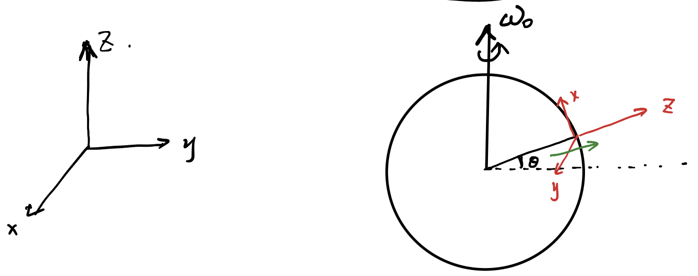
例：傅科摆。摆面如何转。ω 多大，怎么转。



$\vec{F}_{科} = m\cdot 2\vec{v}\times\vec{\omega}_0 = 2m\cdot(v_x\hat{x}+v_y\hat{y})\times(\omega_0\cos\theta\cdot\hat{x}+\omega_0\sin\theta\,\hat{z})$

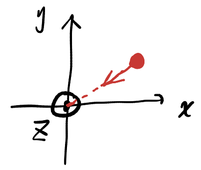
$= 2m\cdot(v_x\cdot\omega_0\cdot\sin\theta(-\hat{y})+v_y\cdot\omega_0\sin\theta\,\hat{x}+v_y\cdot\omega_0\cdot\cos\theta(-\hat{z}))$

看摆面如何转？只看 x, y 方向的力。

$\vec{F}_{科}' = 2m\cdot(v_x\cdot\omega_0\cdot\sin\theta(-\hat{y})+v_y\omega_0\sin\theta\cdot\hat{x})$

$= 2m\cdot\vec{v}\times\vec{\omega}_z$　　$\vec{\omega}_z = \omega_0\sin\theta\cdot\hat{z}$

法一：复数法。



牛二：$m\ddot{x} = -mg\cdot\frac{x}{l}+2m\dot{y}\cdot\omega_z$

$m\ddot{y} = -mg\cdot\frac{y}{l}-2m\dot{x}\cdot\omega_z$

令 $u = x+iy$，$\dot{u} = \dot{x}+i\dot{y}$，$\ddot{u} = \ddot{x}+i\ddot{y}$

①+i② $\Rightarrow \ddot{u} = -\frac{g}{l}u - i2\omega_z\cdot\dot{u}$

猜：$u = u_0 e^{i\omega t}$

$\Rightarrow -\omega^2 u = -\frac{g}{l}u-(2i\omega_z)(i\omega)\cdot u$

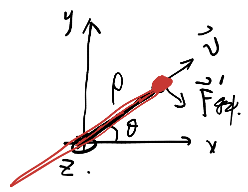
$\omega^2+2\omega\cdot\omega_z-\frac{g}{l}=0 \Rightarrow \omega = -\omega_z\pm\sqrt{\frac{g}{l}+\omega_z^2}$

$\approx -\omega_z\pm\sqrt{\frac{g}{l}}$

$\Rightarrow u = e^{-i\omega_z t}\cdot(A_1 e^{i\sqrt{\frac{g}{l}}t}+A_2\cdot e^{-i\sqrt{\frac{g}{l}}t})$

$\Rightarrow$ 摆面转动 $\Rightarrow \Omega = \omega_z = \omega_0\cdot\sin\theta$

法二：正则角动量：

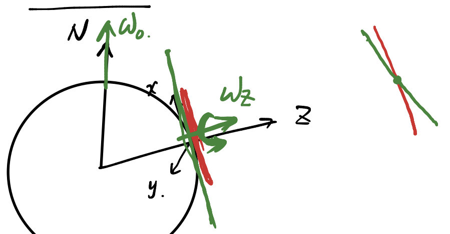


$\frac{dL}{dt} = -2\dot{\rho}m\cdot\omega_z\cdot\rho$

$\Rightarrow \frac{d}{dt}(L+m\omega_z\rho^2)=0 \Rightarrow \mathcal{L} = L+m\omega_z\rho^2$

$m\rho^2\cdot\dot{\theta}+m\rho^2\cdot\omega_z=0 \Rightarrow \dot{\theta} = -\omega_z \Leftrightarrow$ 摆面的转动。

法三：找到惯性系。

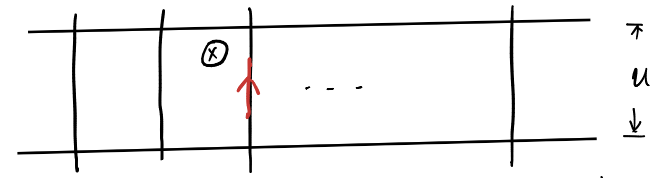


摆面不转。

地球在转 $\to \omega_z$。$\dot{\theta} = -\omega_z$。

动力学问题。　宽度：l。B。光滑。棒子：r, m。

例：



初：第 i 个速度 v_{0i}。相距远，不碰撞。

问：最后第 n 个棒，相对于第 1 棒的位移。

r：阻尼。　L：质量。　C：弹簧。

最后，匀速：对第 i 根：$u = Blv_i - I_i\cdot r$。

节点：$\Sigma I_i = 0$。$\Rightarrow$ 第 i 根，动力学：$I_i\cdot l\cdot B = -m\cdot\dot{v}_i$（力）。

$\Rightarrow$ 总：$\Sigma I_i\cdot l\cdot B = -\Sigma m\dot{v}_i = 0 \Rightarrow$ 动量守恒。

$\Rightarrow v_{if} = \bar{v} = \frac{1}{n}\Sigma v_{0i}$

$\Rightarrow n\cdot u = \Sigma Blv_i - r\Sigma I_i = \Sigma Bl\cdot v_i$

$\Rightarrow u = Bl\cdot\frac{\Sigma v_i}{n} = Bl\bar{v}$

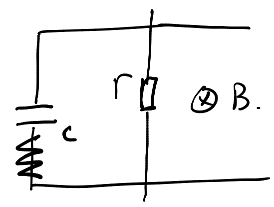
$\Rightarrow$ 第 i 根：$I_i\cdot r = Bl(v_i-\bar{v})$（电学）

$\Rightarrow a_i = -\frac{B^2l^2(v_i-\bar{v})}{mr}$

$\Rightarrow \left[a_n - a_0 = -\frac{B^2l^2(v_0-v_n)}{mr}\right]\cdot dt$

$\Rightarrow \Delta v_n - \Delta v_0 = \frac{B^2l^2(x_0-x_n)}{mr} \Rightarrow$ ✓

法：



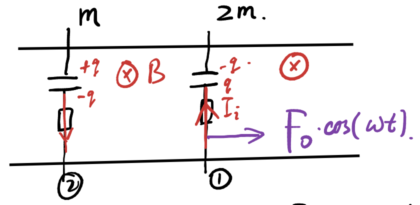
$\Rightarrow \frac{dp}{dt} = -Bq̇l$

$\Rightarrow \frac{d}{dt}(p+Blq)=0$

电学：$Bl\dot{x} = r\dot{q}+\frac{q}{c}$

$\Rightarrow m\dot{x}+Blq = $ 常。

B^2l^2c　$\frac{q}{Blc}$

例: 

初: 静. $Q=0$.

