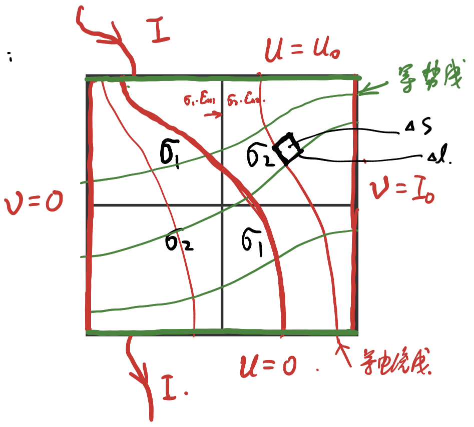
例：



$t\to 0$，厚度，电导率。

$R=\frac{U}{I}=?$

$R=\frac{1}{t\sqrt{\sigma_1\sigma_2}}$

U：等电势线，V：等电流线

$j=\sigma\cdot E$

$\frac{\Delta U}{\Delta S}=E$，$\frac{\Delta V}{\Delta l\cdot t}=j \Rightarrow \frac{\Delta V}{\Delta l}=j\cdot t$

$j\cdot\Delta l=\sigma\cdot E\cdot\Delta l=\Delta I/t$

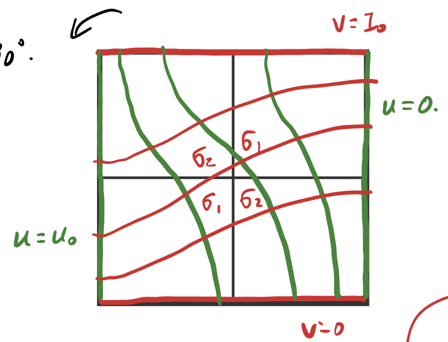
$j=\sigma E \Rightarrow \boxed{\frac{\Delta V}{\Delta l}=t\cdot\sigma\cdot\frac{\Delta U}{\Delta S}}$ ※

u：拉普拉斯方程　$u\perp v$

边界：✓　$\frac{\Delta u}{\Delta s}(\sigma_1)=\frac{\Delta u}{\Delta s}(\sigma_2)$

复变换：$f(x,y)=u+iv$　u：势，v。

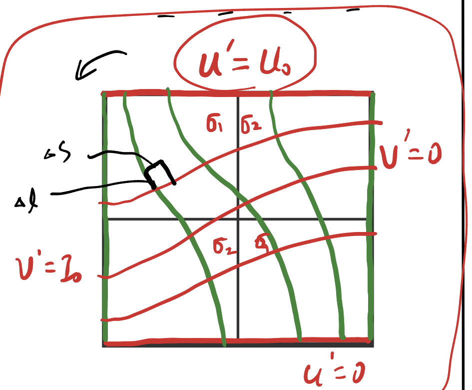
转90°



令：$u'=\frac{u_0}{I_0}\cdot V$

令：$V'=\frac{I_0}{u_0}\cdot u$

令：$\sigma_1\Leftrightarrow\sigma_2$



边界：u' ✓

使 $j'=\sigma' E'$

$\frac{\Delta V'}{t\Delta S}=\sigma'\cdot\frac{\Delta u'}{\Delta l}$

$\frac{I_0}{u_0}\cdot\frac{\Delta u}{t\cdot\Delta S}=\sigma'\cdot\frac{u_0}{I_0}\cdot\frac{\Delta V}{\Delta l}$

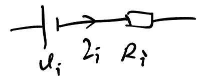
$\frac{I_0}{u_0}\cancel{\frac{\Delta u}{\Delta S}}=\sigma'\cdot\frac{u_0}{I_0}\cdot\sigma\cdot t^2\cdot\cancel{\frac{\Delta u}{\Delta S}}$

$\sigma'\cdot\sigma\equiv\sigma_1\cdot\sigma_2$

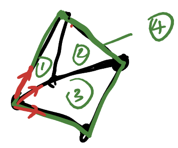
$\Rightarrow \frac{I_0}{u_0}=\sigma_1\cdot\sigma_2\cdot\frac{u_0}{I_0}\cdot t^2 \Rightarrow R_2\ \frac{u_0}{I_0}=\frac{1}{t\sqrt{\sigma_1\sigma_2}}$

电路：基尔霍夫：节点：$\oint\sigma\vec{E}\cdot d\vec{S}=0 \Rightarrow \sum I=0$

回路：$\oint\vec{E}\cdot d\vec{l}=0 \Rightarrow$



网络：



V个顶点　E条边　F个区域

I：$\begin{cases}\text{有}(V-1)\text{个独立电压：}U_i,\ i=1,2,\cdots(V-1)\\ \Rightarrow\text{有}(V-1)\text{个独立节点方程：}\sum_j I_{ij}=0\end{cases}$

