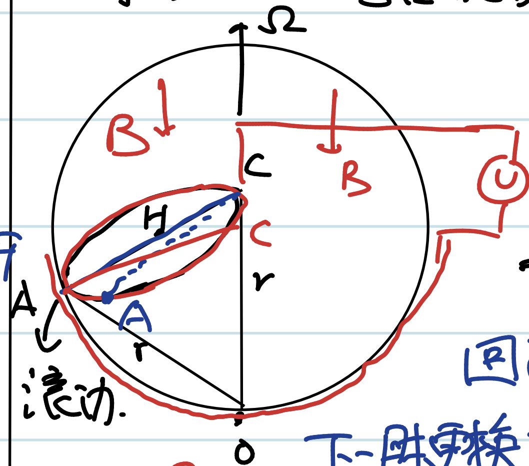
9 电磁感应

9.1 动生. $\int q\vec{v}\times\vec{B}\cdot d\vec{l}$

真实的原子实移动. 不是边界移动！

→只对指定线路有定义. 不是磁场在运动.

例155. 公转角速度 Ω



$\varepsilon_{C-H-A-O}$

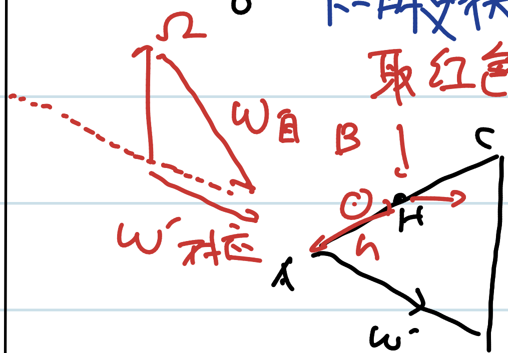
$\vec{v}\times\vec{B}\cdot d\vec{l}\, dt$ $\quad$ 扫过面积

$= \vec{B}\cdot \underbrace{d\vec{l}\times\vec{v}\,dt}_{d\vec{S}}$

回路面积变化不同.

下一时刻更换接触点. $\neq$ 扫过.

取红色扫过面.



$v = \omega' h\cdot\frac{\sqrt{3}}{2}.$

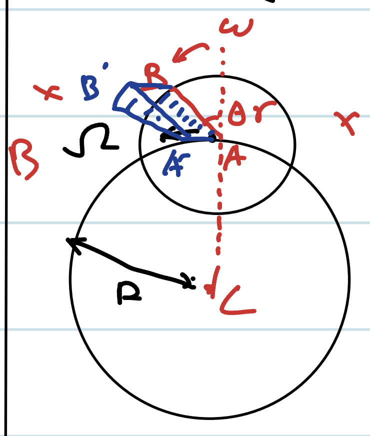
$\varepsilon = \int vB\frac{\sqrt{3}}{2}\,dh$

$= \frac{3}{4}\int_0^r \omega' h\cdot B\,dh$

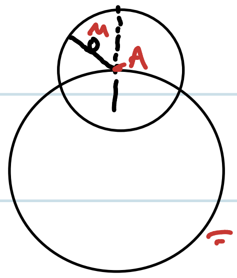
$= \frac{3}{4}\,\Omega\cdot\frac{Br^2}{2}.$

例156. 是真转动 $\int \vec{v}\times\vec{B}\cdot d\vec{l}$ $\quad$ → ΔS → 直接算.

求 $\varepsilon_{AB} = ?$



$\varepsilon = \frac{B\Delta S}{\Delta t} = B\cdot(\Omega R\cdot r\cos\theta + \frac{1}{2}\cdot\omega\cdot r^2).$

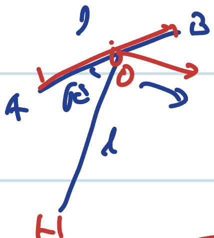


$\vec{v}_M = \vec{v}_A + \vec{\omega}\times\vec{AM}$

$\int (\vec{v}_A + \vec{\omega}\times\vec{AM})\times\vec{B}\cdot d\vec{r}_M$

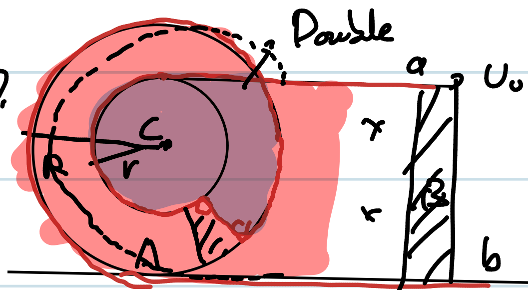
$= \int \vec{v}_A\times\vec{B}\cdot d\vec{r}_M + \int(\vec{\omega}\times\vec{AM})\times\vec{B}\cdot d\vec{r}_M$

$v_A B r\cos\theta$. $\frac{1}{2}\omega r^2 B$.