(1). $V_c(t)$. (2). $I(t)$. 并讨论长时间之后行为.

(3). $V_1(t)$ $V_2(t)$. 讨论长时间后之行为.

(1). 节点: $\sum I_i = 0$ ⇒ 整体: $F_0\cos(\omega t) = 3m\cdot a_c$

⇒ $v_c = \frac{F_0}{3m\omega}\sin(\omega t)$.

(2). 左: 力: $IBl = m\dot{v}_2$ ①

右: 力: $F_0\cos(\omega t) - IBl = 2m\cdot\dot{v}_1$ ②

电学: $\frac{2q}{C} + 2I\cdot r = Bl(v_1 - v_2)$ ③

$\frac{d③}{dt}$ ⇒ $\frac{2}{C}I + 2r\dot{I} = Bl(\dot{v}_1 - \dot{v}_2)$

$\frac{2}{C}I + 2r\cdot\dot{I} = Bl\left(\frac{F_0\cos(\omega t)}{2m} - \frac{IBl}{2m} - \frac{IBl}{m}\right)$

⇒ $I + \frac{4mcr}{4m+3B^2l^2C}\cdot\dot{I} = \frac{BlC\cdot F_0\cdot\cos(\omega t)}{4m+3B^2l^2C}$

① 猜到一个非齐次特解.

② 齐次之通解 ⇒ 非齐次之通解.

先猜特解. $I = I_0\cdot\cos(\omega t+\varphi_0)$.

代入, $I_0\cos(\omega t+\varphi_0) - \frac{4mcrI_0\omega}{4m+3B^2l^2C}\cdot\sin(\omega t+\varphi_0) = \frac{BlC\cdot F_0}{4m+3B^2l^2C}\cdot\cos(\omega t)$

$I_0\cdot\sqrt{1+\left(\frac{4mcr\omega}{4m+3B^2l^2C}\right)^2}\,\cancel{\cos(\omega t}+\alpha+\varphi_0) = \frac{BlC\cdot F_0}{4m+3B^2l^2C}\cancel{\cos(\omega t)}$

⇒ $I_0 = \frac{BlC\cdot F_0}{\sqrt{(4m+3B^2l^2C)^2+(4mcr\cdot\omega)^2}}$

$\varphi_0 = -\alpha = -\arctan\frac{4mcr\omega}{4m+3B^2l^2C}$

再看齐通解: $I + \sigma\cdot\dot{I} = 0$. 令 $I = A\cdot e^{\lambda t}$

⇒ $A + \sigma A\cdot\lambda = 0$ ⇒ $\lambda = -\frac{1}{\sigma}$

⇒ $I = A\cdot e^{-\frac{1}{\sigma}\cdot t}$.

⇒ $I = Ae^{-\frac{1}{\sigma}\cdot t} + I_0\cdot\cos(\omega t+\varphi_0)$.

其中 $\sigma = \frac{4mrC}{4m+3B^2l^2C}$

初: $I(0) = 0$ ⇒ $A = -I_0\cdot\cos(\varphi_0)$.

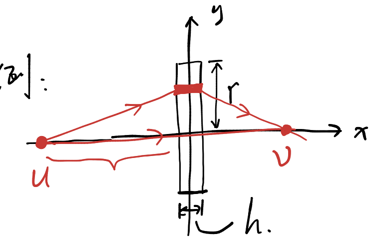
⇒ $I = I_0\cdot\cos(\omega t+\varphi_0) - I_0\cdot\cos\varphi_0\cdot e^{-\frac{1}{\sigma}\cdot t}$

长时间之后: $I = I_0\cos(\omega t+\varphi_0)$

⇒ (3). ...

光学. 几何光学.

等光程聚焦. ↔ 非傍轴.

例: 

$n(y) = n_0 - \alpha y^2$

$h \ll r$.

近轴条件.

$\frac{1}{u}+\frac{1}{v} = \frac{1}{f}$

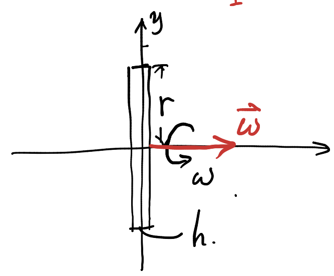
$\sqrt{u^2+y^2} + h\cdot(n_0-\alpha y^2) + \sqrt{v^2+y^2} = C$.

⇒ $u + \frac{y^2}{2u} + v + \frac{y^2}{2v} + hn_0 - h\alpha y^2 = C$.

⇒ $\frac{1}{2u} + \frac{1}{2v} - h\alpha = 0$ ⇒ $\frac{1}{u}+\frac{1}{v} = 2h\alpha \triangleq \frac{1}{f} = \Phi$.

⇒ $\Delta(y) = \Delta(0) - \boxed{\frac{\Phi}{2}}y^2$ $n(y) = n_0 - \alpha y^2$.

⇒ Φ

例: 

气体. T, r.

折射率 $n = 1+\alpha\nu$.

ν: 数密度.

中心: ν_0. 求 f

算. 等效模型 ⇒ ✓

记势.

⇒ M.B分布: $\nu(y) = \nu_0\cdot e^{-\frac{E_p}{kT}}$

$\nu(y) = \nu_0\cdot e^{-\frac{-\frac{1}{2}m\omega^2y^2}{kT}}$

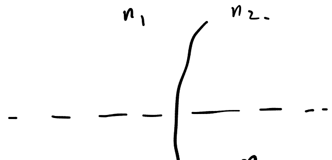
$F^* = \int m\omega^2 r\,dr$

⇒ $\nu_y = \nu_0\left(1+\frac{m\omega^2y^2}{2kT}\right)$. ⇒

⇒ $n_y = 1+\alpha\nu_0 + \alpha\nu_0\frac{m\omega^2}{2kT}y^2$.

⇒ $\Delta(y) = \Delta(0) + \frac{1}{2}\alpha\nu_0\frac{m\omega^2 h}{kT}\cdot y^2$ ⇒ $f = \frac{1}{\Phi} = -\frac{kT}{\alpha\nu_0 m\omega^2 h}$.

一般情形、近轴.