II：设圈电流：$(F-1)$个圈电流

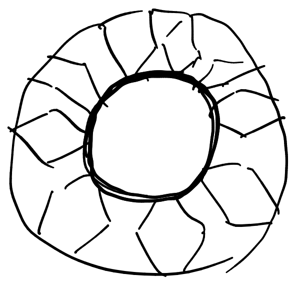
基尔回路：$\sum U_{ij}=0$　$(F-1)$个回路方程

III：设E个电流，约束：节点：$\sum_j I_{ij}=0$，$(V-1)$个

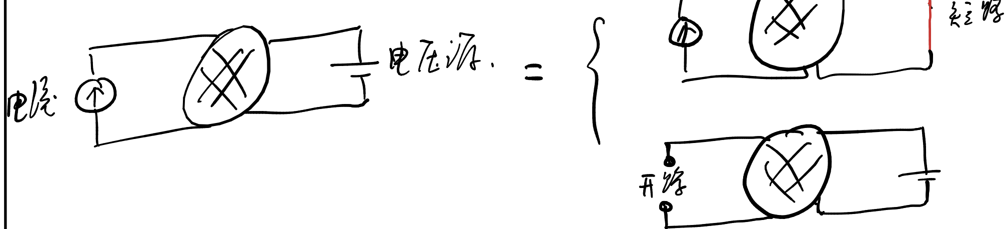
回路方程：$(F-1)$个，$\sum U_{ij}=0$

$\boxed{E=V-1+F-1=V+F-2}$ ← 欧拉公式

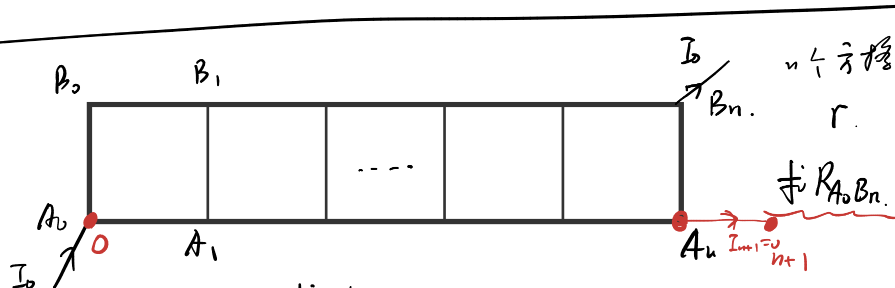
简单多面体



叠加：

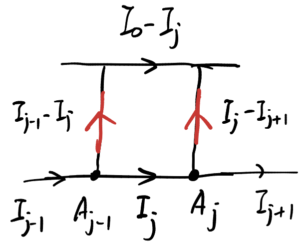


例：



n个方程。r。求$R_{A_0B_n}$

加入I_0，求U，I_j。



$\Rightarrow$ 回路：

$I_j+(I_j-I_{j+1})-(I_0-I_j)-(I_{j-1}-I_j)=0$

$\Rightarrow -I_{j-1}+4I_j-I_{j+1}=I_0$ ← 非齐次 L

令. $I_j = \frac{I_0}{2}+a_j$. $I_{j-1}=\frac{1}{2}+a_{j-1}$

⇒ $a_{j-1}-4a_j+a_{j+1}=0$. ← 齐次. $A\lambda^{j-1}$

令 $a_j = A\lambda^j$ $\quad 1-4\lambda+\lambda^2=0$.

⇒ $\lambda=\frac{4\pm\sqrt{12}}{2}=\boxed{2\pm\sqrt{3}}$.

⇒ 通解: $a_j = A_+\cdot\lambda_+^j + A_-\lambda_-^j$

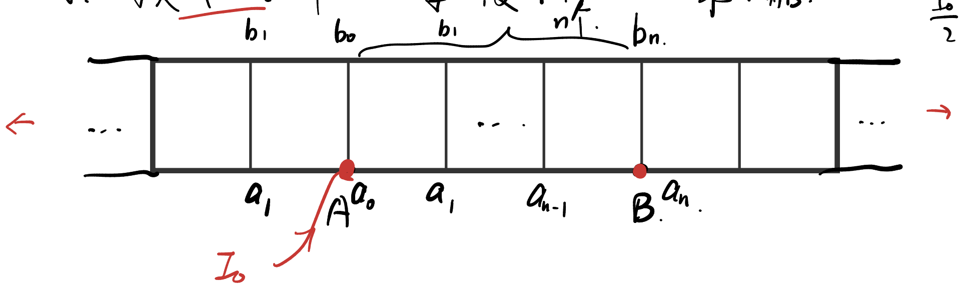
边界: $I_0 = I_0$ ⇒ $I_0=\frac{I_0}{2}+a_0$ ⇒ $a_0=\frac{I_0}{2}$

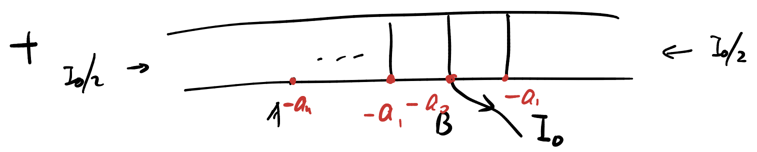
$I_{n+1}=0$ ⇒ $I_{n+1}=\frac{I_0}{2}+a_{n+1}=0$ ⇒ $a_{n+1}=-\frac{I_0}{2}$.

⇒ $A_+=\dots$ $\quad A_-=\dots$ ⇒ $I_j=\dots$

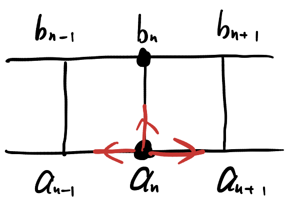
$U_{A_0B_n}=$ ⇒ $R_{A_0B_n}=\frac{U_{A_0B_n}}{I_0}$

例: 换成电压解. 每一段 r. 求 R_{AB}. $\frac{I_0}{2}$





⇒ $U_{AB}=2(a_0-a_n)$.



节点:

$$\begin{cases}(a_n-a_{n-1})+(a_n-a_{n+1})+(a_n-b_n)=0.\\(b_n-b_{n-1})+(b_n-b_{n+1})+(b_n-a_n)=0\end{cases}$$

⇒ $$\begin{cases}-a_{n-1}+3a_n-a_{n+1}-b_n=0\\-a_n-b_{n-1}+3b_n-b_{n+1}=0.\end{cases}$$

先消去 b. ⇒ 递推方程. $b_n=-a_{n-1}+3a_n+a_{n+1}$ $\quad b_{n-1}$、b_{n+1}

⇒ $\boxed{a_{n-2}-6a_{n-1}+10a_n-6a_{n+1}+a_{n+2}=0}$ ← 齐次.

设 $a_n = A\lambda^n$

代入: $1-6\lambda+10\lambda^2-6\lambda^3+\lambda^4=0$.

电流全为0时. ⇒ 电势相等 ⇒ $\lambda=1$.

$(1-\lambda)(1-5\lambda+5\lambda^2-\lambda^3)=0$.

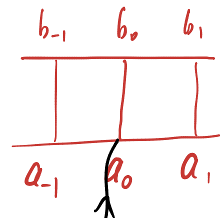
$(1-\lambda)(1-\lambda)(1-4\lambda+\lambda^2)=0$.

⇒ $\lambda_1=1,\ \lambda_2=1,\ \lambda_3=2+\sqrt{3}.\ \lambda_4=2-\sqrt{3}$.

有重根: $a_n=(A_1+nA_2)\cdot 1^n+A_3\cdot\lambda_3^n+A_4\cdot\lambda_4^n$.

边界条件: $n\to\infty$. 不发散: $A_3=0$. $\quad a_{-1}=a_1$, $b_{-1}=b_1$

$n=0$、$n=1$: $\quad b_n=-a_{n-1}+3a_n-a_{n+1}$



⇒ $$\begin{cases}(a_0-a_1)+(a_0-a_1)+(a_0-b_0)=I.\\(b_0-b_1)+(b_0-b_1)+(b_0-a_0)=0.\end{cases}$$

⇒ $a_0=A_1+A_4$. $\quad a_1=A_1+A_2+A_4\cdot\lambda_4$.

⇒ $b_0=-a_1+3a_0-a_1=3A_1+3A_4-2A_1-2A_2-2A_4\lambda_4$.

$b_1=-a_0+3a_1-a_2.=A_1+A_2-A_4+3A_4\lambda_4-A_4\lambda_4^2$.

三元二次方程组. ⇒ 二元一次 ⇒ ✓.

$a_n = A_1+nA_2+A_4\lambda_4^n$.

常数 ↔ 整体提高. 降低电势. 无用

令 $A_1=0$.

令解为: $\begin{pmatrix}a_n\\b_n\end{pmatrix}=\begin{pmatrix}A\\B\end{pmatrix}\cdot\lambda^n$.

⇒ $$\begin{cases}(-1+3\lambda-\lambda^2)A-\lambda B=0\\-\lambda A+(-1+3\lambda-\lambda^2)B=0.\end{cases}$$

要求有非平凡解: $\frac{-1+3\lambda-\lambda^2}{-\lambda}=\frac{-\lambda}{-1+3\lambda-\lambda^2}$

$(-1+3\lambda-\lambda^2+\lambda)(-1+3\lambda-\lambda^2-\lambda)=0$.

