相对论

四维矢量 $x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$ $x_\mu = \begin{pmatrix} ct \\ -x \\ -y \\ -z \end{pmatrix}$ ⇒ $\sum x^\mu x_\mu = (ct)^2 - x^2 - y^2 - z^2$.

: $\begin{pmatrix} ict \\ x \\ y \\ z \end{pmatrix}$ ⇒ $(ict\ \ x\ \ y\ \ z)\begin{pmatrix} ict \\ x \\ y \\ z \end{pmatrix}$ ⇒ $-(ct)^2 + x^2 + y^2 + z^2$

$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$. $(ct\ \ x,\ y\ \ z)$ ⇒ $(ct\ \ x\ \ y\ \ z)\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$.

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

τ: 固有时

x^μ ⇒ 矢量·标量 ⇒ 矢量

$\frac{d\,矢量}{d\,标量}$ ⇒ 矢量

x^μ ⇒ $\boxed{U^\mu = \frac{dx^\mu}{d\tau}}$ 四维速度.

$a^\mu = \frac{dU^\mu}{d\tau}$. (矢量)

$\boxed{p^\mu = m_0 U^\mu}$ 四维动量 (矢量).

S: p^μ S': p'^μ $x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$.

$U^\mu = \frac{dx^\mu}{d\tau}$. $\frac{dx}{d\tau} = \frac{dx}{dt}\cdot\frac{dt}{d\tau} = v\cdot\gamma$.

τ → S:

⇒ $U^\mu = \gamma_1\begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix}$ τ → S': $U'^\mu = \gamma_2\begin{pmatrix} c \\ v_x' \\ v_y' \\ v_z' \end{pmatrix}$

S → S': γ. $\gamma_2\begin{pmatrix} c \\ v_x' \\ v_y' \\ v_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\gamma_1\begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix}$

⇒ $\left.\begin{aligned} \gamma_2 c &= \gamma_1(\gamma c - \beta\gamma v_x). \\ \gamma_2 v_x' &= \gamma_1(-\beta\gamma c + \gamma v_x) \end{aligned}\right\}$ ⇒ $v_x'/c = \frac{v_x/c - \beta}{1 - \beta v_x/c}$

$p^\mu = m_0 U^\mu$ ⇒ $\begin{pmatrix} m_0\gamma c \\ m_0\gamma v_x \\ m_0\gamma v_y \\ m_0\gamma v_z \end{pmatrix} = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$

$E^2/c^2 - (p^2) = E_0^2/c^2 =$ 常.

$$\begin{pmatrix} E'/c \\ p_x' \\ p_y' \\ p_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}.$$

对质心系: 四维动量守恒定理.

$p_1^\mu + p_2^\mu = p_t^\mu$.   可以找到一个系. $p_t^\mu = 0$.

0动量参考系. (质心系). E_{t0}: 相对质心能量.

⇒ $p_t^\mu = \begin{pmatrix} E_t/c \\ p_{tx} \\ p_{ty} \\ p_{tz} \end{pmatrix}$ ⇒ $E_t^2 - p_t^2c^2 = E_{t0}^2$

$v/c \to 0$. $E - \frac{p_t^2}{2m_0} = E_r$.

例. 4D动量解碰撞   换质心系.

① 找到质心系. 地系: p_t^μ.   质心系: $p_t'^\mu$.

⇒ $\begin{cases} E_t'/c = \gamma(E_t/c - \beta p_{tx}) \\ p_{tx}' = \gamma(p_{tx} - \beta E_t/c). \\ p_{ty}' = p_{ty} \\ p_{tz}' = p_{tz}. \end{cases}$

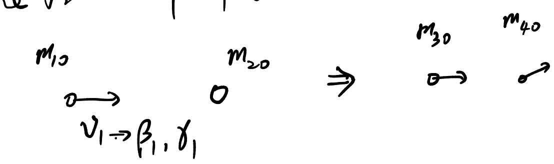
令 $p'_{tx} = p'_{ty} = p'_{tz} = 0$.

⇒ $p_{ty} = 0$. $p_{tz} = 0$.

令 $p_{tx} - \beta E_t/c = 0$

⇒ $\beta = \frac{p_{tx}}{E_t/c}$.