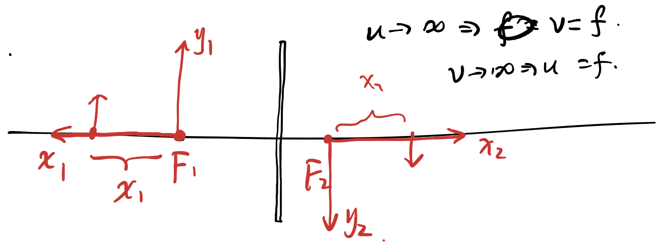
$$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_{后}-n_{前}}{|R|}=\Phi$$

$$\Rightarrow f_1=\frac{n_1}{\Phi},\ f_2=\frac{n_2}{\Phi}$$

牛顿成像公式.

$u\to\infty\Rightarrow v=f$.

$v\to\infty\Rightarrow u=f$.

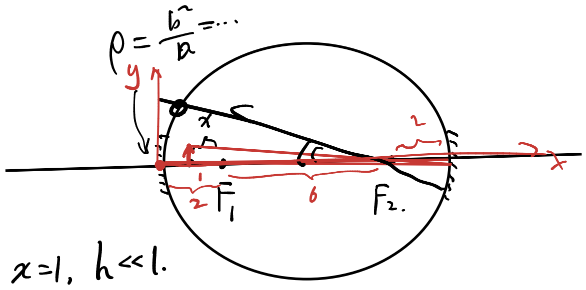


$$\frac{f_1}{u}+\frac{f_2}{v}=1$$

$$\Rightarrow x_1\cdot x_2=f_1f_2,\ \Rightarrow 横向放大:\ \frac{y_2}{y_1}=\frac{n_1\cdot(f_2+x_2)}{n_2\cdot(f_1+x_1)}$$

例：反射成像.

求先右、再左成像.



$a,b,x,h.$

$a=5,\ b=4\Rightarrow c=3.\ x=1,\ h\ll1.$

(1) 逐次成像法 ✓ (2) 用椭圆＜光学性质，几何法.

(2). 光线1: $F_2\to F_1\to F_2$

斜率: $-\frac{h}{7},\ -\frac{h}{7}\cdot2\cdot\frac{1}{8},\ -\frac{h}{7}\cdot\frac{2}{8}\cdot2\cdot\frac{1}{8}$

$$\Rightarrow k_1=-\frac{h}{7\times16}$$

光线2. $F_1\to F_2\to F_1$

$-\frac{h}{1},\ -\frac{h}{1}\cdot8\cdot\frac{1}{2},\ -\frac{h}{1}\cdot8\cdot\frac{1}{2}\cdot8\cdot\frac{1}{2}.$

$$\Rightarrow k_2=-16h.$$

$\Rightarrow$ 光线1: $y=8\times\frac{h}{7\times16}-\frac{h}{7\times16}\cdot x$

光线2: $y=2\times16h-16h\cdot x.$

成像: $x=1.997\Leftrightarrow$ 成像一样.

例：成像中后曲线二次性保持不变.

$f_1:f_2=n_1:n_2$

前：直. $\to$ 后. 直.

前：二次曲线 $\to$ 后. 二次曲线

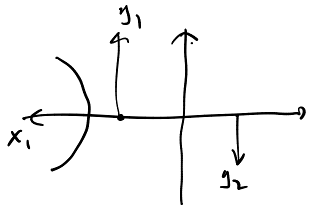
曲线: $f(x_1,y_1)=0$ ✓ 求后: $g(x_2,y_2)=0$ 求.

$$x_1\cdot x_2=f_1f_2.\qquad \frac{y_2}{y_1}=\frac{n_1\cdot(x_2+f_2)}{n_2(x_1+f_1)}$$

$$x_1=\frac{f_1f_2}{x_2}.\ y_1=y_2\cdot\frac{n_2(\frac{f_1f_2}{x_2}+f_1)}{n_1(x_2+f_2)}=y_2\frac{n_2f_1(\cancel{f_2+x_2})}{n_1x_2(\cancel{x_2+f_2})}=\frac{y_2f_2}{x_2}$$

二次 $\to$ 二次.

已知成像之前:

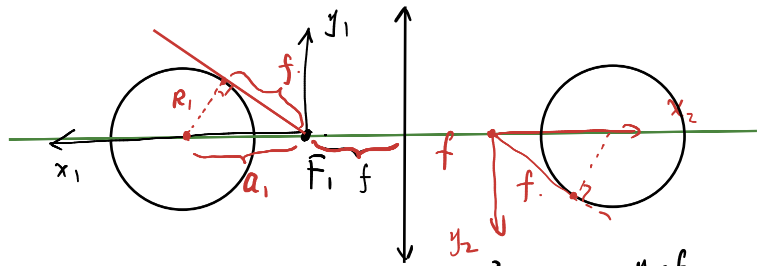


$$ax_1^2+bx_1y_1+cy_1^2+dx_1+ey_1+g=0.$$

$$x_1=\frac{f_1f_2}{x_2},\ y_1=\frac{y_2\cdot f_2}{x_2}$$

$$af_1f_2^2+bf_1f_2^2y_2+cf_2^2y_2^2+d\cdot f_1f_2x_2+ef_2\cdot y_2\cdot x_2+g\cdot x_2^2=0.$$

例：圆 $\to$ 圆. 证明: $f-f$ 关系. 令($f_1=f_2$).



$$(x_1-a_1)^2+y_1^2=R_1^2.\qquad x_1=\frac{f^2}{x_2},\ y_1=\frac{y_2\cdot f}{x_2}$$

$$\left(\frac{f^2}{x_2}-a_1\right)^2+\left(\frac{y_2f}{x_2}\right)^2=R_1^2$$

$$\Rightarrow a_1^2x_2^2-2a_1f^2\cdot x_2+f^4+f^2y_2^2-R_1^2x_2^2=0.$$

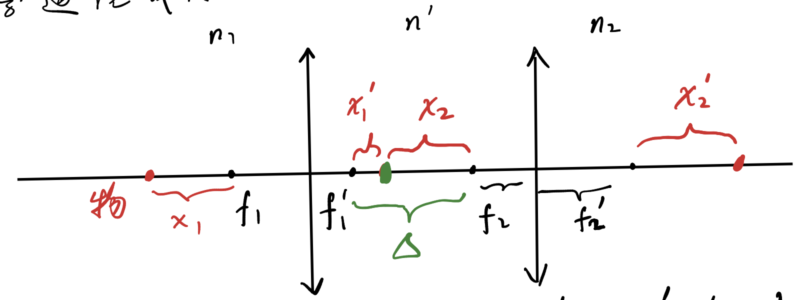
$$(a_1^2-R_1^2)x_2^2+f^2\cdot y_2^2-2a_1f^2\cdot x_2+f^4=0$$

为使 x_2、y_2 是一个圆. $\boxed{a_1^2-R_1^2=f^2}$

$$x_2^2+y_2^2-2a_1\cdot x_2+f^2=0.$$

$$\Rightarrow \boxed{(x_2-a_1)^2+y_2^2=a_1^2-f^2=R_1^2}$$

组合透镜成像公式:



成像: $x_1\cdot x_1'=f_1f_1',\ x_2\cdot x_2'=f_2\cdot f_2',\ x_1'+x_2=\Delta.$

$$\Rightarrow \frac{f_1f_1'}{x_1}+\frac{f_2f_2'}{x_2'}=\Delta.$$

$$\frac{n_1}{u_1}+\frac{n_2}{v_1}=\Phi$$

$$(u_1-f_1)\cdot(v_1-f_2)=f_1\cdot f_2$$

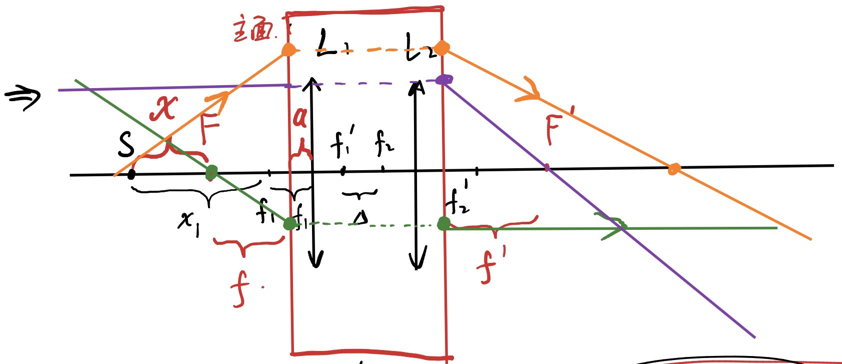
$$x_1\cdot x_2=f_1f_2$$

$$\left(x_1-\frac{f_1f_1'}{\Delta}\right)\cdot\left(x_2'-\frac{f_2f_2'}{\Delta}\right)=\frac{f_1f_2}{\Delta}\cdot\frac{f_1'f_2'}{\Delta}$$

组合透镜: $f=\frac{f_1f_2}{\Delta}.\ f'=\frac{f_1'f_2'}{\Delta}$

x $\quad x'$

$$\Rightarrow \boxed{x\cdot x'=f\cdot f'}$$

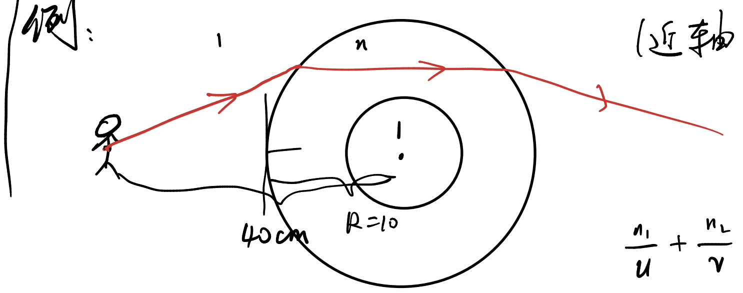


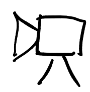
$f=\frac{f_1f_2}{\Delta}$, $f'=\frac{f_1'f_2'}{\Delta}$ $\rightarrow$ $\boxed{x\cdot x'=f\cdot f'}$

$L_1 \Leftrightarrow S$: $x_1+f_1=x+f+a \longleftrightarrow \boxed{a}$ ← 不需要

$$\left(x_1-\frac{f_1f_1'}{\Delta}\right)\cdot\left(x_2'-\frac{f_2f_2'}{\Delta}\right)=\frac{f_1f_2}{\Delta}\cdot\frac{f_1'f_2'}{\Delta}$$

例: $n_水=\frac{4}{3}$. $R=10cm$. $r=4cm$. (近轴成像)





$\frac{n_1}{u}+\frac{n_2}{v}=\frac{n-1}{R}\Leftarrow\Phi$

$\Phi=\frac{\frac{1}{3}}{10}=\frac{1}{30}$

求2个像之位置及放大率.

法一. 用单球面成像:

第一次: $\frac{1}{30}+\frac{n}{v_1}=\frac{\frac{4}{3}-1}{|R|}\Rightarrow\frac{\frac{4}{3}}{v_1}=0\Rightarrow v_1=\infty$

$\Rightarrow v_外=30cm \Rightarrow$ 到球心: 40 cm

法二: 外球成像: $f_1=\frac{1}{\Phi}=30$, $f_1'=\frac{n}{\Phi}=40$.
$f_2=40$. $f_2'=30$.

$\frac{f_1f_1'}{\Delta}=-20$.

$\Rightarrow$ $\Delta=20-40-40=-60$ cm.

$\frac{f_2f_2'}{\Delta}=-20$

$x_1=30-30=0$

$\left(x_1-\frac{f_1f_1'}{\Delta}\right)\cdot\left(x_2-\frac{f_2f_2'}{\Delta}\right)=\frac{f_1f_1'f_2f_2'}{\Delta^2}$.

$\Rightarrow$ $20\times(x_2+20)=20^2\Rightarrow x_2=0$

$\Rightarrow$ 像到球心距离: 40 cm

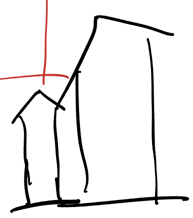
法三: "角动量守恒". 粒子光学. 颗粒

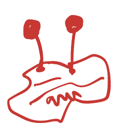
定义: $\boxed{v=c\cdot n}$ → 真空中粒子速度. ← 折射率.

① 直线传播. ② 折射定律.

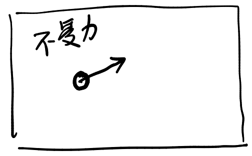
理论: $\boxed{定律. 公理. 原理}$ + 推导.

信仰



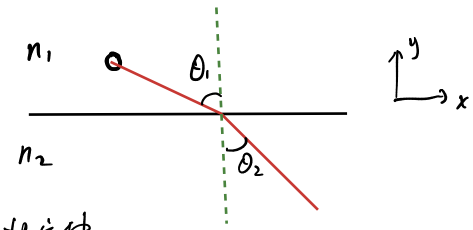


① 直线传播. ✓ 粒子速率v不变



$\frac{\partial E}{\partial x}=0$.

② 折射:



$\frac{\partial E}{\partial y}\neq0$

$\frac{\partial E}{\partial x}=0$

$F_x=0$, 动量守恒 $\Rightarrow$ $\cancel{m\cdot c}\cdot n_1\cdot\sin\theta_1=\cancel{m\cdot c}\cdot n_2\cdot\sin\theta_2$.

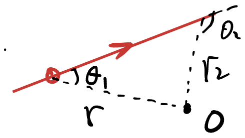
$n_1\cdot\sin\theta_1=n_2\cdot\sin\theta_2\Rightarrow$ ✓.

③ 定义 能量 守恒: $\frac{1}{2}\cdot m(cn)^2+E_p=常$

$\Rightarrow E_p=常-\frac{1}{2}m\cdot c^2\cdot n^2\triangleq-\frac{1}{2}m\,c^2\cdot n^2$

验证: 角动量是否守恒.

