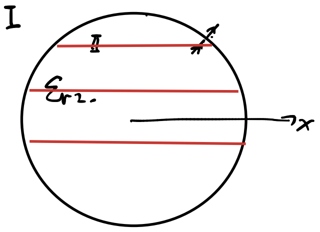
(2)



外：$\varphi_1 = A_0 + \frac{B_0}{r} + (A_1 r + B_1 \frac{1}{r^2})\cos\theta + \Sigma \cdots$

内：$\varphi_2 = A_0' + \frac{B_0'}{r} + (A_1' r + B_1' \cdot \frac{1}{r^2})\cos\theta + \Sigma \cdots$

$r \to \infty$：$\varphi_1 = r \cdot \cos\theta \cdot E_0$.

$\Rightarrow A_0 = 0$. $A_1 = E_0$. $A_n (n \geq 2) = 0$.

$r \to 0$. φ_2 不发散. $\Rightarrow B_n' = 0$. $n \geq 0$.

边界条件：$r = R$. $\begin{cases} \varphi_{1(R)} = \varphi_{2(R)} \\ \varepsilon_{r1} \cdot E_{n1} = \varepsilon_{r2} \cdot E_{n2}. \end{cases}$

$$\begin{cases} \varphi_1 = \frac{B_0}{r} + (E_0 \cdot r + \frac{B_1}{r^2}) \cdot \cos\theta + \sum_{n\geq 2} (B_n \cdot \frac{1}{r^{n+1}}) \cdot P_n(\cos\theta). \\ \varphi_2 = A_0' + A_1' r \cdot \cos\theta + \sum_{n\geq 2} A_n' \cdot r^n \cdot P_n(\cos\theta). \end{cases}$$

$$\Rightarrow \varphi_1 = \frac{B_0}{R} + (E_0 \cdot R + \frac{B_1}{R^2})\cos\theta + \sum_{n\geq 2} (B_n \frac{1}{R^{n+1}}) P_n(\cos\theta) \quad ①.$$

$$\varphi_2 = A_0' + A_1' \cdot R \cdot \cos\theta + \sum_{n \geq 2} (A_n R^n) P_n(\cos\theta)$$

$$E_{n1} = -\left.\frac{\partial \varphi_1}{\partial r}\right|_{r=R} = \frac{B_0}{R^2} + (-E_0 + \frac{2B_1}{R^3})\cos\theta + \sum_{n\geq 2} B_n \cdot \frac{n+1}{R^{n+2}} P_n(\cos\theta)$$

$$E_{n2} = -\left.\frac{\partial \varphi_2}{\partial r}\right|_{r=R} = -A_1' \cos\theta + \sum_{n\geq 2} -A_n' \cdot n R^{n-1} P_n(\cos\theta)$$

$\varepsilon_{r1} \cdot E_{n1} = \varepsilon_{r2} \cdot E_{n2}$. ②.

P_0："1"：φ：$\frac{B_0}{R} = A_0'$. $\Rightarrow B_0 = 0$, $A_0' = 0$.

E_n：$\varepsilon_{r1} \cdot \frac{B_0}{R^2} = 0$.

P_1："$\cos\theta$"：φ：$E_0 \cdot R + \frac{B_1}{R^2} = A_1' \cdot R$.

E_n：$\varepsilon_{r1}(-E_0 + \frac{2B_1}{R^3}) = \varepsilon_{r2} \cdot (-A_1')$.

$$\Rightarrow A_1' = \frac{3\varepsilon_{r1}}{\varepsilon_{r2} + 2\varepsilon_{r1}} E_0, \quad B_1 = \frac{\varepsilon_{r1} - \varepsilon_{r2}}{\varepsilon_{r2} + 2\varepsilon_{r1}} \cdot E_0 \cdot R^3.$$

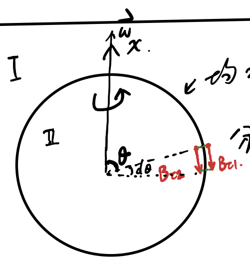
$n \geq 2$：φ：$B_n \cdot \frac{1}{R^{n+1}} = A_n' \cdot R^n$. $\Rightarrow A_n' = B_n = 0$. $n \geq 2$.