⇒ $\underbrace{\lambda_{1,2}=1}_{\text{重根}}$, $\quad\lambda_{3,4}=2\pm\sqrt{3}$.

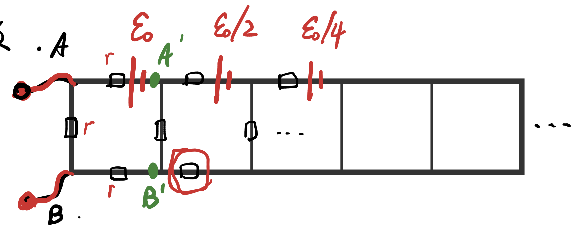
对 $\lambda_{1,2}$. $-1+3\lambda-\lambda^2=1$. ⇒ $\begin{cases}A-B=0\\-A+B=0\end{cases}$ ⇒ $A=B$.

对 $\lambda_{3,4}$. $-1+3\lambda-\lambda^2=-\lambda$ ⇒ $\begin{cases}-\lambda A-\lambda B=0\\-\lambda A-\lambda B=0\end{cases}$ ⇒ $A=-B$.

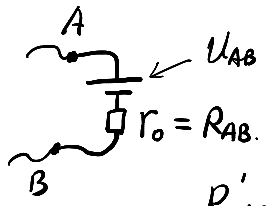
⇒ $\begin{pmatrix}a_n\\b_n\end{pmatrix}=\left[A_1\binom{1}{1}+nA_2\binom{1}{1}\right]\cdot 1^n+A_3\binom{1}{-1}\lambda_3^n+A_4\binom{1}{-1}\lambda_4^n$

再用边界条件.

例. 二端响应函数. r, 求 U_{AB}=?

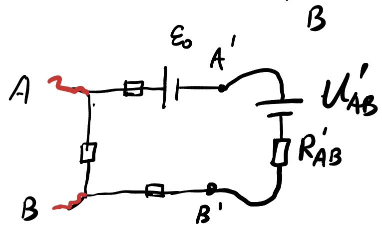


电压源定理: 原电路:



$r_0 = R_{AB}$

原电路:



$R'_{AB} = R_{AB}$

$U'_{AB} = \frac{1}{2}U_{AB}$

$U_{AB} = f(\varepsilon_0)$

$= \bigcirc\,\varepsilon_0$

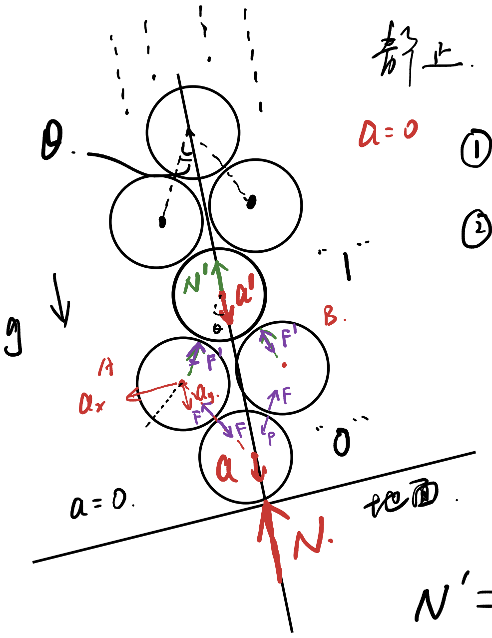
$R_{AB} = (R_{AB}+2r)//r \Rightarrow R_{AB} = (\sqrt{3}-1)r$

$U_{AB} = r\cdot\frac{\varepsilon_0+\frac{1}{2}U_{AB}}{3r_0+R_{AB}} \Rightarrow U_{AB} = \frac{\frac{r}{3r+R_{AB}}\cdot\varepsilon_0}{1-\frac{1}{2}\frac{r}{3r+R_{AB}}}$

力学中的二端响应函数.

光滑球. r, m, θ.

静止. 释放. 求 N



$a=0$

① 地面上看. "0" ⇒ N.

② 在 1 球上看. 非惯性系:

$g' = g - a'$

$N = f(g) = \bigcirc\cdot mg$

$N' = f(g') = \bigcirc\cdot mg'$

$N' = \frac{g-a'}{g}\cdot N$ ⊛

约束: $a_y = \frac{a'}{2}$

约束: a_{A0} 沿球切向 ⇒ $a_x\cdot\sin\theta = a_y\cdot\cos\theta$

⇒ $a_x = \frac{\cos\theta}{\sin\theta}\cdot\frac{a'}{2}$

对"0"球受力: $2F\cdot\cos\theta + mg = N \Rightarrow F = \frac{N-mg}{2\cos\theta}$

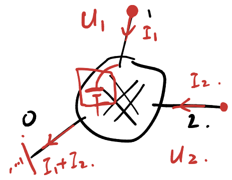
同理: $N' = 2F'\cos\theta$ $F' = \frac{N'}{2\cos\theta}$

⇒ 对A: 竖直: $F'\cdot\cos\theta + mg - F\cdot\cos\theta = m\cdot a_y$ ①

水平: $F'\cdot\sin\theta + F\cdot\sin\theta = m\cdot a_x$ ②

⇒ 解得 ... ✓

三端网络. (线性)



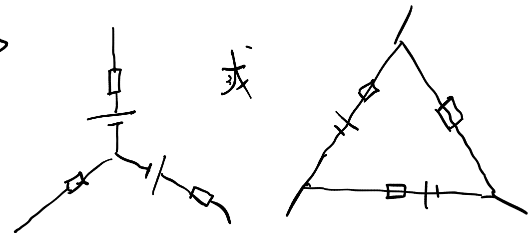
$U_1 = f_1(I_1,I_2) = U_{10} + a_{11}I_1 + a_{12}\cdot I_2$

$U_2 = f_2(I_1,I_2) = U_{20} + a_{21}I_1 + a_{22}\cdot I_2$

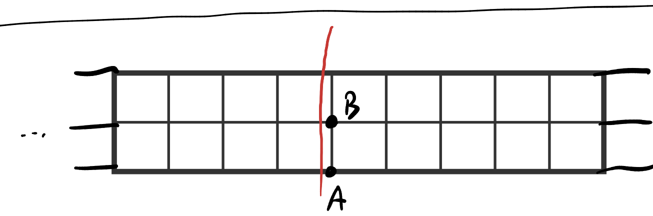
$a_{12} = a_{21}$. ⇒ 三端网络.

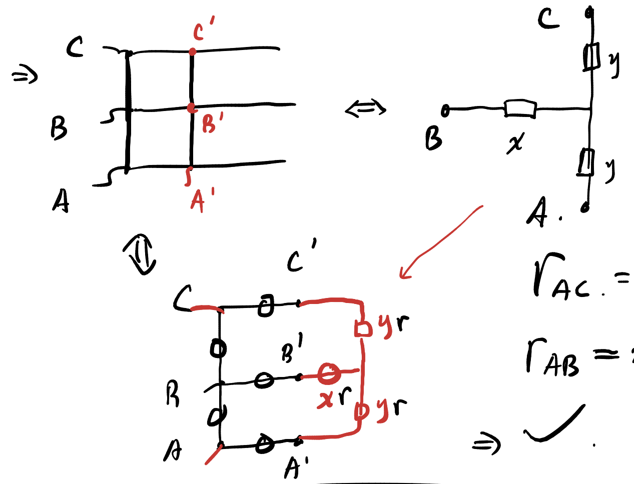
无源: 三个自由度 ⇒ 人 或 △

有源: 三个 + 2个. ⇒



练: ... 求 R_{AB}. r.