② 在质心系中解决问题.



问, 要撞出 v_1 至少为多少?

↕ 质心系中.

前: 地 $\begin{cases} E_t = m_{10}\gamma_1 c^2 + m_{20}c^2 \\ p_t = m_{10}\gamma_1\beta_1 c. \end{cases}$

$\boxed{(E_t/c)^2 - p_t^2} = (E_t'/c)^2 = \boxed{(m_{30}c^2 + m_{40}c^2 + \Delta E)^2}/c^2$.

左 $= (m_{10}\gamma_1 c^2 + m_{20}c^2)^2 - (m_{10}\gamma_1\beta_1 c)^2$

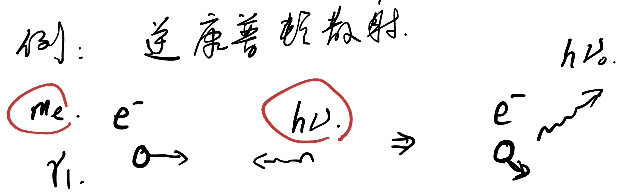
$= m_{10}^2c^4 + m_{20}^2c^4 + 2m_{10}m_{20}c^4\gamma_1$

只需在质心系中 $\Delta E = 0$

⇕ m_{30}、m_{40} 共同运动.

⇒ v_1 最小

例: 逆康普顿散射.



(1). 求出射光子的最大能量.

(2) 设质心系中. 光子 4π 均匀出射. 地系中. 一半的光子集中在 Δθ 的角度内. 求 Δθ=?

(3). Δθ 角上出射光子频率是多少?

例：逆康普顿散射.

m_e e^- γ_1 $h\nu$ ⇒ e^- $h\nu_0$

(1). 求出射光子的最大能量.

(2) 设质心系中，光子 4π 均匀出射. 在地系中，一半的光子集中在 $\Delta\theta$ 的角度内. 求 $\Delta\theta=?$

(3). $\Delta\theta$ 角上出射光子频率是多少？

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(1). 地系：e^- γ $h\nu$　　后：$h\nu_0$

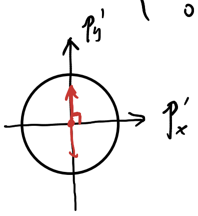
质心系：p'　$h\nu'=p'c$ ⇒ p'　$h\nu'$

碰撞：只改变方向

① 找质心系　$\beta = \frac{P_{tx}}{E_t/c} = \frac{m_e c^2\sqrt{\gamma_1^2-1}-h\nu}{m_e\gamma_1 c^2+h\nu}$

⇒ 碰前　$h\nu' = h\nu\sqrt{\frac{1+\beta}{1-\beta}}$　光子：$\begin{pmatrix} h\nu'/c \\ -h\nu'/c \\ 0 \\ 0\end{pmatrix} = \begin{pmatrix}\gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & 0 \\ & & 1 & \\ 0 & & & 1\end{pmatrix}\begin{pmatrix} h\nu/c \\ -h\nu/c \\ 0 \\ 0\end{pmatrix}$

碰后：$h\nu'$　θ'

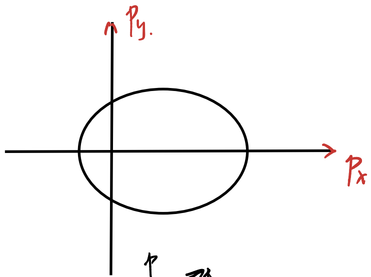


$\begin{pmatrix} h\nu'/c \\ h\nu'/c\cdot\cos\theta' \\ h\nu'/c\cdot\sin\theta' \\ 0\end{pmatrix}$

回到地系：$\begin{pmatrix} h\nu_0/c \\ p_x \\ p_y \\ 0\end{pmatrix} = \begin{pmatrix}\gamma & \gamma\beta & & 0 \\ \gamma\beta & \gamma & & \\ & & 1 & \\ 0 & & & 1\end{pmatrix}\begin{pmatrix} h\nu'/c \\ h\nu'/c\cdot\cos\theta' \\ h\nu'/c\cdot\sin\theta' \\ 0\end{pmatrix}$