E_n：$\varepsilon_{r1} \cdot B_n \cdot \frac{n+1}{R^{n+2}} = -A_n' \cdot n \cdot R^{n-1}$

$$\Rightarrow \varphi_1 = \underbrace{r E_0 \cdot \cos\theta}_{匀强} + \frac{\varepsilon_{r1} - \varepsilon_{r2}}{2\varepsilon_{r1} + \varepsilon_{r2}} \cdot \underbrace{\frac{E_0 \cdot R^3}{r^2}}_{偶极子} \cdot \cos\theta. \quad r \geq R.$$

$$\varphi_2 = \frac{3\varepsilon_{r1}}{2\varepsilon_{r1} + \varepsilon_{r2}} \cdot E_0 \cdot \underbrace{r \cdot \cos\theta}_{匀强}.$$

例：I



均匀带电，σ 以 ω 自旋转. 求 $\vec{B}(\vec{r})$ 分布.

分别讨论内、外. 无电流.

$\nabla \cdot \vec{E} = 0$ $\boxed{\nabla \times \vec{B} = 0}$ ← 定义一个磁标势.

使得. $\vec{B} = -\nabla \varphi_m \Rightarrow \nabla^2 \varphi_m = 0$.

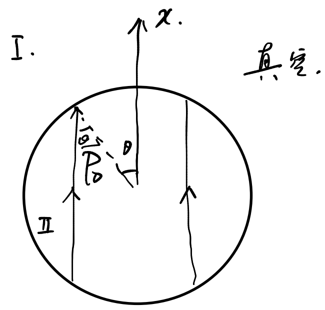
猜：外：$\varphi_1 = A_0 + \frac{B_0}{r} + (A_1 r + B_1 \frac{1}{r^2})\cos\theta$

内：$\varphi_2 = A_0' + \frac{B_0'}{r} + (A_1' r + B_1' \frac{1}{r^2})\cos\theta$

例：球形驻极体（永电体）.

$\vec{P}_0 = P_0 \hat{x}$. $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}_0}$ $\Rightarrow D_{n1} = D_{n2}$



外：$\varphi_1 = A_0 + \frac{B_0}{r} + A_1 \cdot r \cdot \cos\theta + \frac{B_1}{r^2} \cdot \cos\theta$.

内：$\varphi_2 = A_0' + \frac{B_0'}{r} + A_1' \cdot r \cdot \cos\theta + \frac{B_1'}{r^2} \cos\theta$.

$r \to \infty$：有限 $\Rightarrow A_0 = 0$. $A_1 = 0$.

$r \to 0$：φ_2 不能炸 $\Rightarrow B_0' = B_1' = 0$.

$$\Rightarrow \begin{cases} \varphi_1 = \frac{B_0}{r} + \frac{B_1}{r^2} \cdot \cos\theta. \\ \varphi_2 = A_0' + A_1' \cdot r \cdot \cos\theta. \end{cases}$$

$r \to R$. φ 连续：$\varphi_1 = \varphi_2$. $D_{n1} = D_{n2}$.

$$\frac{B_0}{R} + \frac{B_1}{R^2}\cos\theta = A_0' + A_1' R \cdot \cos\theta.$$

$$D_{n1} = -\left.\frac{\partial \varphi_1}{\partial r}\right|_{r=R} = \frac{B_0}{R^2} + \frac{2B_1}{R^3}\cos\theta.$$

$$D_{n2} = -\varepsilon_0 \left.\frac{\partial \varphi_2}{\partial r}\right|_{r=R} + P_0(R, \theta处) \cdot n.$$

$$= -\varepsilon_0 A_1' \cos\theta + P_0 \cdot \cos\theta.$$

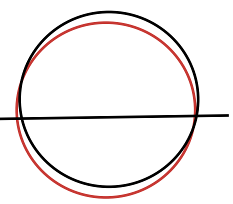
$\Rightarrow$ 0阶：P_0："1" $\Rightarrow$ $\begin{cases} B_0/R = A_0' \\ B_0/R^2 = 0. \end{cases}$ $\Rightarrow B_0 = 0$.