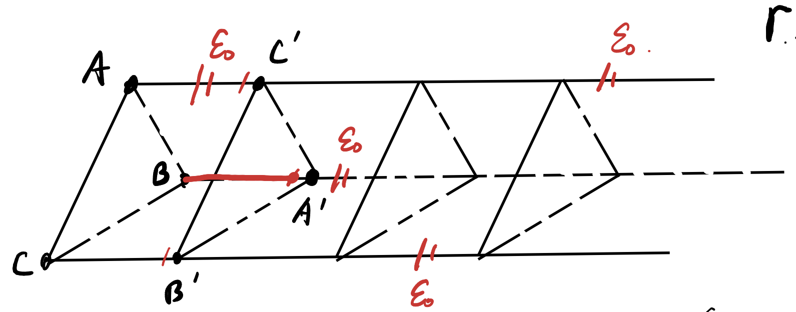


$r_{AC} = 2y = (2y+2)//2$.

$r_{AB} = x+y = \cdots$

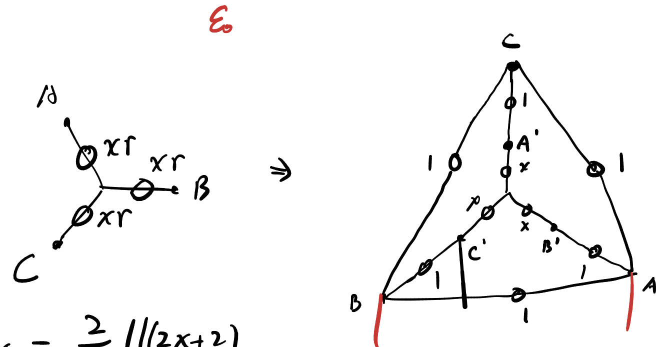
⇒ ✓

例: r.



求 U_{AC}.

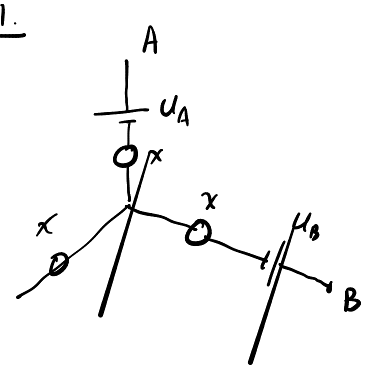
电阻网络:



⇒ $R_{AB}/r = 2x = \frac{2}{3}//(2x+2)$

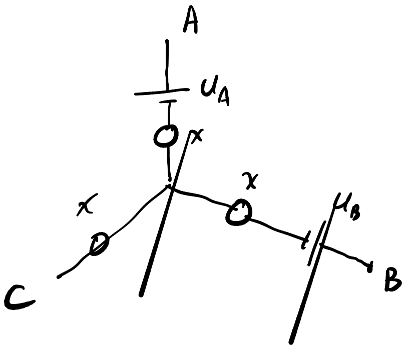
⇒ $x = \frac{-3+\sqrt{21}}{2}$

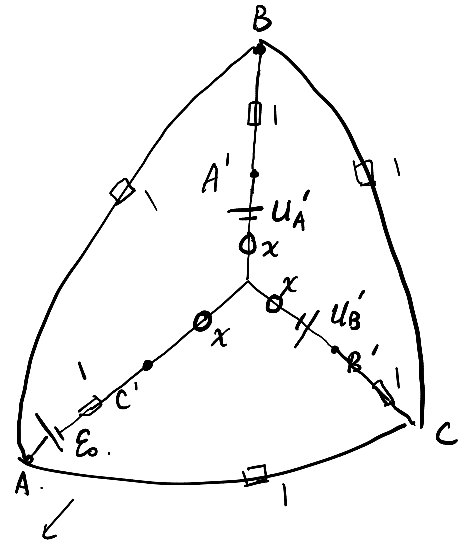
再入电源:



又. 原网络可以为:

$U_A' = U_A \quad U_B' = U_B$





$\Rightarrow U_{AC} = U_A.$

$U_{BC} = U_B.$

叠加原理:

只有 $\mathcal{E}_0$: $\begin{cases} U_{AC1} = 1\cdot \mathcal{E}_0 \cdot \left(\frac{x+2}{2} + x + 1\right)^{-1} \cdot \frac{1}{2} \\ U_{BC1} = 0. \end{cases}$ $\lambda.$

只有 U_A: $\begin{cases} U_{AC2} = 0. \\ U_{BC2} = 1\cdot U_A \left(\frac{x+2}{2} + x + 1\right)^{-1} \cdot \frac{1}{2} \end{cases}$

只有 U_B: $\begin{cases} U_{AC3} = -1\cdot U_B \cdot \left(\frac{x+2}{2} + x + 1\right)^{-1} \cdot \frac{1}{2} \\ U_{BC3} = -1\cdot U_B \cdot \left(\frac{x+2}{2} + x + 1\right)^{-1} \cdot \frac{1}{2}. \end{cases}$

$\Rightarrow U_{AC} = U_A = \lambda \mathcal{E}_0 + 0 - \lambda\cdot U_B$ ①

$U_{BC} = U_B = 0 + \lambda U_A - \lambda U_B$ ②

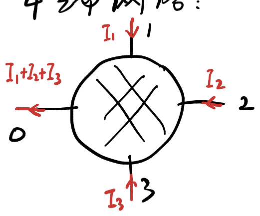
$\Rightarrow (1+\lambda) U_B = \lambda\cdot(\lambda\mathcal{E}_0 - \lambda U_B).$

$\Rightarrow U_B = \frac{\lambda^2 \mathcal{E}_0}{1+\lambda+\lambda^2}.$

$\Rightarrow U_A = \frac{(\lambda+\lambda^2+\lambda^3)\mathcal{E}_0 - \lambda^3\mathcal{E}_0}{1+\lambda+\lambda^2} = \frac{(\lambda+\lambda^2)\mathcal{E}_0}{1+\lambda+\lambda^2}.$

$\Rightarrow U_{AC} = U_A = \cdots$

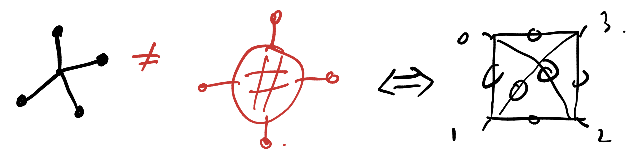
4端网络:



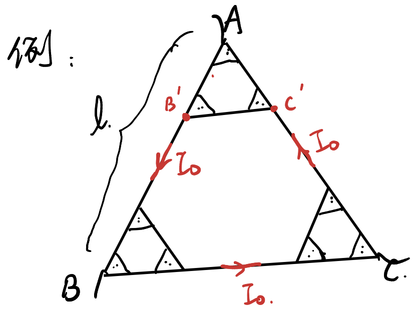
$\begin{cases} U_1 = U_{10} + a_{11} I_1 + \boxed{a_{12}} I_2 + a_{13} I_3. \\ U_2 = U_{20} + \boxed{a_{21}} I_1 + a_{22} I_2 + a_{23} I_3. \\ U_3 = U_{30} + a_{31} I_1 + a_{32} I_2 + a_{33} I_3. \end{cases}$

电阻网络: 自由度为6

试:



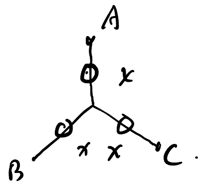
例:



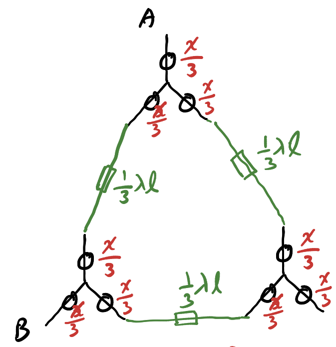
(1). 已知. 单位长电阻 λ. 求 R_{AC}.

(2). $B = kt$. 求 I_0.

(1) 原图:



$x \to x\cdot\lambda l$



$\Rightarrow R_{AC} = 2x. = \frac{2}{3}x + \left(\frac{2}{3}x + \frac{1}{3}\right) // \left(\frac{4}{3}x + \frac{2}{3}\right).$

$\Rightarrow x = \frac{1}{4}\lambda l. \Rightarrow R_{AC} = 2x = \frac{1}{2}\lambda l.$

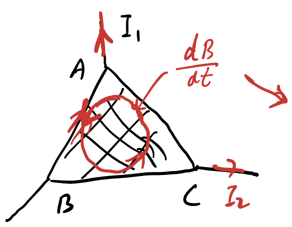
无量纲化: $x = \frac{1}{4}.$

(2). 从最外层等效到内层之关系时. 只能看电流.电压.不包含电动势.

只看电阻产生的电压降. 输入出均是 I.

E_{BA}: 从 B→A. $\Sigma I r$.

E_{BC}: 从 B→A. $\Sigma I\cdot r$.



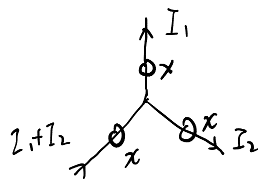
$E_{BA} = a_{11} I_1 + a_{12} I_2 + E_{10}$

$E_{BC} = a_{21} I_1 + a_{22} I_2 + E_{20}.$

$\frac{dB}{dt}$

a_{ij} 与 E_{10}, E_{20}. 相互独立.(叠加原理)

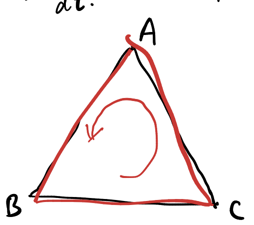
先 a_{ij}: 无 $\frac{dB}{dt}$ ⇒



$a_{11} = \frac{1}{2} = a_{22}.$

$a_{12} = a_{21} \neq \frac{1}{4}.$

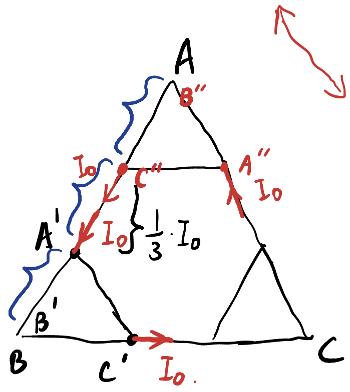
由 $\frac{dB}{dt}$ 产生部分: $\mathcal{E}_0 = k\cdot S_\triangle = k\cdot\frac{\sqrt{3}}{4} l^2.$



$E_{BA} = E_{10} = -\frac{1}{3}\mathcal{E}_0.$

$E_{BC} = E_{20} = \frac{1}{3}\mathcal{E}_0.$

⇒ 对于最外层: $\begin{cases} E_{BA} = \frac{1}{2}I_1 + \frac{1}{4}I_2 - \frac{1}{3}\mathcal{E}_0 \\ E_{BC} = \frac{1}{4}I_1 + \frac{1}{2}I_2 + \frac{1}{3}\mathcal{E}_0. \end{cases}$



对次外层: $x \to \frac{1}{3}x$, $S \to \frac{1}{9}S$, $\mathcal{E} \to \frac{1}{9}\mathcal{E}_0$

$E_{B'A'}' = \frac{1}{6}I_1 + \frac{1}{12}I_2 - \frac{1}{27}\mathcal{E}_0$

$E_{B'C'}' = \frac{1}{12}I_1 + \frac{1}{6}I_2 + \frac{1}{27}\mathcal{E}_0.$

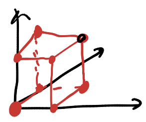
求最外层.总电压降.

$\begin{cases} E_{B''C''} = \frac{1}{12}\cdot(-I_0) + \frac{1}{6}\cdot I_0 + \frac{1}{27}\mathcal{E}_0. \\ E_{C''A'} = \frac{1}{3}\cdot I_0 \\ E_{A'B'} = -E_{B'A'} = -\left[\frac{1}{6}\cdot(-I_0) + \frac{1}{12}\cdot I_0 - \frac{1}{27}\mathcal{E}_0\right] \end{cases}$

$\Rightarrow E_{A\to B\to C\to A} = 3\cdot(E_{B''C''} + E_{C''A'} + E_{A'B'}) \overset{\text{基二}}{=} \mathcal{E}_0.$

$\Rightarrow I_0 = \frac{14}{27}\mathcal{E}_0. \Rightarrow I_0 = \frac{14}{27}\cdot\frac{\mathcal{E}_0}{\lambda l}$

例：N维立方体，r，求R_M.



顶点：$(0,0,\cdots 0)$ N ; $(1,1,\cdots 1)$ N

相邻：

C_N^1. $(1,0,\cdots 0)$或$(0,1,0,\cdots 0)$或 $\cdots(0,0,\cdots 0,1)$.

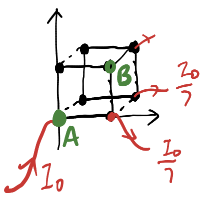
定义.顶点之间的间隔：M：$A\Rightarrow B$.最少步数. $1\le M\le N$.

A: $(0,0,\cdots 0)$. B点：$(0,0,\cdots 0,\overbrace{1,1,\cdots 1}^{M个1})$

利用叠加原理：

(1) 从A输入I_0. 从其余点平均各输出一份电流：$\frac{I_0}{2^N-1}=\Delta I$. $\to U_{AB1}$

对N维立方体：维数：N. 顶点数：2^N.



点边个数：$\frac{N\times 2^N}{2}$个边.