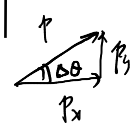
$$\Rightarrow \begin{cases} h\nu_0/c = \gamma\,(h\nu'/c + \beta\cdot h\nu'/c\cdot\cos\theta') \\ p_x = \gamma\,(\beta\cdot h\nu'/c + h\nu'/c\cdot\cos\theta') \\ p_y = h\nu'/c\cdot\sin\theta' \end{cases} \Rightarrow$$

$\sin^2\theta' + \cos^2\theta' = 1$



(1) 令 $\theta'=0$ ⇒ $h\nu_0/c = \cdots$

(2) 令 $\theta'=90^\circ$ ⇒ $\cos\theta=0$，$\sin\theta=1$.



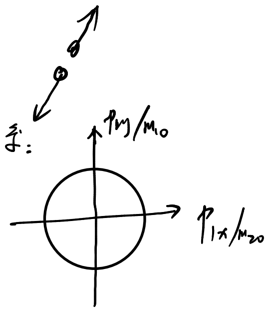
⇒ $\cos\Delta\theta = \frac{p_x}{p} = \frac{p_x}{h\nu_0/c} = \beta$.　$\beta\to 1$.　$\Delta\theta\to 0$.

(3). ⇒ $h\nu_0/c = \gamma h\nu'/c = \cdots$

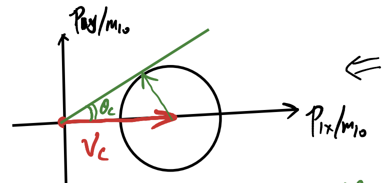
例：碰撞最大偏转角的问题.

m_{10} m_{20} v_1　弹性碰撞. 求碰后 m_{10} 最大偏转角. $m_{10} > m_{20}$.　(1) 经典　(2) 狭相.

(1). 质心系中：m_{10}：$\frac{m_{20}}{m_{10}+m_{20}}\cdot v_1$　m_{20}：$\frac{m_{10}}{m_{10}+m_{20}}\cdot v_1$ ⇒ 质心系：



$V_c = \frac{m_{10}c}{m_{10}+m_{20}}$



⇒ $\sin\theta_c = \frac{m_{20}}{m_{10}}$ ⇒ $\tan\theta_c = \frac{m_{20}}{\sqrt{m_{10}^2-m_{20}^2}}$

(2). 狭相；m_{10} γ_1 β_1　m_{20}　$\begin{pmatrix} m_{20}c \\ 0 \\ 0 \\ 0\end{pmatrix}$

① 找到质心系.　$E_{总} = \gamma_1\cdot m_{10}c^2 + m_{20}\cdot c^2$.

$P_{总} = \gamma_1\cdot\beta_1\cdot m_{10}\cdot c$

⇒ $\beta = \frac{P_{总}}{E_{总}/c} = \frac{\gamma_1\beta_1\cdot m_{10}c}{\gamma_1 m_{10}c + m_{20}c}$

碰前质心系：p'　p' ⇒ $\begin{pmatrix} E_2'/c \\ P_x \\ 0 \\ 0\end{pmatrix}$

⇒ $|p'| = |P_x| = \beta\gamma\cdot m_{20}\cdot c$.

碰后质心系：m_{10}　θ'　p'　p'　m_{20}　　m_1 后：$\begin{pmatrix} E_1'/c \\ p'\cdot\cos\theta' \\ p'\cdot\sin\theta' \\ 0\end{pmatrix}$

碰后地系：　$E_1'/c = \gamma\gamma_1\cdot m_{10}\cdot c - \beta\gamma\cdot\gamma_1\cdot\beta_1\cdot m_{10}\cdot c$

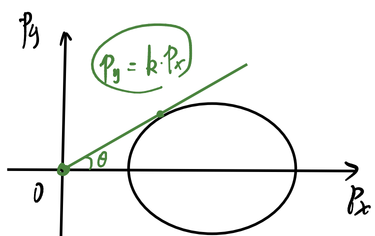
$\begin{pmatrix} E_1/c \\ P_x \\ P_y \\ 0\end{pmatrix} = \begin{pmatrix}\gamma & \gamma\beta & & \\ \gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1\end{pmatrix}\begin{pmatrix} E_1'/c \\ p'\cos\theta' \\ p'\sin\theta' \\ 0\end{pmatrix}$