1阶：P_1："$\cos\theta$" $\Rightarrow$ $\begin{cases} \frac{B_1}{R^2} = A_1' \cdot R \\ \frac{2B_1}{R^3} = -\varepsilon_0 A_1' + P_0 \end{cases}$

$$\Rightarrow B_1 = \frac{P_0}{3\varepsilon_0} \cdot R^3, \quad A_1' = -\frac{P_0}{3\varepsilon_0}$$

$$\Rightarrow \varphi_1 = -\frac{P_0}{3\varepsilon_0} \cdot \frac{R^3}{r^2} \cdot \cos\theta. \leftarrow 偶极.$$

$$\varphi_2 = -\frac{P_0}{3\varepsilon_0} \cdot r \cdot \cos\theta \leftarrow 匀强$$



$r \to \infty$：$\varphi_1 = 0$. $\Rightarrow A_0 = A_1 = 0$.

$r \to 0$：φ_2 不能炸 $\Rightarrow B_0' = B_1' = 0$.

$\Rightarrow \varphi_1 = \frac{B_0}{r} + \frac{B_1}{r^2}\cos\theta$, $\varphi_2 = A_0' + A_1' \cdot r \cdot \cos\theta$.

边界：$r = R$ $B_{n1} = B_{n2}$. $B_{\tau 1}$, $B_{\tau 2}$.

$(B_{\tau 1} - B_{\tau 2}) \cdot \cancel{R \cdot d\theta} = \mu_0 \cdot dI = \mu_0 \cdot \frac{\cancel{2\pi} \cdot R \cdot \sin\theta \cdot \cancel{R d\theta} \cdot \sigma}{\cancel{2\pi}/\omega}$

$B_{\tau 1} - B_{\tau 2} = \omega \cdot \mu_0 \cdot \sigma \cdot R \cdot \sin\theta$.

$\Rightarrow B_n = -\frac{\partial \varphi_m}{\partial r}$. $B_\tau = -\frac{1}{r}\frac{\partial \varphi_m}{\partial \theta}$.

$\varphi_1 = \frac{B_0}{r} + \frac{B_1}{r^2}\cos\theta$, $\quad \varphi_2 = A_0' + A_1'\cdot r\cdot\cos\theta$.

$\Rightarrow$ n: $\frac{B_0}{R^2} + \frac{2B_1}{R^3}\cos\theta = -A_1'\cdot\cos\theta$.

τ: $\frac{B_1}{R^3}\cdot\sin\theta - \left(\frac{1}{R}\cdot A_1'\cdot R\cdot\sin\theta\right) = \omega\cdot\mu_0\cdot\sigma\cdot R\cdot\sin\theta$.

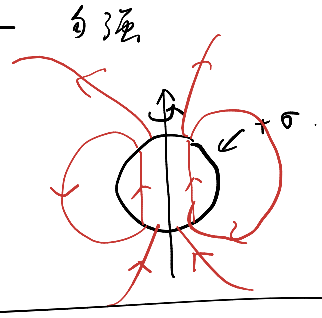
$\Rightarrow$ $\frac{B_0}{R^2} = 0$. $\quad -\frac{2B_1}{R^3} = A_1'$

$\frac{B_1}{R^3} - A_1' = \omega\mu_0\cdot\sigma\cdot R$.