$(\vec{\omega}\times\vec{HO})\times\vec{B}\cdot\vec{AB}$

例17



$\varepsilon_{ab}=?$

μ大

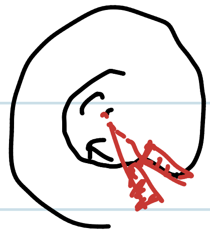
① $v_c = v_0\frac{R}{R+r}$ $\Delta S_1 = -v_0\Delta t(r+R)$

$\Delta S_2 = \frac{1}{2}\omega\Delta t(R^2-r^2)$

$\varepsilon = B\frac{\Delta S_1+\Delta S_2}{\Delta t} = \left[-v_0(r+R)+\frac{1}{2}\frac{v_0R}{R+r}(R^2-r^2)\right]B$

② $\vec{v}_M = \vec{v}_c + \vec{v}_{cM}$ → 转动.

平动.



$\int \vec{v}_c\times\vec{B}\cdot d\vec{r}_M$ → $\frac{1}{2}\omega(R^2-r^2)B$.

$=\vec{v}_c\times\vec{B}\cdot\int d\vec{r}_M$

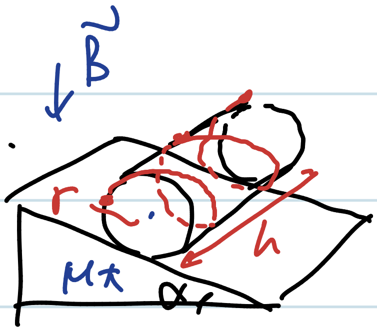
$= v_c B(R+r)$.

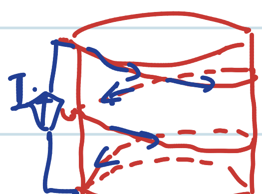
$\varepsilon = -\vec{B}\cdot\frac{\Delta\vec{S}}{\Delta t} = \frac{d\vec{B}\cdot\vec{m}}{dt\, I}$ ..

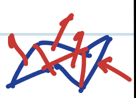
定电流, $\frac{\Delta\vec{B}}{\Delta t}=0$

$\vec{S}=?$

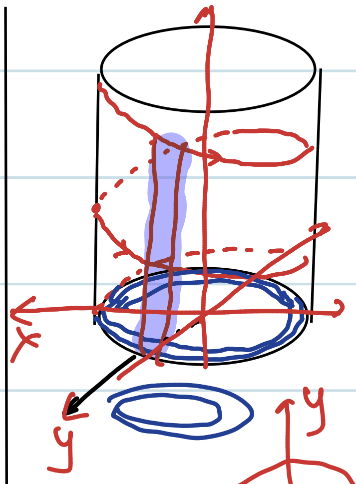
例158.

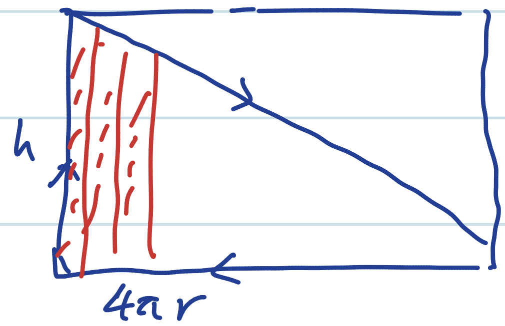






求力平衡条件. $E_p = -\vec{m}\cdot\vec{B}$





$d\theta$: $ds = r\,d\theta\cdot\frac{\theta}{4\pi r}\cdot h$.

$\hat{s} = (\cos\theta, \sin\theta)$

$\vec{m}_{侧} = I\cdot\int_0^{4\pi} r\frac{\theta h}{4\pi r}(\cos\theta\hat{x}+\sin\theta\hat{y})\,d\theta$

$\int_0^{4\pi}\theta(\cos\theta\hat{x}+\sin\theta\hat{y})\,d\theta$

$+\vec{M}_{底}=2\pi r^2\hat{z}$

$=\int_0^{4\pi}\theta\, e^{i\lambda\theta}d\theta$

$\int_0^{4\pi} e^{i\lambda\theta}d\theta = \frac{1}{i\lambda}e^{i\lambda\theta}\Big|_0^{4\pi}$

$= \frac{e^{i\lambda\theta}-1}{i\lambda}$

$\frac{\partial}{\partial\lambda}\int_0^{4\pi}e^{i\lambda\theta}d\theta = \int_0^{4\pi} i\theta e^{i\lambda\theta}d\theta = \frac{i\theta e^{i\lambda\theta}}{i\lambda} - \frac{e^{i\lambda\theta}-1}{i\lambda^2}\Big|_0^{4\pi}$

令λ=1

$\theta\to\theta+d\theta$.

$\rho\to\rho-k\,d\theta$.

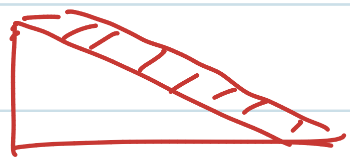
铁丝. 底. 铁丝单位角质量相同

重力转 $mg\cdot r\,d\theta\sin\alpha$

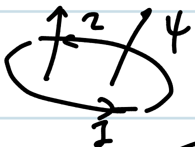
电源作功，贮能 $-\vec{m}\cdot\vec{B}$

$\int u\,dq = \int u\frac{dq}{dt}dt = \int 2\,d\psi$ → 构成面积

平动. 转动.