(2) 从B输出I_0, 从其余顶点各入一份：$\frac{I_0}{2^N-1}=\Delta I$.

$\Downarrow U_{AB2}$.

$U_{AB1}=U_{AB2}$. $\Rightarrow U_{AB}=2U_{AB1} \Rightarrow R_M=R_{AB}=\frac{2U_{AB1}}{I+\Delta I}$.

从A $(0,0,\cdots 0)$. 出发 $\begin{cases}(1,0,0\cdots 0)\\(0,1,0\cdots 0)\\(0,0,1,\cdots 0)\end{cases}$

↓间隔为1点.

"1"：点个数：C_N^1个; 边个数 ($0\to1$)：C_N^1个

↓间隔为2点

"2"：点个数：C_N^2个; 边个数 ($1\to2$)：$(N-1)\cdot C_N^1$

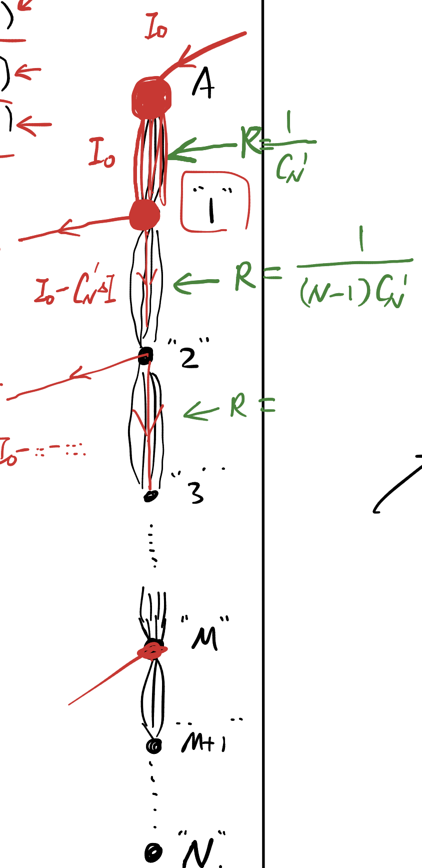
↓

"3"：点个数：C_N^3个; 边个数 $(N-2)\cdot C_N^2$.

⋮

"M"：点个数，C_N^M个，边个数：$(N-M+1)\cdot C_N^{M-1}$

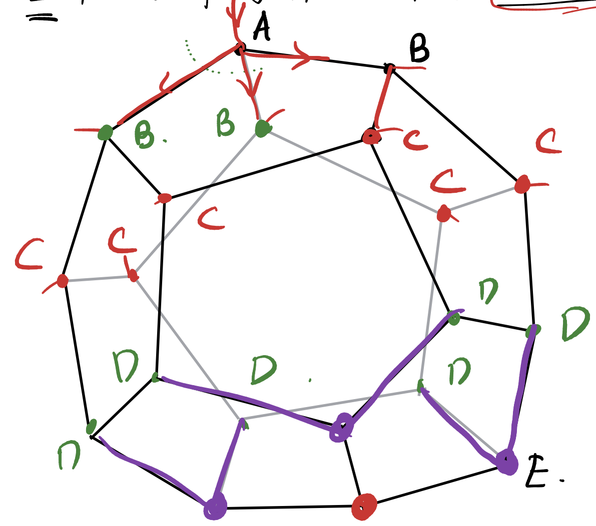
"M+1"：$\cdots\cdots$



$\Rightarrow U_{AB1} = \sum_{i=0}^{M-1}\frac{r}{C_N^i\cdot(N-i)}\cdot\left(I_0-\sum_{j=1}^{i}C_N^j\cdot\frac{I_0}{2^N-1}\right).$

$\Rightarrow R_{AB} = \frac{2U_{AB1}}{I_0+\frac{1}{2^N-1}I_0}.$

例：正十二面体.每条边：r，求任2点之间电阻.



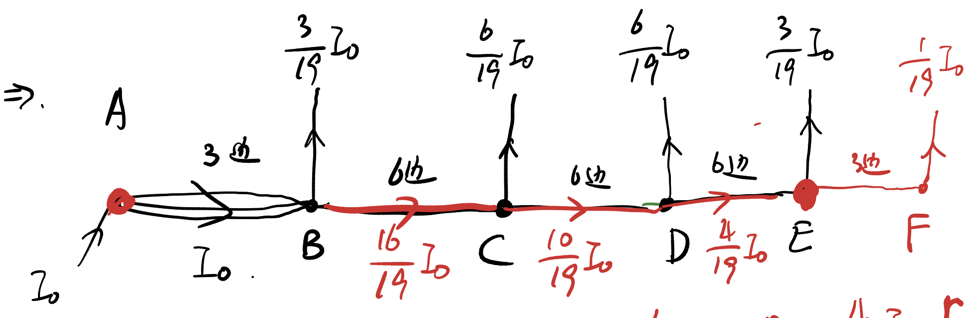
求R_{AB}. R_{AE}.

(1) 从A注入I_0. 从余下顶点 出 $\frac{1}{19}I_0 \Rightarrow U_{AE1}$

(2) 从E出I_0. 从余下顶点 入 $\frac{1}{19}I_0 \Rightarrow U_{AE2}$

$\Rightarrow U_{AE}=U_{AE1}+U_{AE2}=2U_{AE1}.$

$\Rightarrow$



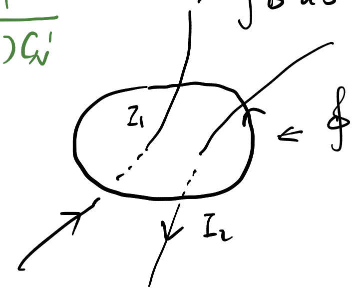
$\Rightarrow U_{AE1} = I_0\cdot\frac{r}{3}+\frac{16}{19}I_0\times\frac{1}{6}r+\frac{10}{19}I_0\times\frac{r}{6}+\frac{4}{19}I_0\times\frac{r}{6}.$

$\Rightarrow \cdots \Rightarrow R_{AE}=\frac{17}{15}r.$

静磁场： B-S. $I\to B$.

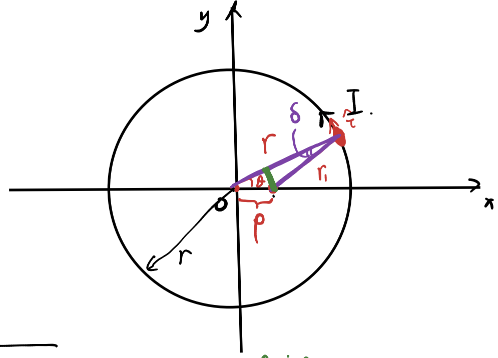
$d\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{I\cdot d\vec{l}\times\hat{r}}{r^2}.$

$\Rightarrow \oint\vec{B}\cdot d\vec{s}=0.$ $\oint\vec{B}\cdot d\vec{l}=\mu_0\cdot\sum I_{内}$.



$\Leftarrow \oint\vec{B}\cdot d\vec{l}=\mu_0\cdot(I_1-I_2).$

例：$\rho \ll r$. 求 $B(\rho)$



保留一阶 ⇒ 没了.