⇒ $P_x = \beta\gamma(\gamma\gamma_1 - \beta\gamma\gamma_1\beta_1)\cdot m_{10}\cdot c + \gamma\beta\gamma\cdot m_{20}\cdot c\cdot\cos\theta'$

$= \underbrace{\beta\gamma^2\gamma_1(1-\beta\beta_1)\cdot m_{10}\cdot c}_{A} + \underbrace{\gamma^2\cdot\beta\cdot m_{20}\cdot c}_{a}\cdot\cos\theta'$

$P_y = \underbrace{\beta\gamma\cdot m_{20}\cdot c}_{b}\cdot\sin\theta'$

⇒ $\left(\frac{P_x - A}{a}\right)^2 + \left(\frac{P_y}{b}\right)^2 = 1$.



令 $P_y = kP_x$

$\left(\frac{1}{a^2}+\frac{k^2}{b^2}\right)P_x^2 + \left(-2\cdot\frac{A}{a^2}\cdot P_x\right) + \left(\frac{A}{a}\right)^2 - 1 = 0$.

相切：$\Delta=0$ ⇒ $4\left(\frac{A}{a^2}\right)^2 - 4\left[\left(\left(\frac{A}{a}\right)^2-1\right)\cdot\left(\frac{1}{a^2}+\frac{k^2}{b^2}\right)\right] = 0$

⇒ $k = \tan\theta = \sqrt{\frac{b^2}{A^2-a^2}}$

$\Rightarrow\ k=\sqrt{\dfrac{b^2}{A^2-a^2}} \overset{\text{暴算}}{=\!=} \cdots \sqrt{\dfrac{m_{20}^2}{m_{10}^2-m_{20}^2}} \Rightarrow$

5 经典情况一致

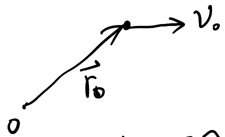
电磁.

叠加原理. 

$f(x_1,x_2)$

$f(\lambda_1 x_1,\lambda_2 x_2)=\lambda_1 f(x_1,0)+\lambda_2 f(0,x_2)$

线性 ⇒ 叠加原理

例: 平抛.  讨

$\vec r_t(\vec r_0,\vec v_0)\ \Rightarrow\ \vec r_t(0,0)=-\frac12 g\cdot t^2\hat y$

$\vec r_t(0,v_0\hat x)\ \Rightarrow\ \vec r_t(0,v_0\hat x)=v_0\cdot t\cdot\hat x-\frac12 gt^2\hat y$

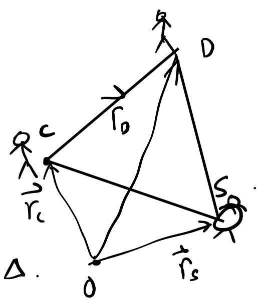
$\Rightarrow \vec r_t(0,v_0\hat x+v_y\hat y)=v_0 t\hat x+v_y\cdot t\cdot\hat y-\frac12 gt^2\hat y$

$\vec r_t(\vec r_0,0)=\vec r_0-\frac12 gt^2\hat y$

$\vec f_t(\vec r_0,\vec v)=\vec r_t(\vec r_0,\vec v)+\frac12 gt^2\hat y$

$\vec f_t(\vec r_0,\lambda\vec v_0)=\vec f_t(\vec r_0,0)+\lambda\vec f_t(0,\vec v_0)$

$|\vec a_s|\cdot(\vec v_c,\vec v_D,\vec a_c,\vec a_D)$

或 

正△.

$|\vec a_c|=a_0$, $|\vec a_D|=a_0$

$|\vec v_c|=v_0$, $|\vec v_D|=v_0$. 求$|\vec v_s|$至少多少

才能保持上. 若是线性

$\vec v_s(\vec v_c,\vec v_D)\xleftarrow{} \vec v_s(\vec v_c,0)+\vec v_s(0,\vec v_D)$

$\Rightarrow |\vec r_c-\vec r_D|=|\vec r_c-\vec r_s|=|\vec r_D-\vec r_s|$

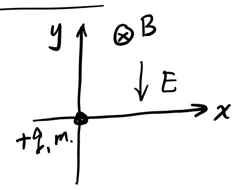
$\Rightarrow (\vec r_c-\vec r_D)\cdot(\vec r_c-\vec r_D)=(\vec r_c-\vec r_s)\cdot(\vec r_c-\vec r_s)$

$\Rightarrow 2(\vec r_c-\vec r_D)\cdot(\vec v_c-\vec v_D)=2(\vec r_c-\vec r_s)\cdot(\vec v_c-\boxed{\vec v_s})$

常　　常　　线性.