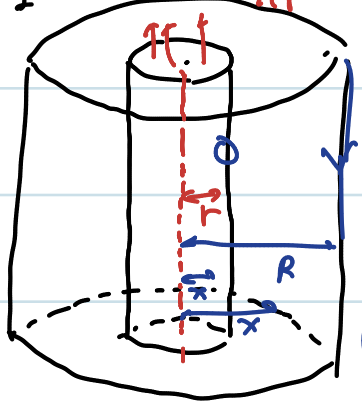
9.2 磁能.



$\psi=LI.$

$j=\frac{I}{\pi r^2}$ 均匀

例159



单位长度电感.

x处磁场

$B(x)=\frac{\mu_0\pi x^2 j}{2\pi x}$; $x<r$

$B(x)=\frac{\mu_0\pi r^2 j}{2\pi x}$; $R>x>r$

按电流大小作加权平均. ψ 为什么.

$x\sim x+dx.\quad dI=2\pi x\,dx\,j$

$$\frac{\psi(x)}{l}=\int_x^R B(\xi)\,d\xi$$

$$=\frac{\mu_0 j}{2}\left(\frac{R^2}{2}-\frac{x^2}{2}\right)+\frac{\mu_0 r^2 j}{2}\ln\frac{R}{r}$$

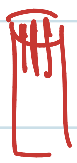
$$\frac{\psi_总 I}{l}=\int\frac{\psi(x)}{l}\,dI$$

$$=\frac{\mu_0 j}{2}\int_0^r\left[\left(\frac{R^2}{2}-\frac{\xi^2}{2}\right)+\ln\frac{R}{r}\right]2\pi\xi\,d\xi$$

$$\frac{\psi_总}{l}\cancel{I}=\frac{\mu_0 I}{2\pi r^2}\left[\frac{R^2}{2}R-\frac{R^3}{6}+\ln\frac{R}{r}(R-r)\right]$$

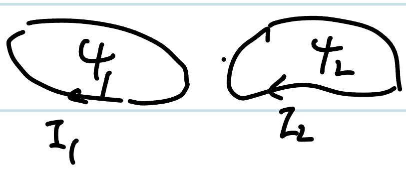
← 对于回路.

进电路 $dW=\int U\,dq$



$$=\int\psi\frac{dq}{dt}=\int\psi\,dI$$

例160. 互感



线性叠加 自感 互感.

$\psi_1(I_1,I_2)=a_{11}I_1+a_{12}I_2$

$\psi_2(I_1,I_2)=a_{21}I_1+a_{22}I_2$

$a_{12}=a_{21}$

$$\psi=\int\vec B\cdot d\vec S=\int(\nabla\times\vec A)\cdot d\vec S$$

$$=\int\vec A\cdot d\vec l\ \ni\oint\oint\frac{\mu_0\,d\vec l_1}{4\pi r}\cdot d\vec l_2$$

$$dW=\int(\psi_1 dI_1+\psi_2 dI_2)$$

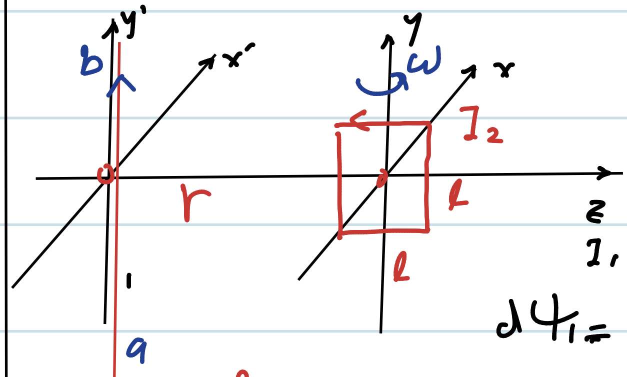
$$\int U_1dq_1+U_2dq_2$$

I_1: $0\to I_a\to I_a$; I_2: $0\to 0\to I_b$

I_1: $0\to 0\to I_a$; I_2: $0\to I_b\to I_b$

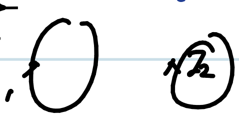
$a_{12}=a_{21}$

唯一性定理. W一样. $a_{21}=a_{12}$



I_2 向y轴以ω转动

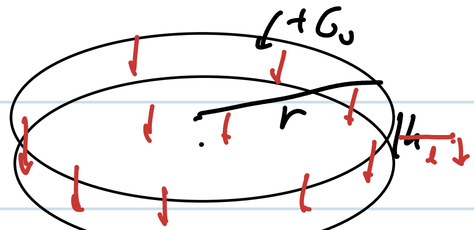
求t时. $\varepsilon_1=?$



$r>\frac{l}{2}$.

$$d\psi_1=L_1\frac{dI_1}{dt}+M_{12}\frac{dI_2}{dt}+dM_{12}\,I_2$$

$\frac{dM_{12}}{dt}=?=\frac{dM_{21}}{dt}$



$E=?$

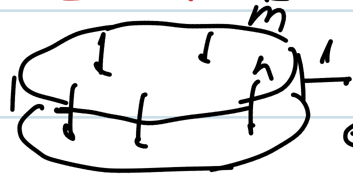
$r\gg l\gg h$

$l\gg h$.