① 均匀介质.



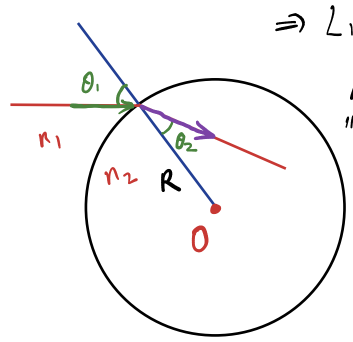
$L_1=m\cdot c\cdot n\cdot r\cdot\sin\theta_1$

$L_2=m\cdot c\cdot n\cdot r_2\cdot\sin\theta_2$

正弦定理: $\frac{r_1}{\sin\theta_2}=\frac{r_2}{\sin\theta_1}$

$\Rightarrow L_1=L_2$.

②.



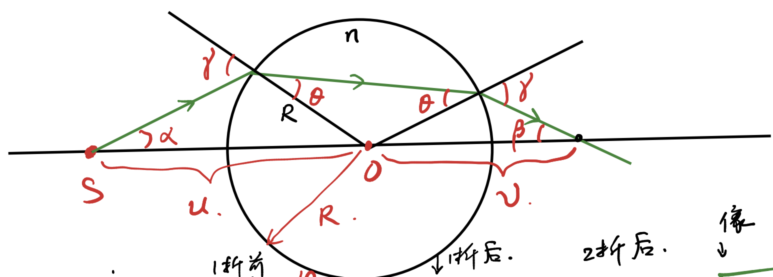
$L_1=m\cdot c\cdot n_1\cdot R\cdot\sin\theta_1$.

$L_2=m\cdot c\cdot n_2\cdot R\cdot\sin\theta_2$.

由 $n_1\cdot\sin\theta_1=n_2\cdot\sin\theta_2$

$\Rightarrow L_1=L_2$.

$\Rightarrow$ 此题情景中. 角动量守恒. 可以用.



初 1折前 1折后. 2折后. 像

40 $\boxed{u\cdot1\cdot\alpha}=R\cdot1\cdot\gamma=R\cdot n\cdot\theta=R\cdot1\cdot\gamma=\boxed{v\cdot1\cdot\beta}$

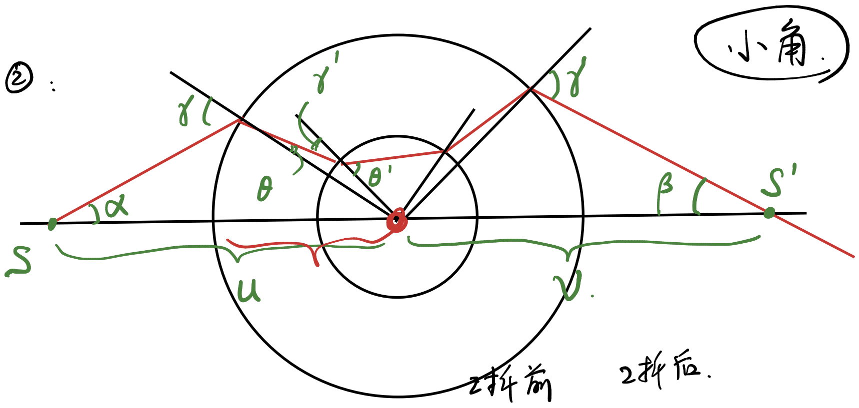
10

$\Rightarrow\gamma=4\alpha$. $\theta=3\alpha$. 求β. 即可求v.

光线总偏转: $\alpha+\beta=2(\gamma-\theta)=2\alpha$.

$\Rightarrow\beta=\alpha$. $\Rightarrow \underline{v=40\text{ cm}}$.

② :



4cm　2折前　2折后.

角放大率:

$\boxed{u\cdot 1\cdot\alpha} = R\cdot 1\cdot\gamma = R\cdot n\cdot\theta = r\cdot n\cdot\gamma' = r\cdot 1\cdot\theta' = \boxed{v\cdot 1\cdot\beta}$.

$\gamma = 4\alpha,\quad \theta = 3\alpha,\quad \gamma' = 7.5\alpha,\quad \theta' = 10\alpha.$

总偏转: $\alpha+\beta = 2(\gamma-\theta) - 2(\theta'-\gamma')$

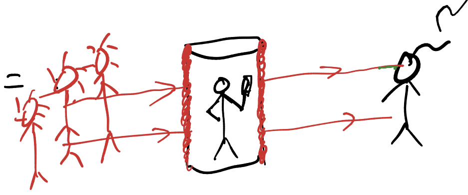
$= 2\alpha - 5\alpha = -3\alpha$.

$\Rightarrow \beta = -4\alpha. \Rightarrow v = -10\,\text{cm}.$

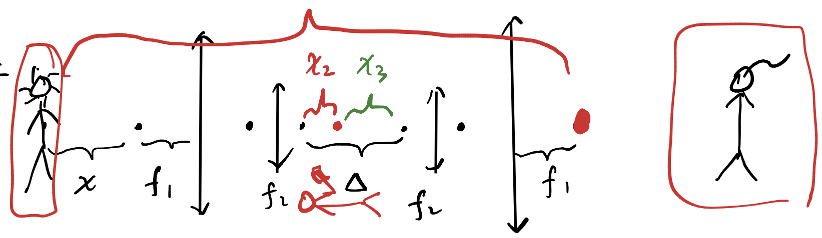
如何隐身?

法一: 特效大法好.

法二



法三



前2次: $x\cdot x_1 = f_1^2$.　$-x_1\cdot x_2 = f_2^2$.

后2次: $x_3\cdot x_3' = f_2^2$,　$+x_3'\cdot[+(x+4(f_1+f_2)+\Delta)] = f_1^2$

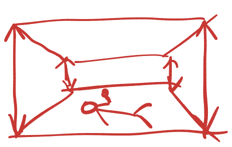
$x_2 + x_3 = \Delta$.

$\Rightarrow -\frac{f_1^2}{x}\cdot x_2 = f_2^2 \Rightarrow x_2 = -\frac{f_2^2}{f_1^2}\cdot x.$

$x_3 = \frac{f_2^2}{x_3'} = \frac{f_2^2}{f_1^2}\cdot(x+4(f_1+f_2)+\Delta)$

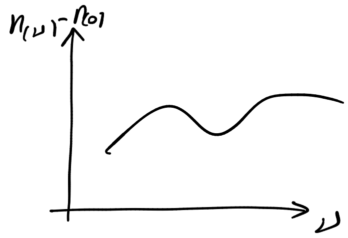
$x_2+x_3=\Delta \Rightarrow \cancel{-x} + (\cancel{x}+4(f_1+f_2)+\Delta) = \frac{f_1^2}{f_2^2}\cdot\Delta.$

$\Rightarrow$ 无求 $\Delta = \frac{4f_2^2}{f_1-f_2}$ $\Rightarrow$ ✓



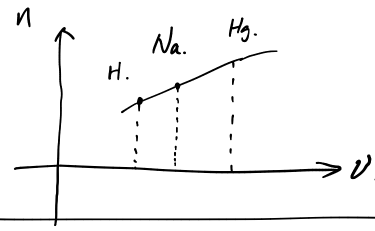
$\Rightarrow$ ✓

如何消除色差.