$\Rightarrow B_1 = -\frac{1}{3}\mu_0\omega\cdot\sigma R^4$, $\quad A_1' = \frac{2}{3}\mu_0\omega\sigma\cdot R$

$\Rightarrow \varphi_1 = -\frac{1}{3}\mu_0\cdot\omega\sigma\cdot\frac{R^4}{r^2}\cdot\cos\theta$ ← 偶极

$\varphi_2 = \frac{2}{3}\mu_0\cdot\omega\sigma\cdot R\cdot r\cdot\cos\theta$ ← 匀强



电像法.

唯一性定理. $\boxed{\text{内}: \rho_f(\vec r)}$. 边界: $\begin{cases} \varphi\|_{\text{边界}} \text{ ← 第一类} & \Rightarrow \vec E(\vec r) \text{ 有唯一解} \\ \text{或 } \frac{\partial\varphi}{\partial n}\|_{\text{边界}} \text{ ← 第二类} \end{cases}$

不求证明: $\varphi_1(\vec r)$、$\varphi_2(\vec r)$ 是解.

$\varphi = \varphi_1 - \varphi_2 \Rightarrow \nabla^2\varphi = 0$. 边界: $\varphi\|_{\text{边}} = 0$ 或 $\frac{\partial\varphi}{\partial n}\|_{\text{边}} = 0$

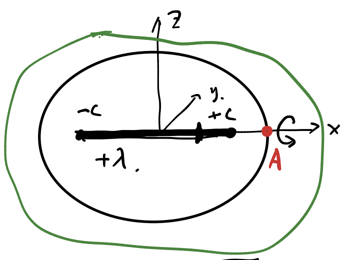
$\int\nabla\cdot(\varphi\nabla\varphi)dV = \oint\underset{\text{边}}{\varphi}\,\underset{\text{边}}{\nabla\varphi}\cdot\underbrace{d\vec S}_{\text{在边界上}} = 0$.

$\int[(\nabla\varphi)^2 + \varphi\underset{0}{\nabla^2\varphi}]dV = \int(\nabla\varphi)^2 dV = 0$.

$\Rightarrow \nabla\varphi = 0$. $\Rightarrow \varphi = 0$.

例: 长轴旋转椭球. 电容. (短轴) $\quad a$, b, c.

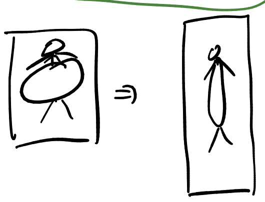
$\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$.



$\varphi_A = \int_{-c}^{c}\frac{\lambda\cdot dx}{4\pi\varepsilon_0\cdot(a-x)} = \frac{\lambda}{4\pi\varepsilon_0}\ln\frac{a+c}{a-c}$.