电容极一致等价

$\Phi=\frac{\sigma\pi r^2 h}{\pi r^2}=\sigma h\to E$

$E\to B.$ $\quad m\to B.$

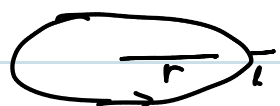


$B=?$ $\quad E=\frac{q}{4\pi\epsilon r^2}$ $\quad B=\frac{q_m\mu}{4\pi r^2}$

$\sigma h\to m$ $\quad\vec E\to\vec B.$

$\frac{1}{\epsilon}\to\mu$

$l\gg h$.

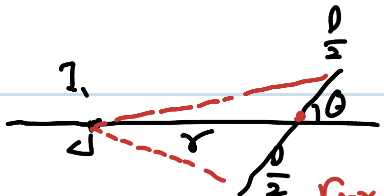


$B=\frac{\mu_0}{2\pi}\frac{I\cdot\pi r^2}{l\cdot\pi r^2}$

$l\ll r$ $\quad=\frac{\mu_0\,\pi r^2\cdot m}{2\pi\,l\cdot\pi r^2}$

$I\pi r^2=M.$

$E=\frac{\mu_0\pi r^2\cdot\sigma h}{\epsilon_0\,2\pi\,l\cdot\pi r^2}$



$M_{21}=\frac{\psi_2}{I_1}$

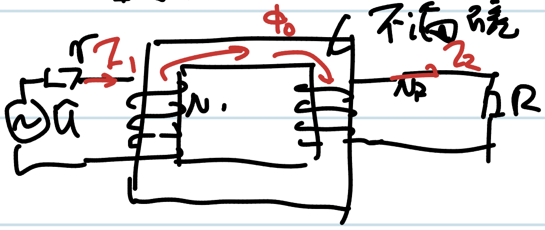
$$M_{21}=\frac{1}{I_1}\int_{r_-}^{r_+}l\cdot\frac{\mu_0 I_1}{2\pi\xi}\,d\xi.$$

$$M_{21}=\frac{l\mu_0}{2\pi}\cdot\ln\sqrt{\frac{r^2+(\frac{l}{2})^2+2r\frac{l}{2}\cos\theta}{r^2+(\frac{l}{2})^2-2r\frac{l}{2}\cos\theta}}$$

$$\varepsilon_1=\frac{dM_{21}}{dt}\cdot I_2$$

$$=I_2\frac{l\mu}{2\pi\cdot 2}\left[\frac{-2rl\sin\theta\,\omega}{r^2+(\frac{l}{2})^2+2rl\cos\theta}-\frac{2rl\sin\theta\,\omega}{r^2+(\frac{l}{2})^2-2rl\cos\theta}\right]$$

9.4 磁路



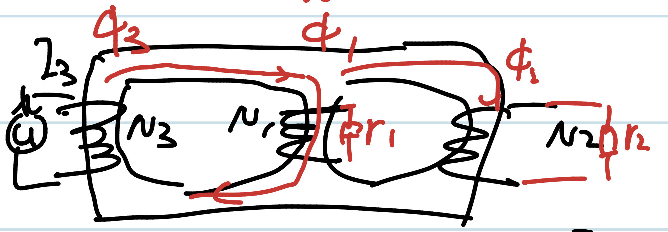
$R=?$

$\varepsilon_1 = N_1\frac{d\phi_0}{dt}$, $\varepsilon_2 = N_2\frac{d\phi_0}{dt}$ $\frac{\varepsilon_1}{\varepsilon_2}=\frac{N_1}{N_2}$

$\Delta E=0$ $I_1\varepsilon_1 = I_2\varepsilon_2$.

$\tilde{U} = \tilde{I}_1 r + \frac{N_1}{N_2}\tilde{I}_2 r = \underline{\tilde{I}_1 r + (\frac{N_1}{N_2})^2 \tilde{I}_1 R}$.

準.



$\frac{\varepsilon_1}{\varepsilon_2}=\frac{N_1}{N_2}$

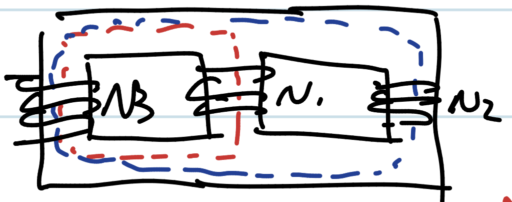
$\frac{I_3}{I_2}=\frac{N_2}{N_3}\cdots$ ✗

$\phi_3=\phi_1+\phi_2$

$\frac{\varepsilon_3}{N_3}=\frac{\varepsilon_1}{N_1}+\frac{\varepsilon_2}{N_2}$ $\underline{\varepsilon_3 I_3 = \varepsilon_1 I_1 + \varepsilon_2 I_2}$

$\oint \vec{H}\cdot d\vec{l} = I_{内}$ $\oint \vec{B}\cdot d\vec{S}=0$ ϕ ε

$\oint \vec{E}\cdot d\vec{l} = \varepsilon$ $\oint \vec{j}\cdot d\vec{S}=0$ I I



$BL \sim \frac{\phi}{S}\cdot l$

$\oint \vec{H}\cdot d\vec{l} = \Sigma I_内 = \oint \frac{\vec{B}}{\mu_0\mu_r}\cdot d\vec{l}$ 磁导率.