再求导 $\Rightarrow 2(\vec v_c-\vec v_D)\cdot(\vec v_c-\vec v_D)+2(\vec r_c-\vec r_D)(\vec a_c-\vec a_D)=\cdots$

$\Rightarrow \vec a_s(\vec v_c,\vec v_D,\vec a_c,\vec a_D)$ 不是线性.不能叠加.

例. 

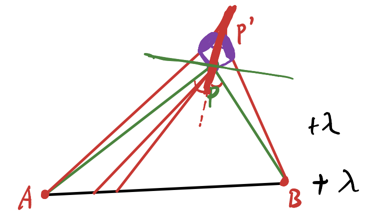
$\vec r_0=0.$

$m\dfrac{d\vec v}{dt}=q\vec E+q\vec v\times\vec B$

$=q(\vec u+\vec v)\times\vec B$ 其中 $\vec u\times\vec B=\vec E$

$\Rightarrow m\dfrac{d(\vec u+\vec v)}{dt}=q(\vec u+\vec v)\times\vec B$ 二个 总.

$\dfrac{d\vec u}{dt}=0.$ 一个.

例: 均匀带电线段 

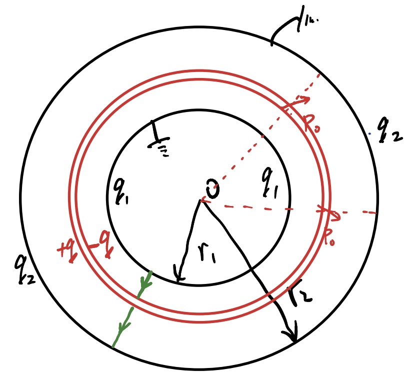
$\vec E_{r(q)}=\dfrac{kq}{r^2}\hat r.$

$\Rightarrow \vec E$: 双曲线.

$\Rightarrow \varphi$: 椭圆.

$\vec E$ 沿角平分线. $\Rightarrow AP'-BP'=AP-BP$ ⇒ 双曲线.

例: 叠加原理 + 恢复对称性.



在距O点l处放置一个电偶极子$\vec p_0$. 与$\vec l$夹θ.

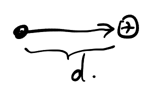
求内外壳上感应电荷总量.

$\vec p_0\to q_1,q_2$. $\vec p_0'\to q_1',q_2'$

$\vec p_0+\vec p_0'\to q_1'+q_1,\ q_2'+q_2$.

①.用叠加. ②用对称. ⇒ $\vec p_0$切向排一圈. ⇒ 偶极壳.

厚度 $d\cdot\cos\theta$.

$\vec p_0=q\cdot\vec d$ 

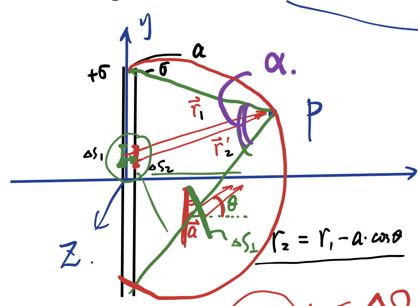
$\Rightarrow$ 中层. $\Delta U=-\dfrac{q_1-q}{4\pi\varepsilon_0 l^2}\cdot d\cos\theta$

内+外: $\Delta U'=-\dfrac{q_1}{4\pi\varepsilon_0}\displaystyle\int_{r_1}^{l}\frac{dr}{r^2}+\left(-\frac{q_1}{4\pi\varepsilon_0}\int_{l}^{r_2}\frac{dr}{r^2}\right)$

$\Rightarrow \Delta U+\Delta U'=0 \Rightarrow q_1=\cdots$ ✓. $q_2=-q_1=\cdots$

例: 电偶极层对某点P电势.