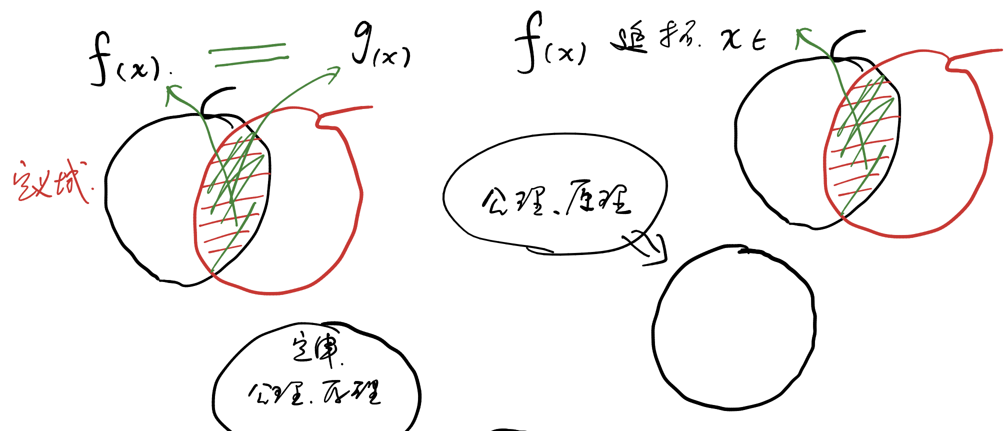
高斯 $\Rightarrow Q = \lambda\cdot 2c$

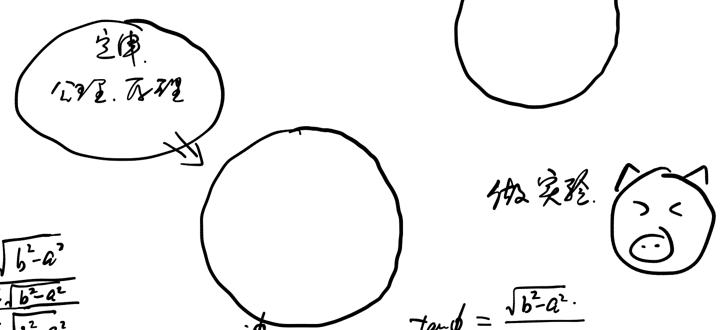
$\Rightarrow C = \frac{8\pi\varepsilon_0\cdot\sqrt{a^2-b^2}}{\ln\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}}$



$C = \frac{8\pi\varepsilon_0\cdot\sqrt{a^2-b^2}}{\ln\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}}$

短轴旋转椭球. 令 $a<b$. 强行解析延拓





$C = \frac{8\pi\varepsilon_0\cdot i\sqrt{b^2-a^2}}{\ln\frac{a+i\sqrt{b^2-a^2}}{a-i\sqrt{b^2-a^2}}} \Rightarrow A\cdot e^{i\phi}$, $A\cdot e^{-i\phi}$. $\quad \tan\phi = \frac{\sqrt{b^2-a^2}}{a}$.

$\Rightarrow C = \frac{8\pi\varepsilon_0\cdot i\sqrt{b^2-a^2}}{\ln e^{2i\phi}} = \frac{8\pi\varepsilon_0\cdot i\sqrt{b^2-a^2}}{2i\phi} = \frac{4\pi\varepsilon_0\sqrt{b^2-a^2}}{\arctan\frac{\sqrt{b^2-a^2}}{a}}$.

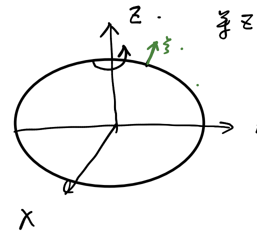
令 $a\leftrightarrow b$, $b\leftrightarrow a$. $\Rightarrow C = \frac{4\pi\varepsilon_0\sqrt{a^2-b^2}}{\arctan\frac{\sqrt{a^2-b^2}}{b}}$

椭球坐标

$\vec r = x\hat x + y\hat y + z\hat z$.

等 x: 平面.
等 y: 平面
等 z: 平面

令 $\begin{cases} x = c\cdot \mathrm{ch}\xi\cdot\cos\eta\cdot\cos\phi \\ y = c\cdot\mathrm{ch}\xi\cdot\cos\eta\cdot\sin\phi \\ z = c\cdot\mathrm{sh}\xi\cdot\sin\eta. \end{cases}$



$\eta: [-\frac{\pi}{2}, \frac{\pi}{2}]$ $\quad \phi: [0, 2\pi)$.

等 ξ 面. $\cos^2\eta = \frac{x^2+y^2}{c^2\,\mathrm{ch}^2\xi}$, $\quad \sin^2\eta = \frac{z^2}{c^2\,\mathrm{sh}^2\xi}$

$\Rightarrow \left(\frac{\sqrt{x^2+y^2}}{c\cdot\mathrm{ch}\xi}\right)^2 + \left(\frac{z}{c\cdot\mathrm{sh}\xi}\right)^2 = 1 \Rightarrow$ 椭圆 一簇共焦

等 η 面: $\mathrm{ch}^2\xi - \mathrm{sh}^2\xi = 1 \Rightarrow \left(\frac{\sqrt{x^2+y^2}}{c\cdot\cos\eta}\right)^2 - \left(\frac{z}{c\cdot\sin\eta}\right)^2 = 1$. 双曲线