⇒ 保留二阶

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int \frac{r d\theta \cdot \hat{\tau}\times\hat{r}_1}{r^2+\rho^2-2r\rho\cos\theta}$$

$$\delta \doteq \frac{\rho\sin\theta}{r} + o(2).$$

$$|\hat{\tau}\times\hat{r}_1| = \cos\delta = 1-\frac{\delta^2}{2} = 1-\frac{1}{2}\left(\frac{\rho\sin\theta}{r}+o(2)\right)^2.$$

$$= 1-\frac{1}{2}\cdot\left(\frac{\rho^2\sin^2\theta}{r^2}+o(4)+o(3)\right)$$

$$= 1-\frac{1}{2}\cdot\frac{\rho^2\sin^2\theta}{r^2}.$$

或：$\cos\delta = \dfrac{-\rho^2+r^2+(r^2+\rho^2-2r\rho\cos\theta)}{r\cdot\sqrt{r^2+\rho^2-2r\rho\cos\theta}}$ $\qquad (1+x)^n = 1+nx+\frac{n(n-1)}{2}\cdot x^2.$

$\doteq \cdots$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} r d\theta\left(1-\frac{1}{2}\frac{\rho^2\sin^2\theta}{r^2}\right)\cdot\frac{1}{r^2}\cdot\left(1+\frac{\rho^2}{r^2}-\frac{2\rho}{r}\cos\theta\right)^{-1}$$

$$= \frac{\mu_0 I}{4\pi r}\int_0^{2\pi} d\theta\left(1-\frac{1}{2}\frac{\rho^2}{r^2}\sin^2\theta\right)\cdot\left[1+(-1)\cdot\left(\frac{\rho^2}{r^2}-\frac{2\rho}{r}\cos\theta\right)+\frac{(-1)\cdot(-2)}{2}\left(\frac{2\rho}{r}\cos\theta\right)^2\right]$$

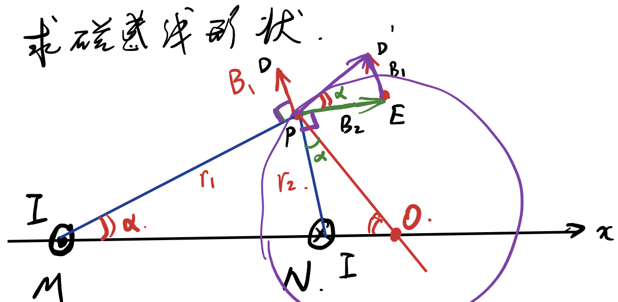
$$= \frac{\mu_0 I}{4\pi r}\int_0^{2\pi} d\theta\left(1-\frac{1}{2}\frac{\rho^2}{r^2}\sin^2\theta\right)\left[1+\frac{2\rho}{r}\cos\theta-\frac{\rho^2}{r^2}+\frac{4\rho^2}{r^2}\cos^2\theta\right]$$

$$= \frac{\mu_0 I}{4\pi r}\int_0^{2\pi} d\theta\left[1+\underbrace{\frac{2\rho}{r}\cos\theta}_{0}-\frac{\rho^2}{r^2}+\underbrace{\frac{4\rho^2}{r^2}\cos^2\theta}_{\frac{1}{2}\cdot 2\pi}-\underbrace{\frac{1}{2}\frac{\rho^2}{r^2}\sin^2\theta}_{\frac{1}{2}\cdot 2\pi}\right]$$

$$= \frac{\mu_0 I}{4\pi r}\cdot\left(2\pi-\frac{\rho^2}{r^2}\cdot 2\pi+\frac{4\rho^2}{r^2}\cdot\pi-\frac{1}{2}\cdot\frac{\rho^2}{r^2}\cdot\pi\right)$$

$= \cdots$ ✓.

例：求磁感线形状.



$PE = \dfrac{\mu_0 I}{2\pi r_2}$ $\quad ED' = \dfrac{\mu_0 I}{2\pi r_1}$ ⇒ $\dfrac{PE}{ED'} = \dfrac{r_1}{r_2}$. 又. $\angle E = \angle MPN$

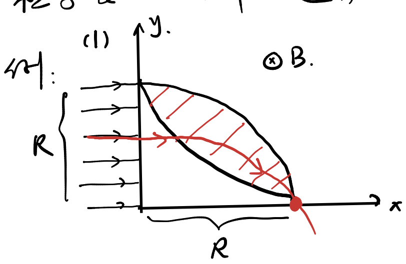
⇒ $\boxed{\triangle PED' \backsim \triangle MPN.}$

$\angle O = \angle O$, $\angle OPN = \angle OMP$ ⇒ $\triangle MOP \backsim \triangle PON$.

$\dfrac{MO}{PO} = \dfrac{PO}{NO}$ ⇒ $MO\cdot NO = PO^2$ ⇒ Apollonius 圆.

粒子在磁场中之运动. $\quad v_0 \Rightarrow R = \dfrac{mv_0}{qB}$.

例：(1)



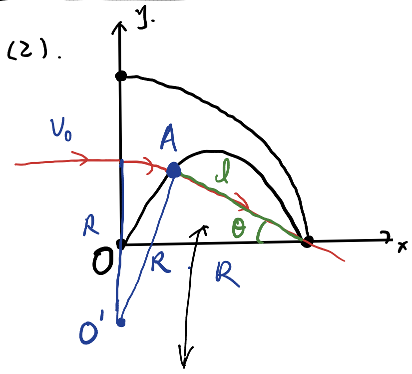
求最小之磁场面积.

$$S = 2\cdot\left(\frac{\pi}{4}R^2-\frac{1}{2}R^2\right) = \left(\frac{\pi}{2}-1\right)\cdot R^2.$$

$\boxed{\text{思考题}}$ 证明. 所需之最小磁场面积为恒定. $S = \left(\frac{\pi}{2}-1\right)\cdot R^2$.

Yth. ⇒ 证明.

(2).



A: $\begin{cases} x = R\cdot\sin\theta \\ y = l_{(\theta)}\cdot\sin\theta\end{cases}$

$x = R - l\cdot\cos\theta = R\cdot\sin\theta$.

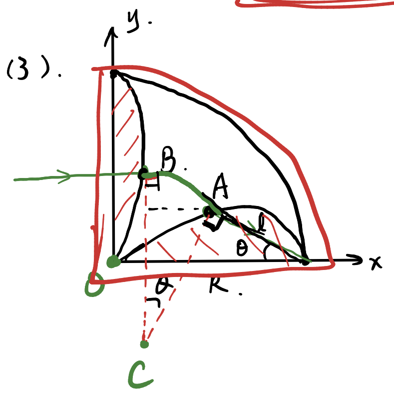
$\Rightarrow l = \dfrac{1-\sin\theta}{\cos\theta}\cdot R$.

$$S_B = \int y_A\cdot dx_A = \int l\sin\theta\cdot R\, d(\sin\theta)$$

$$= \int_0^{\frac{\pi}{2}}\frac{(1-\sin\theta)\cdot\sin\theta}{\cancel{\cos\theta}}\cdot R^2\cancel{\cos\theta}\, d\theta.$$

$$= \boxed{R^2\left(1-\frac{\pi}{4}\right)} \Rightarrow S = \frac{\pi}{4}R^2 - S_B = \boxed{\left(\frac{\pi}{2}-1\right)\cdot R^2}$$

(3).