$\sigma,-\sigma$. 沿z方向无限延伸.



求P点处φ电势.

$\Delta\varphi_p=\Delta\varphi^{+\sigma}_{p(\vec r_1)}-\Delta\varphi^{-\sigma}_{p(\vec r_2)}$

$=\Delta\varphi_{p(\vec r_1)}-\Delta\varphi_{p(\vec r_1-\vec a)}$

$=\nabla(\Delta\varphi_{p(\vec r)})\cdot\vec a$

$=\nabla\left(\dfrac{\sigma\cdot\Delta S}{4\pi\varepsilon_0 r}\right)\cdot\vec a$

$=-\dfrac{\sigma\cdot\Delta S}{4\pi\varepsilon_0}\cdot\dfrac{\hat r\cdot\vec a}{r^2}$

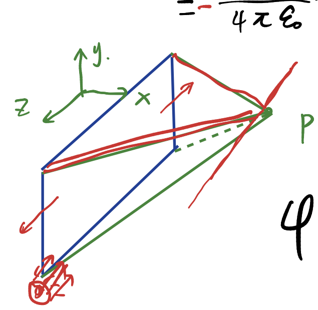
$=-\dfrac{a\cdot\sigma(\Delta S\cdot\cos\theta)}{4\pi\varepsilon_0\cdot r^2}$

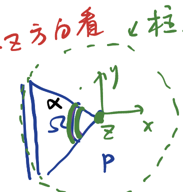
$\Delta\varphi_p=-\dfrac{a\cdot\sigma}{4\pi\varepsilon_0}\dfrac{\Delta S_\perp}{r^2}$ ← $\Delta\Omega$

$\Rightarrow \varphi_p=-\dfrac{a\sigma}{4\pi\varepsilon_0}\displaystyle\int d\Omega.$

$=-\dfrac{a\sigma}{4\pi\varepsilon_0}\cdot\Omega$ ← 立体角.

向-z方向看 ← 柱.





$\Omega=\dfrac{\alpha}{2\pi}\cdot4\pi=2\alpha$

$\varphi_p=-\dfrac{a\sigma}{4\pi\varepsilon_0}\cdot\Omega=-\dfrac{a\sigma}{4\pi\varepsilon_0}\cdot2\alpha \Rightarrow$ 等势线 ⇓ 等α线 ⇓ 圆

$0\sim\pi$

例. 求证：范德瓦尔斯力 $\propto \frac{1}{r^7}$. $\varphi \propto \frac{1}{r^6}$.

某种因素. 产生一个 p_0. $\vec{p}' \propto \vec{E}$ $\vec{p}' = \alpha \cdot \vec{E}$. 常

$$\vec{E}_{A\to B} = \frac{2 \cdot p_0}{4\pi\varepsilon_0 r^3} \cdot \hat{r} \Rightarrow \vec{p}' = \alpha \cdot \frac{2p_0}{4\pi\varepsilon_0 \cdot r^3} \hat{r}$$

$\vec{p}'$ 对A的作用力: $\boxed{\vec{E}_{B\to A}} = \frac{2p'}{4\pi\varepsilon_0 r^3} \cdot \hat{r}$. 对$p_0$的作用.

$\Rightarrow$ p_0 的受力: $F = \frac{2p_0'}{4\pi\varepsilon_0} \cdot \left(\frac{1}{r^3} - \frac{1}{(r+l)^3}\right) \cdot q$.

$F = p \boxed{\frac{\partial E_x}{\partial x}} \Rightarrow p_0 \cdot ○ \cdot \cancel{\frac{d(\frac{1}{r^3})}{dr}}$

对场点位置进行操作.

$$\Rightarrow F = \frac{2p_0'}{4\pi\varepsilon_0} \cdot \frac{1}{r^3}\left(1 - 1 + \frac{3l}{r}\right) \cdot q = \frac{6○ \cdot \frac{1}{r^3}}{4\pi\varepsilon_0 r^3} \cdot \frac{p_0}{r} = ○ \cdot \frac{1}{r^7}$$

静电问题通解.

① 对称性高 + 高斯. → 猜得.

② 电像 → 猜得.