$d\vec r = \sigma\hat x + \sigma\hat y + \sigma\hat z = dx\hat x + dy\hat y + dz\hat z$

定义: $\hat\xi = \frac{\partial\vec r}{\partial\xi}\Big/\left|\frac{\partial\vec r}{\partial\xi}\right|$, $\quad \hat\eta = \frac{\partial\vec r}{\partial\eta}\Big/\left|\frac{\partial\vec r}{\partial\eta}\right|$, $\quad \hat\phi = \frac{\partial\vec r}{\partial\phi}\Big/\left|\frac{\partial\vec r}{\partial\phi}\right|$

$$\hat{\xi} = \frac{\partial\vec r}{\partial\xi} \Big/ \left|\frac{\partial\vec r}{\partial\xi}\right| = \frac{\left(c\cdot sh\xi\cdot\cos\eta\cdot\cos\phi\cdot\hat x + c\cdot sh\xi\cdot\cos\eta\cdot\sin\phi\cdot\hat y + c\cdot ch\xi\cdot\sin\eta\cdot\hat z\right)}{\left|c\cdot sh\xi\cdot\cos\eta\cdot\cos\phi\cdot\hat x + c\cdot sh\xi\cdot\cos\eta\cdot\sin\phi\cdot\hat y + c\cdot ch\xi\cdot\sin\eta\cdot\hat z\right|}$$

$$= \frac{\circ\hat x + \circ\hat y + \circ\hat z}{\sqrt{sh^2\xi\cdot\cos^2\eta + ch^2\xi\cdot\sin^2\eta}} \qquad \hat\eta,\ \hat\phi$$

整理. 令. $d\vec r = h_\xi\cdot d\xi\cdot\hat\xi + h_\eta\cdot d\eta\cdot\hat\eta + h_\phi\cdot d\phi\cdot\hat\phi$.

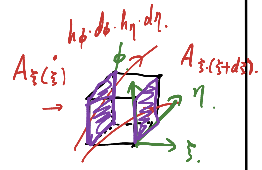
于是: $h_\xi = h_\eta = c\cdot\sqrt{sh^2\xi\cos^2\eta + ch^2\xi\cdot\sin^2\eta}$, $h_\phi = c\cdot ch\xi\cdot\cos\eta$

梯度定义: $\nabla f\cdot d\vec r = df$.

$$\Rightarrow \nabla = \frac{1}{h_\xi}\cdot\frac{\partial}{\partial\xi}\hat\xi + \frac{1}{h_\eta}\frac{\partial}{\partial\eta}\hat\eta + \frac{1}{h_\phi}\cdot\frac{\partial}{\partial\phi}\hat\phi$$

$$df = \frac{\partial f}{\partial\xi}\hat\xi + \frac{\partial f}{\partial\eta}\hat\eta + \frac{\partial f}{\partial\phi}\hat\phi = \nabla_\xi f\cdot h_\xi\cdot\hat\eta + \nabla_\eta f\cdot h_\eta\cdot\hat\eta + \cdots$$

$$\nabla_\xi = \frac{\partial}{h_\xi\cdot\partial\xi} \qquad \nabla_\eta = \frac{1}{h_\eta}\frac{\partial}{\partial\eta}, \quad \cdots$$



散度定义: $\nabla\cdot\vec A\, dV = \sum\vec A\cdot d\vec S$ 边界上. 边界上.

$$\Rightarrow \nabla\cdot\vec A = \frac{1}{h_\xi\cdot h_\eta\cdot h_\phi}\left(\frac{\partial(h_\eta h_\phi A_\xi)}{\partial\xi} + \frac{\partial(h_\phi h_\xi\cdot A_\eta)}{\partial\eta} + \frac{\partial(h_\xi h_\eta A_\phi)}{\partial\phi}\right)$$