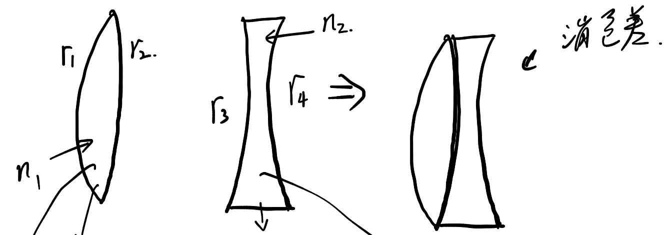
小范围内:

$n(\nu) = n(\nu_0) + \frac{dn}{d\nu}\cdot\Delta\nu.$



定义色散率: $\frac{n_{Hg}-n_H}{n_{Na}-1} = \Delta.$ $\Rightarrow \frac{\Delta n}{n-1}$

透镜:



$\frac{1}{f} = \Phi$　$\frac{1}{f_1} = \Phi_1$　$\frac{1}{f_2} = \Phi_2$.

$\Phi_1 = \frac{1}{\frac{n_1-1}{r_1}+\frac{n_1-1}{r_2}}$　$\Phi_2 = \frac{1}{\frac{1-n_2}{r_3}+\frac{1-n_2}{r_4}}$

$\Rightarrow f_1 = \frac{\frac{r_1 r_2}{r_1+r_2}}{n_1-1}$　$f_2 = -\frac{\frac{r_3 r_4}{r_3+r_4}}{n_2-1}$

$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \frac{df}{f^2} = \frac{df_1}{f_1^2}+\frac{df_2}{f_2^2}$.　要求 $df=0$.

$\Rightarrow \frac{df_1}{f_1^2}+\frac{df_2}{f_2^2} = 0$ ⊛

$\Rightarrow df_1 = -\frac{\frac{r_1 r_2}{r_1+r_2}}{(n_1-1)^2}\cdot dn_1 = -f_1\cdot\frac{dn_1}{n_1-1}$

$df_2 = \frac{\frac{r_3 r_4}{r_3+r_4}}{(n_2-1)^2}\cdot dn_2 = -f_2\cdot\frac{dn_2}{n_2-1}$

$-\frac{1}{f_1}\cdot\frac{dn_1}{n_1-1} - \frac{1}{f_2}\cdot\frac{dn_2}{n_2-1} = 0.$

$\Updownarrow\ \Delta_1$　$\Updownarrow\ \Delta_2$.

$\Rightarrow$ $\boxed{f_2\cdot\Delta_1 + f_1\cdot\Delta_2 = 0}$

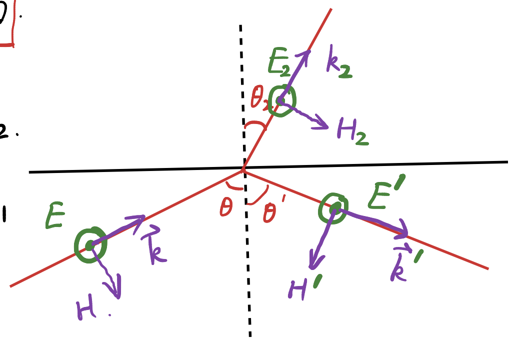
折射时, 振幅有关系.

入射. $\vec{E}$.　出射E.　反射. E.

(1). S光.　E ⊥ 入射面.

(1). S光. $\boxed{E \perp 入射面}$

$\vec{E} \times \vec{H} \parallel \vec{k}$



边界条件:

$E_{\parallel}$ 连.

$\Rightarrow E + E' = E_2$. ①

$\theta = \theta'$.

$H_{\parallel}$ 连续. $H\cdot\cos\theta - H'\cdot\cos\theta' = H_2\cdot\cos\theta_2$. ②

$H = \sqrt{\frac{\varepsilon}{\mu}}\cdot E$. 取 $\boxed{\mu = \mu_0}$. $\mu_r = 1$

② $\Rightarrow \sqrt{\varepsilon_1}\cdot(E - E')\cdot\cos\theta_1 = \sqrt{\varepsilon_2}\cdot E_2\cdot\cos\theta_2$.

$v_{光} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_r\mu_r}\cdot\sqrt{\varepsilon_0\mu_0}} = n\cdot c$. $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$

$\sqrt{\varepsilon} = n\cdot\sqrt{\varepsilon_0}$. $\Rightarrow \sqrt{\varepsilon_1} : \sqrt{\varepsilon_2} = n_1 : n_2$.

$$\Rightarrow \begin{cases} \dfrac{E'}{E} = \dfrac{n_1\cdot\cos\theta_1 - n_2\cdot\cos\theta_2}{n_1\cdot\cos\theta_1 + n_2\cdot\cos\theta_2} \\ \dfrac{E_2}{E} = \dfrac{2n_1\cdot\cos\theta_1}{n_1\cdot\cos\theta_1 + n_2\cdot\cos\theta_2} \end{cases} \quad S光.$$

利用折射定律: $n_1\cdot\sin\theta_1 = n_2\cdot\sin\theta_2$.

$$\frac{E'}{E} = \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_1+\theta_2)}$$

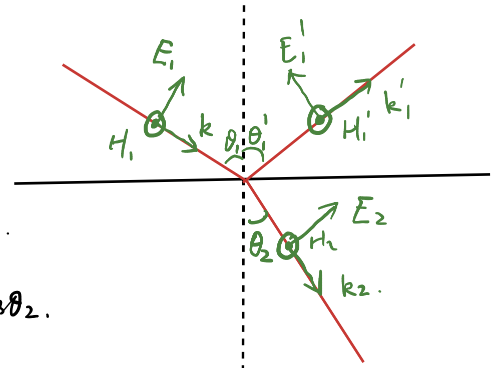
$n_2\theta_2 = n_1\theta_1 \Rightarrow \theta_2 = \frac{n_1}{n_2}\theta_1$

$\Rightarrow$ 半波损失: $n_2 > n_1$ $\theta_2 < \theta_1$.

$$\frac{E'}{E} = \lim_{\theta_1\to 0} -\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)} = \lim_{\theta_1\to 0} -\frac{\frac{n_2-n_1}{n_2}\cdot\theta_1}{\frac{n_1+n_2}{n_2}\cdot\theta_1} = -\boxed{\frac{n_2-n_1}{n_1+n_2}} = \frac{n_2-n_1}{n_1+n_2}e^{i\pi}$$

半波损.