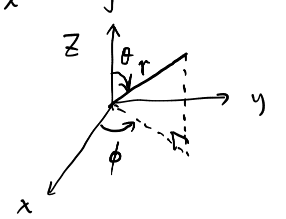
③ Laplace. 观察通解形式 → 猜得.

无电荷: $\nabla \cdot \vec{E} = 0 \Rightarrow \nabla^2 \varphi = 0$.

有电荷: $\nabla \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \nabla^2 \varphi = -\rho/\varepsilon_0$.

直: $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \Rightarrow \varphi = \sum_k e^{i(k_x x + k_y y + k_z z)}$ $k_x^2 + k_y^2 + k_z^2 = 0$.

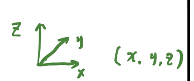
球坐标:



(r, θ, φ).

$$\varphi = \sum \left(A_n r^n + B_n \frac{1}{r^{n+1}}\right) P_n^m(\cos\theta) \cdot (C_m \cos(m\phi) + D_m \sin(m\phi)).$$

勒让德多项式. (多项式).



(x, y, z)

ϕ对称: $\varphi = \sum \left(A_n r^n + B_n \frac{1}{r^{n+1}}\right) P_n(\cos\theta)$. $n=0$ $n=1$ $n=2$

$P_0(x) = 1$, $P_1(x) = x$. $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$, $P_3(x) = \cdots$

$$\varphi_{(r,\theta)} = \left(A_0 + B_0 \cdot \frac{1}{r}\right) + \left(A_1 r + B_1 \cdot \frac{1}{r^2}\right) \cdot \cos\theta + \cdots$$

常 点电荷 匀强场. 电偶极 电四极 ...

再用边界条件 确定 A_n、B_n.

边界: ① $\varphi(\vec{r})$, $\vec{r} \in$ 边界上

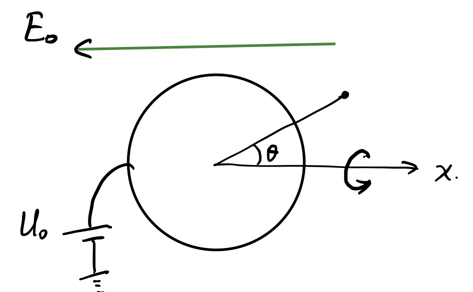
② $\frac{\partial \varphi}{\partial n}(\vec{r}) = \vec{E}_n(\vec{r})$, $\vec{r}$ 在边界上.

$\varepsilon_{r1} E_{n1} = \varepsilon_{r2} \cdot E_{n2}$.

对均匀. 线性. 各向同性. 无色散介质

例. 匀强电场中. 球形区域. 求电势分布.

(1) 金属球



球外 → ∞ 空间: $\varphi = \left(A_0 + \frac{B_0}{r}\right) + \left(A_1 r + B_1 \frac{1}{r^2}\right) \cos\theta + ○$

$r \to \infty$: $\varphi = E_0 \cdot r \cdot \cos\theta$ = $\varphi = A_0 + A_1 r \cos\theta + \sum_{n\geq 2} A_n r^n P_n(\cos\theta)$

$A_0 = 0$. $A_1 = E_0$. $A_n (n \geq 2) = 0$.

$B_2 \cdot \frac{1}{R^3} \cdot \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$

$r \to R$. $\varphi = \boxed{U_0}$, $+ \square \cos\theta$

$$\varphi = \boxed{\frac{B_0}{R}} + \boxed{\left(E_0 R + \frac{B_1}{R^2}\right)} \cos\theta + \sum B_n \frac{1}{r^{n+1}} P_n(\cos\theta)$$

对比 $P_0(\cos\theta)$ $P_1(\cos\theta)$ $P_2(\cos\theta)$

$B_n = 0\ (n \geq 2)$.

$\Rightarrow \frac{B_0}{R} = U_0 \Rightarrow B_0 = U_0 \cdot R$. $E_0 R + \frac{B_1}{R^2} = 0$

$\Rightarrow B_1 = -E_0 R^3$.

$$\Rightarrow \varphi = \frac{U_0 R}{r} + \left(E_0 r + \frac{-E_0 R^3}{r^2}\right) \cdot \cos\theta + 0.$$

感应电荷 原场. 电偶极的影响.

(2). 介质情形.