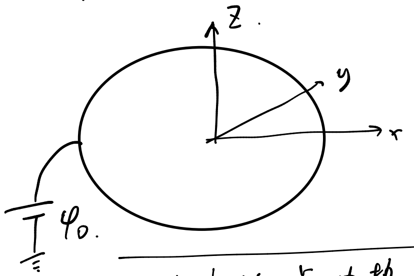
⇒ 拉普拉斯算子:

$$\nabla^2 = \frac{1}{h_\xi^2}\left(\frac{\partial^2}{\partial\xi^2} + \frac{1}{h_\phi}\left(\frac{\partial h_\phi}{\partial\xi}\right)\cdot\frac{\partial}{\partial\xi}\right) + \frac{1}{h_\eta^2}\left(\frac{\partial^2}{\partial\eta^2} + \frac{1}{h_\phi}\left(\frac{\partial h_\phi}{\partial\eta}\right)\frac{\partial}{\partial\eta}\right) + \frac{1}{h_\phi^2}\cdot\frac{\partial^2}{\partial\phi^2}$$

严格求解.

题: 令旋转椭球: $\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1$.

短轴旋转椭球. 电势为 φ_0. 孤立金属球. 求其电容.



球外任一点电势: $\varphi(\xi,\eta,\phi) = \frac{\varphi_0\cdot\arctan\frac{1}{sh\xi}}{\arctan\frac{1}{sh\xi_0}}$

表面任一点电场: $|\vec E(\xi_0,\eta,\phi)| = \frac{\varepsilon_0\cdot\varphi_0}{c\sqrt{sh^2\xi_0\cos^2\eta + ch^2\xi_0\sin^2\eta}}\cdot\frac{1}{\arctan\frac{1}{sh\xi_0}\cdot ch\xi_0}$

$$\Rightarrow C_{短} = \frac{4\pi\varepsilon_0\cdot C}{\arctan\frac{1}{sh\xi_0}}$$

猜: 可分离. 令: $\varphi = R(\xi)H(\eta)\cdot\Phi(\phi)$.

边界: $\varphi|_{\xi=\xi_0} = \varphi_0$. $\varphi|_{\xi\to\infty} = 0$.

$\varphi(\boxed{\xi_0},\eta,\phi) = R(\xi_0)\cdot H(\eta)\cdot\Phi(\phi) = \varphi_0$ 常.

要求: $H(\eta)$、$\Phi(\phi)$ 常系数. 令其为1.

$\Rightarrow \varphi(\xi,\eta,\phi) = R(\xi)$.

解: $\nabla^2\varphi = 0 \Rightarrow \nabla^2 R(\xi) = 0$

$$\Rightarrow \cancel{\frac{1}{h_\xi^2}}\left(\frac{\partial^2 R(\xi)}{\partial\xi^2} + \frac{1}{h_\phi}\left(\frac{\partial h_\phi}{\partial\xi}\right)\frac{\partial R(\xi)}{\partial\xi}\right) = 0$$

$$R''(\xi) + \frac{1}{\cancel{c}\cdot ch\xi\cdot\cancel{\cos\eta}}\cdot\cancel{c}\cdot sh\xi\cdot\cancel{\cos\eta}\cdot R'(\xi) = 0.$$

$\frac{dR'(\xi)}{d\xi}$ "

$$\Rightarrow \frac{dR'}{R'} = -\frac{sh\xi\cdot d\xi}{ch\xi} = -\frac{d\,ch\xi}{ch\xi}$$

$$\Rightarrow R' = \frac{C_1}{ch\xi} \Rightarrow \frac{dR}{d\xi} = \frac{C_1}{ch\xi}$$

$$\Rightarrow \varphi(\xi) = R(\xi) = C_2 + C_3\arctan\frac{1}{sh\xi}$$

边界: $\xi\to\infty$. $\varphi(\xi) = 0$. $\Rightarrow C_2 = 0$.