$\frac{\phi\cdot l}{\mu_0\mu_r S}$ ⇒ 磁阻 $\phi\cdot R_m = \Sigma I_内$

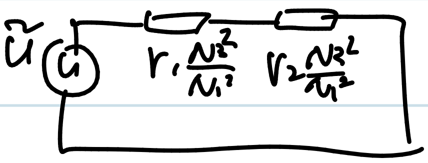
不漏磁 $\mu_r\to\infty$ ⇒ $R_m\to 0$. $\Sigma I_内\to 0$

$N_3 I_3 = N_1 I_1$. $N_3 I_3 = N_2 I_2$

$\frac{\varepsilon_3}{N_3}=\frac{\varepsilon_1}{N_1}+\frac{\varepsilon_2}{N_2}$

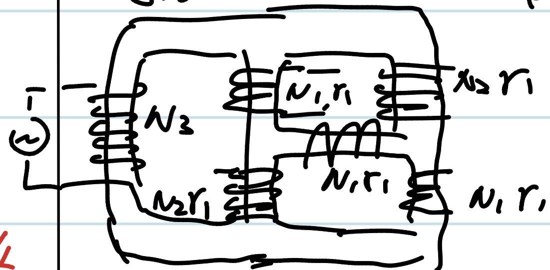
$\varepsilon_3 = \frac{N_3}{N_1}\cdot I_1 r_1 + \frac{N_3}{N_2} r_2$

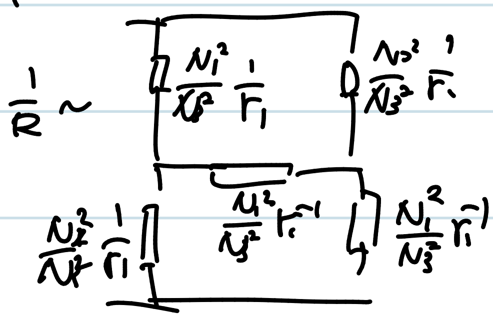
$= \frac{N_3}{N_1}\cdot\frac{N_3}{N_1} I_3 r_1 + \frac{N_3}{N_1}\cdot\frac{N_3}{N_1} I_3\cdot r_2$.



等

例161. 求功率=?



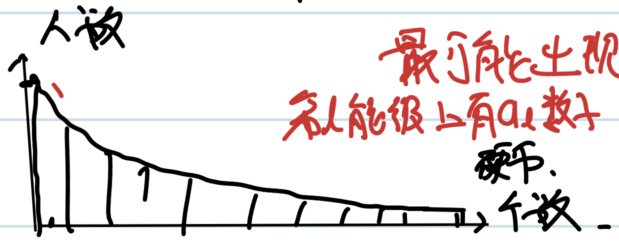


10. 建模 迁移类比

例162

有100个人,每个人有100硬币. 每次随机 硬币给一个人

$t\to\infty$, 硬币分布? 趋向于不均匀分布.



最可能出现的宏观态.

各人能级上有a_i数目 $\{a_i\}$.

不变.

硬币→能量. 人→粒子.

ΣN不变 ΣE不变. $\boxed{自由作用}$

$\underline{终态. M\text{-}B 分布}$?

能多少硬币 ↔ 级别). 一个能级上粒子数.

$\beta=\frac{1}{kT}$.

∴ ε_i: $P_i = Ae^{-\beta\varepsilon_i}$

$\Sigma P_i = 1$. $\sum_{i=1}^{N} Ae^{-\beta\varepsilon_i}=1$ ①

每个人有m个ε_0. $\sum_{i=1}^{N} A\cdot e^{-\beta i\varepsilon_0}\cdot i\varepsilon_0 = \bar{\varepsilon} = m\varepsilon_0$

$A.\beta$定了.

一个确定的硬币出现在一个确定的人上概率 $\frac{1}{N}$.

等概率.

$\{a_i\}$宏观态. →(一个微观态概率). 微观态个数

$(\frac{1}{N})^{Nm}$ $\Omega_{\{a_i\}}$

$\Omega_{\{a_i\}} = \Pi\, C_N^{a_1}\cdot C_{N-a_1}^{a_2}\cdot C_{N-a_1-a_2}^{a_3}\cdots\cdots C_{a_N}^{a_N}$

a_1 a_2 个人

1能级.

$=\Pi\cdot\frac{N!}{a_1!(N-a_1)!}\cdot\frac{(N-a_1)!}{a_2!(N-a_2)!}\cdots\cdots$

$=\Pi\frac{N!}{\Pi a_i!}$

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$\ln\Omega = A - \sum \ln a_i!$ $\ln a_i! \approx a_i\ln a_i - a_i$

$\ln\Omega_{\{a_i\}} = A' - \sum_i (a_i\ln a_i - a_i)$ $S = k\ln\Omega r$

$m_1 \; m_2 \; m_3 \; m_4$ $S = -\sum n_i \ln n_i$ □.