(2) $E \parallel$ 入射面时. P光.



边界: $E_{\parallel}$ 连续

$H_{\parallel}$ 连续.

$\Rightarrow E_1\cdot\cos\theta_1 - E_1'\cdot\cos\theta_1 = E_2\cdot\cos\theta_2$.

$H_1 + H_1' = H_2$. $H_1 = \sqrt{\frac{\varepsilon_1}{\mu_0}}\cdot E_1$

$\Rightarrow n_1\cdot(E_1 + E_1') = n_2\cdot E_2$. $n_1\cdot\sin\theta_1 = n_2\cdot\sin\theta_2$

$$\Rightarrow \text{P光} \begin{cases} \dfrac{E_1'}{E_1} = \dfrac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cdot\cos\theta_1 + n_1\cdot\cos\theta_2} = \dfrac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)} \\ \dfrac{E_2}{E_1} = \dfrac{2n_1\cdot\cos\theta_1}{n_2\cos\theta_1 + n_1\cdot\cos\theta_2} \end{cases}$$

布儒斯特角: 当 $\theta_1+\theta_2 = 90^\circ$ 时 $\Rightarrow \frac{E_1'}{E_1} = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)} \to 0$

$\Rightarrow$ 只有S光. 只有$\perp$于入射面. 反射光也为完全偏振.

全反射: $n_1 > n_2$. $\sin\theta_2 = \frac{n_1}{n_2}\cdot\sin\theta_1 > 1$. 时.

$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$

$\cos\theta_2 = i\cdot\sqrt{\left(\frac{n_1}{n_2}\cdot\sin\theta_1\right)^2 - 1}$.

P光: $$\frac{E_1'}{E_1} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cdot\cos\theta_1 + n_1\cdot\cos\theta_2} = \frac{n_2\cdot\cos\theta_1 - in_1\sqrt{\left(\frac{n_1}{n_2}\cdot\sin\theta_1\right)^2-1}}{n_2\cdot\cos\theta_1 + in_1\sqrt{\left(\frac{n_1}{n_2}\cdot\sin\theta_1\right)^2-1}}$$

$$= \frac{\cancel{A}\cdot e^{-i\varphi}}{\cancel{A}\cdot e^{i\varphi}} = e^{-2i\varphi} \to 模为 1$$

辐角: -2φ.

其中: $\varphi = \arctan\dfrac{n_1\sqrt{\left(\frac{n_1}{n_2}\cdot\sin\theta_1\right)^2-1}}{n_2\cdot\cos\theta_1}$.

$\Rightarrow \boxed{E_1' = E_1\cdot e^{-2i\varphi}} \leftrightarrow$ 能量全反射

S光同理: $\Rightarrow E' = E\cdot e^{-2i\varphi'}$

P光. 折射: $$\frac{E_2}{E_1} = \frac{2n_1\cdot\cos\theta_1}{n_2\cos\theta_1 + n_1\cdot\cos\theta_2} = \frac{2n_1\cdot\cos\theta_1}{n_2\cos\theta_1 + i\cdot n_1\cdot\sqrt{\left(\frac{n_1}{n_2}\sin\theta_1\right)^2-1}}$$

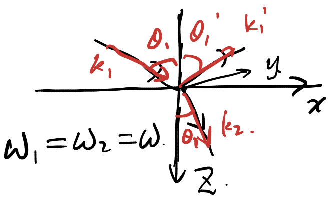
$= A\cdot e^{i\varphi_2}$. $A \neq 0$.

$E_2 \neq 0$

隐失波.
光传播.

$$\tilde E = E_0 \cdot e^{i(\vec k\cdot\vec r-\omega\cdot t)}$$

$$\tilde E_1 = E_1 e^{i(\vec k_1\cdot\vec r_1-\omega_1 t)}$$
$$\tilde E_1' = E_1' e^{i(\vec k_1'\cdot\vec r_1'-\omega_1 t)}$$
$$\tilde E_2 = E_2 e^{i(\vec k_2\cdot\vec r_2-\omega_2 t)}$$



$\omega_1=\omega_2=\omega$

$k_{1x}=k_{1x}'=k_{2x}$

$\Rightarrow k_{1x}=k_1\sin\theta_1$, $k_{1x}'=k_1'\sin\theta_1'$, $k_{2x}=k_2\sin\theta_2$

$\Rightarrow k_1=k_1'=\frac{n_1\omega}{c}$ $\Leftarrow$ $\lambda_1\cdot k_1=2\pi=T\cdot\omega$

$k_2=\frac{n_2\omega}{c}$

当全反射时. $n_1\sin\theta_1=n_2\sin\theta_2 \Rightarrow \sin\theta_2=\frac{n_1}{n_2}\sin\theta_1>1$

$\Rightarrow k_2^2=k_{2x}^2+k_{2z}^2 \Rightarrow k_{2z}=\sqrt{k_2^2-k_{2x}^2}$

$\sin\theta_c=\frac{n_2}{n_1}$

$$k_{2z}=\sqrt{\left(\frac{n_2}{n_1}\right)^2k_1^2-k_1^2\sin^2\theta_1}$$
$$=ik_1\sqrt{\sin^2\theta_1-\left(\frac{n_2}{n_1}\right)^2}=ik_1\sqrt{\sin^2\theta_1-\sin^2\theta_c}$$
$$=i\cdot\frac{2\pi}{\lambda_1}\sqrt{\sin^2\theta_1-\sin^2\theta_c}$$

$$\Rightarrow E_2=E_2\, e^{i(k_{2z}z+k_{1x}x-\omega t)}$$
$$=\boxed{E_2\, e^{-\frac{2\pi}{\lambda_1}\sqrt{\sin^2\theta_1-\sin^2\theta_c}\cdot z}}\cdot \boxed{e^{i(k_{1x}x-\omega t)}}$$

行波

沿界面传播.

隐失波.

穿透深度: dz 处. E_2 变成 e^{-1} 倍.

$$dz=\frac{1}{|k_{2z}|}=\frac{\lambda_1}{2\pi\sqrt{\sin^2\theta_1-\sin^2\theta_c}}$$

偏振.

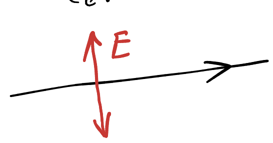
器件: 偏振片. (起偏器. 检偏器)

$\frac14\lambda$波片. $\frac12\lambda$波片.