A: $\begin{cases} x_A = R - l\cdot\cos\theta \\ y_A = l\cdot\sin\theta\end{cases}$ $\qquad l = l(\theta).$

C: $\begin{cases} x_C = R - l\cdot\cos\theta - R\cdot\sin\theta \\ y_C = l\cdot\sin\theta - R\cdot\cos\theta\end{cases}$

B: $\begin{cases} x_B = R - l\cos\theta - R\cdot\sin\theta \\ y_B = l\sin\theta - R\cos\theta + R\end{cases}$

$$\Rightarrow S_B = \int x_B\cdot dy_B + \int y_A\cdot dx_A$$

$$= \int(R - l\cos\theta - R\sin\theta)\cdot d(l\sin\theta - R\cos\theta).$$

$$+ \int l\sin\theta\cdot d(-l\cos\theta).$$

$$= -\int l\sin\theta\cdot d(l\cos\theta) + \int\Big[R\,d(l\sin\theta) - R^2 d\cos\theta - l\cos\theta\, d(l\sin\theta)$$
$$+ l\cos\theta\cdot R\, d\cos\theta - R\sin\theta\, d(l\sin\theta) + R^2\sin\theta\, d\cos\theta\Big]$$

$l^{(0)}$: $\displaystyle\int_0^{\frac{\pi}{2}} -R^2 d\cos\theta + R^2\sin\theta\, d\cos\theta = R^2 - \int_0^{\frac{\pi}{2}} R^2\sin\theta\sin\theta\, d\theta$

$$= R^2 - \frac{\pi}{4}R^2 = \left(1-\frac{\pi}{4}\right)R^2.$$

$l^{(1)}$: $\displaystyle\int_0^{\frac{\pi}{2}} R\, d(l\sin\theta) + l\cos\theta\cdot R\, d\cos\theta - R\sin\theta\, d(l\sin\theta)$

$$= \int_0^{\frac{\pi}{2}} R(1-\sin\theta)\, d(l\sin\theta) + l\cos\theta\cdot R\, d\cos\theta.$$

$$\overset{\text{分部}}{=} R(1-\sin\theta)\cdot l\sin\theta\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} l\sin\theta\, d(+R\sin\theta) + \int_0^{\frac{\pi}{2}} l\cos\theta\, R\, d\cos\theta$$

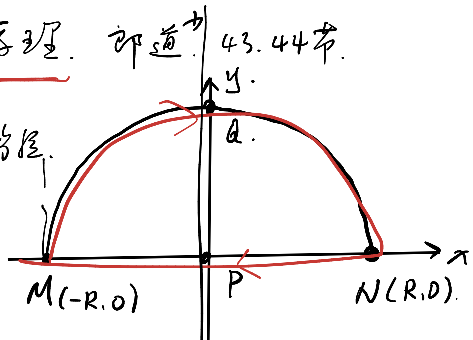
$$= \int_0^{\frac{\pi}{2}} l\cdot\sin\theta\cos\theta\cdot R\, d\theta - \int_0^{\frac{\pi}{2}} l\cdot\cos\theta\cdot R\cdot\sin\theta\, d\theta = 0$$

$l^{(2)}$: $\displaystyle -\int l\sin\theta\, d(l\cos\theta) - \int l\cos\theta\, d(l\sin\theta)$

$$\overset{\text{分部}}{=} -l\sin\theta\, l\cos\theta\Big|_0^{\frac{\pi}{2}} + \int l\cos\theta\cdot d(l\sin\theta) - \int l\cos\theta\, d(l\sin\theta)$$

$= 0$.

(4) yth.大神. 莫培督原理. 即道 43.44节.



$M \to N$. 速度分布 了解 路径.

分析作用量:

$$\boxed{S_0 = \int \sum p_i dq_i = 常}$$

$$L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) + q\vec{v}\cdot\vec{A}$$

$$\vec{p} = m\vec{v} + q\vec{A}$$

$$\Rightarrow S_0 = \int (m\vec{v} + q\vec{A})\cdot \underbrace{d\vec{l}}_{\vec{v}\cdot dt}$$

$$= \int m\vec{v}\cdot\vec{v}\,dt + \int q\vec{A}\cdot d\vec{l}$$

$$= m v_0^2 t + \int q\vec{A}\cdot d\vec{l}$$

$$S_{MPN} = m v_0^2 \cdot \frac{2R}{v_0} + \int_{MPN} q\vec{A}\cdot d\vec{l}$$

$$S_{MQN} = m v_0^2 \cdot \frac{\pi R}{v_0} + \int_{MQN} q\vec{A}\cdot d\vec{l}$$

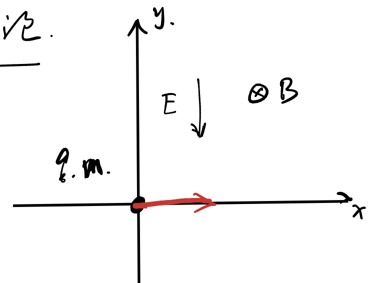
$$\Rightarrow -\int_{MQN} q\vec{A}\cdot d\vec{l} + \int_{MPN} q\vec{A}\cdot d\vec{l} = \oint q\vec{A}\cdot d\vec{l} = m v_0^2 \cdot \frac{R}{v_0}(\pi-2)$$

$$\frac{m v_0}{qB} = R \qquad = qBR^2(\pi-2).$$

又: $\oint \vec{A}\cdot d\vec{l} = \iint \vec{B}\cdot d\vec{S} = B\cdot 2S.$

$$\Rightarrow S = R^2\left(\frac{\pi}{2}-1\right)$$

例: 正交电场, 磁场. 考虑相对论.



$\vec{B} = -B\hat{z}$, $\vec{E} = -E\cdot\hat{y}$.

原点静止释放. 求轨迹.

(1) 经典情形. $\frac{dp_x}{dt} = -qBv_y \Rightarrow \frac{d}{dt}(p_x + qBy) = 0$

$\mathcal{P}_x = p_x + qBy \Rightarrow \mathcal{P}_x$ 守恒量: 正则动量.

$$\begin{cases} p_x + qBy = 0 \ ① \Rightarrow p_x \\ E = 0 + -qE\cdot y = \frac{p_x^2+p_y^2}{2m} \Rightarrow p_y \end{cases} \Rightarrow \frac{dy}{dx} = \frac{p_y}{p_x}$$

$\Rightarrow$ 积分 $\Rightarrow$ 轨迹

(2) 相对论情形. $\Rightarrow p_x + qBy \equiv 0$.

能量: $E = m_0c^2 - qEy = \sqrt{(m_0c^2)^2 + (p_x^2+p_y^2)c^2}$

$p_x, p_y \Rightarrow$ 轨迹 $\Rightarrow$.