设 $\{a_i\}$ 使 $\ln\Omega_{\{a_i\}}$ max.

$\sum a_i = N$ ①

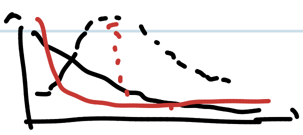
$\sum i a_i = Nm$ ② 熵最大原理.

令 $S' = \ln\Omega + \alpha\sum_l a_l + \beta\sum_l l a_l$

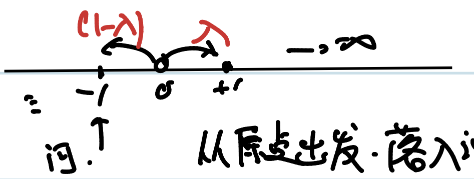
$\frac{\partial S'}{\partial a_l} = 0$ H定理

$\Rightarrow -\ln a_l + \alpha + \beta\cdot l = 0$.

$a_l = e^{-\alpha-\beta l}$. α, β, 由①.②定.



例.163. 随机游走



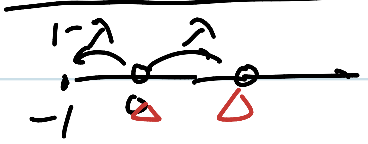
从原点出发.落入河中概率

$P(x,t) = \lambda P(x-1,t-1) + (1-\lambda)P(x+1,t+1)$

$x\ge1$.

$x=0$ $P(0,t) = (1-\lambda)P(1,t)$.

$P(x,0) = \begin{cases}1 & x=0\\ 0 & x\neq0\end{cases}$, 对时间 $t\to\infty$



落入河 $f(\lambda)$.

$f(\lambda) = (1-\lambda) + \lambda\cdot g(\lambda)$

0→-1. 从+1出发走到-2

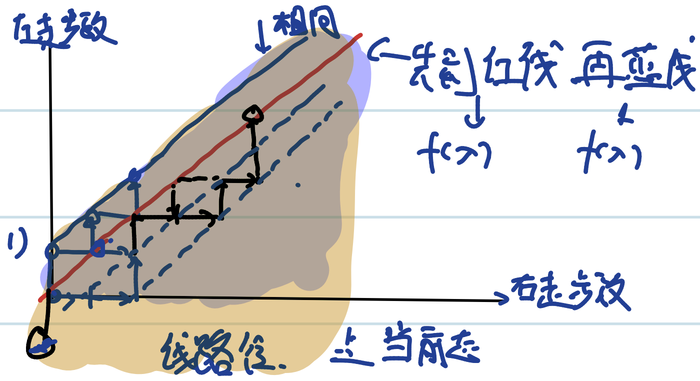
$g(\lambda) = f(\lambda)\cdot f(\lambda)$ 0→-1 -1→-2.

$f(\lambda) = (1-\lambda) + \lambda f^2(\lambda)$ 解得

$\lambda f^2(\lambda) - f(\lambda) - (1-\lambda) = 0$

$(\lambda f(\lambda) - (1-\lambda))(f(\lambda) - 1)$

$f(\lambda) = \begin{cases}1 & \lambda<1/2\\ \frac{1-\lambda}{\lambda} & \lambda>1/2\end{cases}$



0→-1. 所有态.

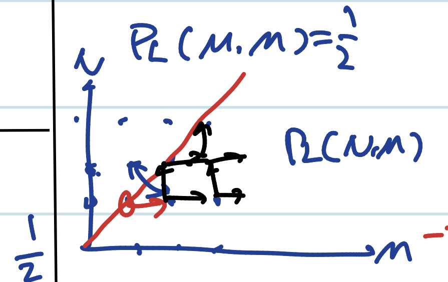
0→-2

-M $\frac12$ $\frac12$ 0 N 河.

Bake 到M概率. $P_L(M,N) = ?$

$P_L(M,N) + P_R(M,N) = 1$

$P_R(M,N) = P_L(N,M)$



$P(1,1) = \frac12$

$P(2,1) = ?$

-2 -1 0 1

先到-1. 再到-2.

$P_L(2,1) = P_L(1,1)\cdot[\frac12 + \frac12 P_L(2,1)]$

解得: $P_L(2,1) = \frac13$. $P_L(1,2) = \frac23$

$P(3,1)$ -3 -2 -1 0 1

$P_L(3,1) = P_L(2,1)(\frac12 + P_L(2,2))$ $P_L(3,1) = \frac14$.

$P_L(4,1) = P_L(3,1)(\frac12 + P_L(2,3))$ 递推方程解

← 再估一次

$P_L(M,N) = \frac{N}{M+N}$ ✓

$\frac12$ $\frac12$ 先分别尝.到.右,左

↑支点. 支平衡.