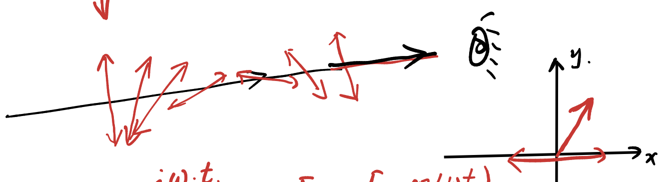
自然光: 各方向有且均匀、随机

部分偏振:

线偏光: E



圆偏光

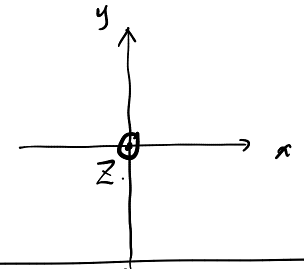


$$\tilde E=E\, e^{i\omega t} \Leftarrow \begin{cases}E_x=E_0\cos(\omega t)\\ E_y=E_0\cos(\omega t\pm\frac{\pi}{2})\end{cases}$$

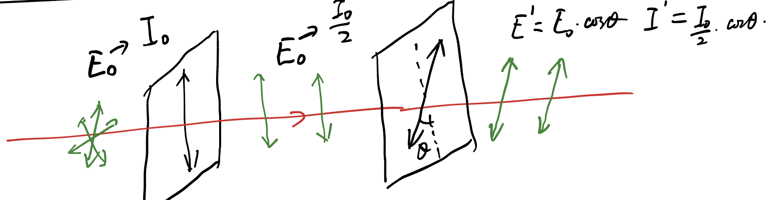
椭圆偏振

$E_x=E_0\cos(\omega t)$

$E_y=E_0'\cos(\omega t\pm\frac{\pi}{2})$.

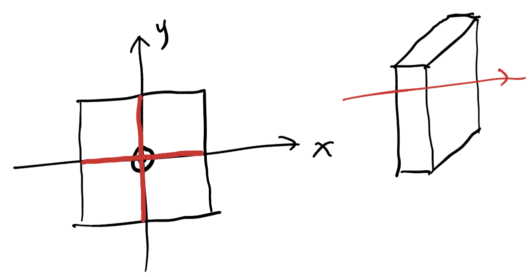


偏振片:



马吕斯定律: $\frac{I'}{I}=\cos^2\theta$.

$\frac12\lambda$波片 $\frac14\lambda$波片. 只对某波长严格.

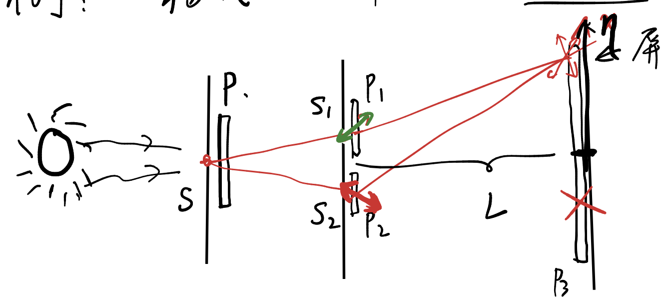


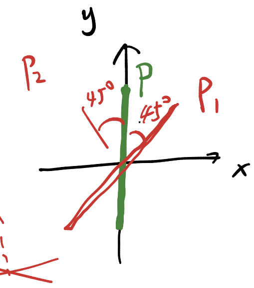
x、y. 有额外相差. 光程差

$\Delta=\frac12\lambda+\lambda\cdot n \Leftrightarrow \frac12\lambda$.

$\Delta=\frac14\lambda+\lambda\cdot n \Leftrightarrow \frac14\lambda$片.

例: 杨氏双缝干涉. ← 偏振片. 求如下情形时. 屏. 条纹如何变化.





(1) 只放P. (2) P, P_1, P_2, 且

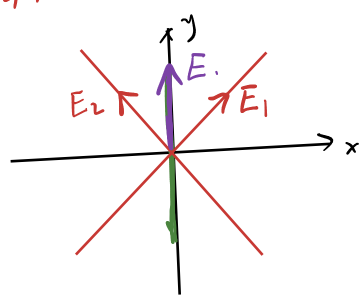
(3) 接(2). 再放P_3 且 $P_3 // P$.

(4). 接(3). $P_3 \perp P$ 自然光 ⇒ ↔ ↕ 无稳定相差 ⇒ 无条纹.

(5)在(3)中间. 去掉P. ⇒ 无条纹.

(1). 有条纹: 光强 ⇒ 一半.

(2).



$E_1(\eta)=E_1\cdot e^{-ik\sqrt{L^2+\eta^2}-i\omega t}$

$E_2(\eta)=E_2\cdot e^{-ik\sqrt{L^2+(\eta+d)^2}-i\omega t}$ 双缝 间距.

$\Rightarrow E_1(\eta)=E_1\cos(\omega t)$ ①

$E_2(\eta)=E_2\cos(\omega t+\varphi(\eta))$ ②

随η而改变

只改变偏振状态.

不改变光强.

⇒ 无条纹.

$\varphi(\eta)=\pm\frac{\pi}{2}$时 ⇒ 圆偏.

$\varphi(\eta)=n\pi$. ⇒ 线偏.

$\varphi(\eta)$: 椭偏光.

(3)

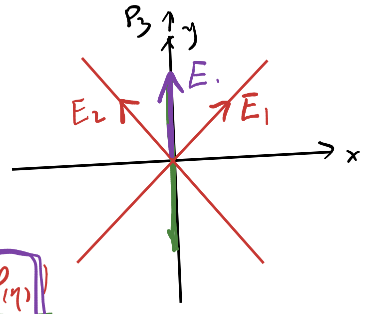
$\Rightarrow \begin{cases} E_1(\eta) = E_1 \cdot \cos(\omega t) \\ E_2(\eta) = E_2 \cdot \cos(\omega t + \varphi(\eta)) \end{cases}$

$P_2 P_3$

$\begin{cases} E_1'(\eta) = E_1 \cdot \frac{\sqrt{2}}{2} \cdot \cos(\omega t) \\ E_2'(\eta) = E_2 \cdot \frac{\sqrt{2}}{2} \cdot \cos(\omega t + \boxed{\varphi(\eta)}) \end{cases}$

同向 $\Rightarrow E_总' = E_1' + E_2'(\eta) \Rightarrow$ 条纹

有条纹，光强变为 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$



$2 \cdot \cos(\omega t)$

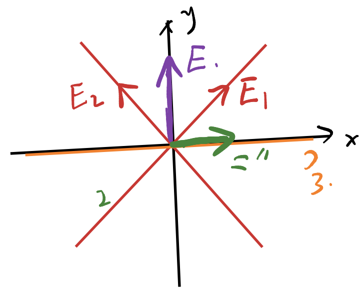
( )

$E_2''(\eta) = E_2 \cdot \frac{\sqrt{2}}{2} \cdot \cos(\omega t + \eta)$

同向

$\Rightarrow$ 有条纹，强度：$\frac{1}{8}$

条纹有平移，平移半个条纹间距



(5)