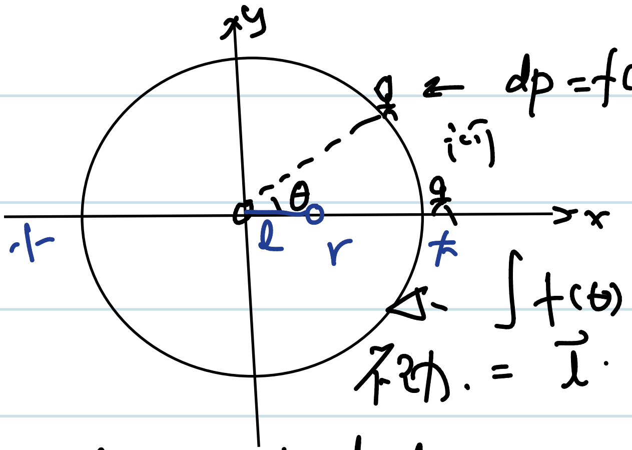
再走一次.支平衡. 直接不动.平衡

最后:

$P_L(M,N)\cdot m$. $P_R(M,N)\, m$ 杠杆平衡.

$\Rightarrow P_L(M,N) = \frac{N}{M+N}$

二维游走



$dp = f(\theta)\,d\theta$

$f(\theta) = \frac{1}{2\pi}$

$\int f(\theta)\,d\theta \cdot \vec{r}(\theta)$

不动 $= 1$

$f(x,y,t)\,dx\,dy$ $\quad 1 \ll l, r$

$\Delta t\ \frac{\partial f}{\partial t}(x,y,t) = -f + \frac{1}{4} f(x+1,y,t-1)$

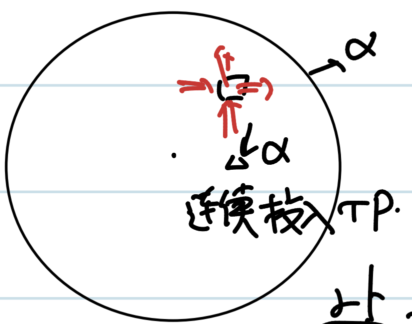
$+ \frac{1}{4} f(x-1,y,t-1) + \frac{1}{4} f(x,y+1,t-1) + \frac{1}{4} f(x,y-1,t-1)$

$\Rightarrow$ 微分. $\frac{\partial f}{\partial t} = \frac{l^2}{\Delta t}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$

$\frac{\partial f}{\partial t} = D\,\nabla^2 f$ 扩散.

信时. 要求只是 $t \to \infty$ 时概率分布

在时间轴上平铺



$\vec{I}(\vec{x},t) \to \vec{I}(\vec{x},0)$

$t \to \infty$ 稳定 $\quad \alpha - \frac{d\alpha}{dt}$

连续接入 TP. $dx\,dy$ 上 $f(x,y)\,dx\,dy = dN$

$\frac{\partial f}{\partial t} = 0$

$\Delta t = \lambda : \left[\frac{1}{4} f(x-\Delta x, y) - \frac{1}{4} f(x,y)\right]\Delta y$ 流进 流出

$+\left[\frac{1}{4} f(x, y-\Delta y) - \frac{1}{4} f(x,y)\right]\Delta x$

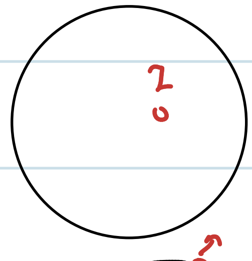
$+\left[\frac{1}{4} f(x+\Delta x, y) - \frac{1}{4} f(x,y)\right]\Delta y$

$+\left[\frac{1}{4} f(x, y+\Delta y) - \frac{1}{4} f(x,y)\right]\Delta x$

$= \Delta x \cdot \Delta y \cdot \nabla^2 f$

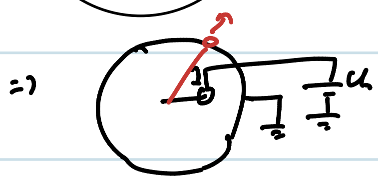
浓度 $f \to$ 电势. $\quad \nabla\cdot[\nabla f] \to$ 电荷变化 $\to 0$.

$\nabla f \to$ 电流 $\quad \nabla^2 \varphi = 0$

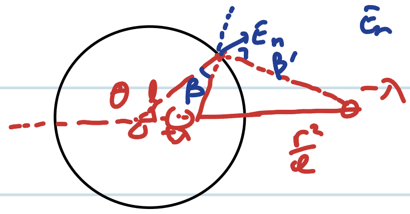


边界 $f \to 0$

(f 连续. 外 $f \to 0$)

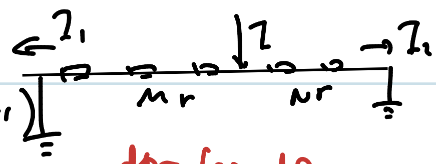


$\Rightarrow$ 电像法.



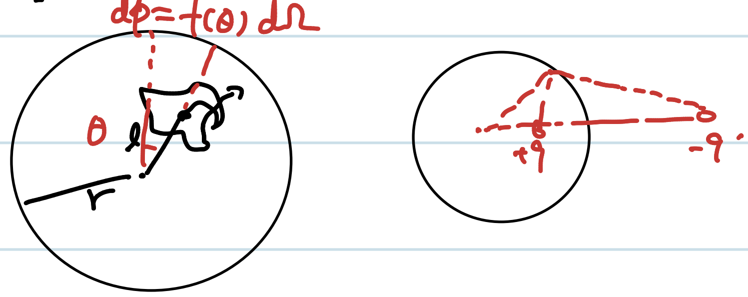
$E_n = \frac{\lambda}{2\pi\varepsilon_0}\frac{\cos\beta}{\sqrt{r^2+l^2-2lr\cos\theta}} + \frac{\lambda}{2\pi\varepsilon_0}\frac{\cos\beta'}{\sqrt{r^2+(\frac{r^2}{l})^2-2\cos\theta\frac{r^2}{l}}}$

$\propto f(\theta)$

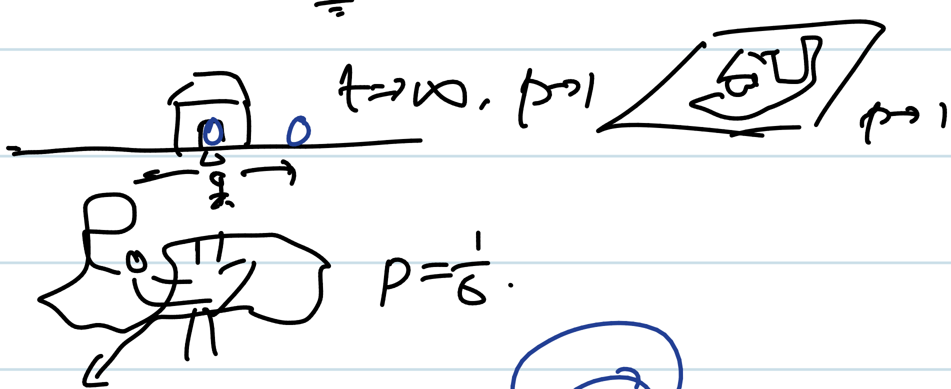


$I_i = \frac{N}{M+N} I$

$dp = f(\theta)\,d\Omega$



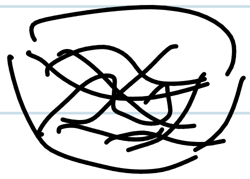
求流被贴到概率. $\frac{r}{l}$.



$t \to \infty,\ p \to 1$

$p = \frac{1}{6}$.

面条



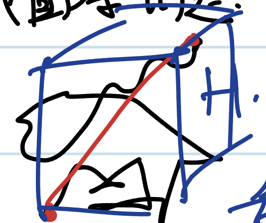
每条长L. 截S. $V=nSL$

每根面条与多少根面条接触

$N(L) = AL^{\alpha}$. 求$\alpha=?$

软 $\hat{n}(s) \to \hat{n}(s+l)$ 软 $\overline{\hat{n}(s)\cdot\hat{n}(s+l)}=0$

面条形态 ⇒ 随机走的人



$\overline{\vec{H}}=0$ $\overline{\vec{H}^2}\neq 0$

每N步.

$\overline{H_N^2} = \overline{(\vec{H}_{N-1} + l\cdot\hat{n})^2}$ $\overline{\quad\quad}=0$ 软

$= \overline{\vec{H}_{N-1}^2} + l^2 + \overline{\vec{H}_{N-1}\cdot\hat{n}l}$

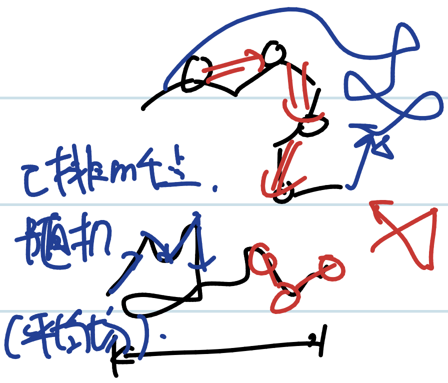
$\overline{H_N^2} = Nl^2$. $\sqrt{\overline{H_N^2}} = \sqrt{N}l$.

在H的体积中 面条有随机走 概率接触

$V \propto \sqrt{\overline{H_N^2}}^{\,3}$. 接触数 $\propto V\cdot n$.

$= \underbrace{N^{3/2}}_{\text{走N步}}\cdot n = L^{3/2}\cdot n$

高分子. 形态 ⇌ 随机游走

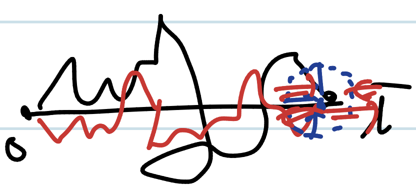


平均大小 $\propto N^{1/2}$.

高分子构位自配

长度为l的态出现概率.

Ω_l ← 长l微观态数.



$\Omega(\vec{L},N) = \int \Omega(\vec{L}+l\hat{n}, N-1)\cdot\frac{d\Omega}{4\pi}$

$= \int\left[\Omega(L) + \nabla\Omega\cdot l\hat{n} + \frac{1}{2}l\hat{n}\hat{n}\nabla\nabla\Omega\right]\frac{d\Omega}{4\pi}$

$= l^2\nabla^2\Omega(\vec{L}, N-1)$

Ω 扩散方程

$\Omega_l \propto Be^{-\frac{Al^2}{N}}$ $\prod_m (1-m\cdot n/6)$. ← 随机

$\llcorner\ e^{-\frac{n l_0^2}{N^2}}$

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