$\xi\to\xi_0$, $\varphi(\xi_0) = \varphi_0$.

$$\Rightarrow \varphi(\xi,\eta,\phi) = \frac{\varphi_0\arctan\frac{1}{sh\xi}}{\arctan\frac{1}{sh\xi_0}}$$

$$E_n = -\nabla_\xi\varphi = -\frac{1}{h_\xi}\frac{\partial\varphi}{\partial\xi}\Big|_{\xi=\xi_0} = \frac{\varphi_0}{h_\xi\cdot\arctan\frac{1}{sh\xi_0}}\cdot\frac{1}{ch\xi_0}$$

$\sigma = \varepsilon_0\cdot E_n$.

$$\Rightarrow dQ = \varepsilon_0\cdot\frac{\varphi_0}{\cancel{h_\xi}\cdot\arctan\frac{1}{sh\xi_0}}\cdot\frac{1}{ch\xi_0}\cdot\cancel{h_\eta}\cdot d\eta\cdot h_\phi\cdot d\phi.$$

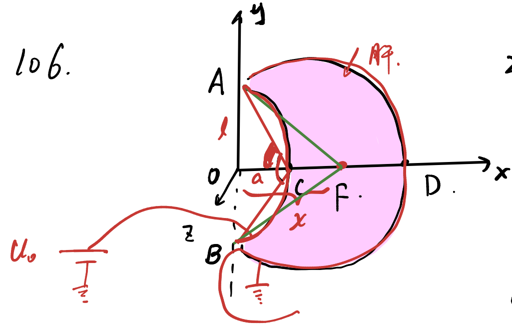
$$= \boxed{\frac{\varepsilon_0\cdot\varphi_0\cdot c^2}{\arctan\frac{1}{sh\xi_0}}}\cdot\cos\eta\cdot d\eta\cdot d\phi.$$

$$\Rightarrow Q = \int dQ = \circ\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\eta\cos\eta\cdot\int_0^{2\pi} d\phi = \frac{2\varepsilon_0\cdot\varphi_0\cdot c^2\cdot 2\pi}{\arctan\frac{1}{sh\xi_0}}$$

$$\Rightarrow C_{短} = \frac{4\pi\varepsilon_0\cdot C}{\arctan\frac{1}{sh\xi_0}} \leftarrow \quad a = c\cdot ch\xi_0,\ b = c\cdot sh\xi_0.$$

与一致.

例. 106.



2个圆柱. 无限长.

求 F处之电场.

$A(0,l)$, $B(0,-l)$, $C(a,0)$, $D(b,0)$, $F(l,0)$.

电像法, 猜: $\sigma \cdot h$.

$$\varphi_C - \varphi_D = U_0 \Rightarrow \varphi_C = \frac{\sigma h}{4\pi\varepsilon_0} \cdot 2\theta_C$$

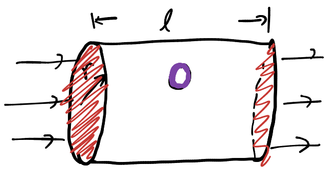
$$\Rightarrow \varphi_C - \varphi_D = \frac{\sigma h}{4\pi\varepsilon_0} \cdot 4\left(\arctan\frac{l}{a} - \arctan\frac{l}{b}\right) = U_0$$

$$\Rightarrow \varphi(x) = \frac{\sigma h}{4\pi\varepsilon_0} \cdot 4\left(\arctan\frac{l}{x}\right)$$

$$\Rightarrow \varphi(x) = \frac{U_0}{\left(\arctan\frac{l}{a} - \arctan\frac{l}{b}\right)} \cdot \arctan\frac{l}{x}$$

$$\Rightarrow E(x) = -\frac{\partial\varphi}{\partial x} = \textcircled{\,} \cdot \frac{-\frac{l}{x^2}}{1+\frac{l^2}{x^2}} \Rightarrow \checkmark$$

例:



σ (电导率). $R_0 = \frac{l}{\sigma \pi R^2}$

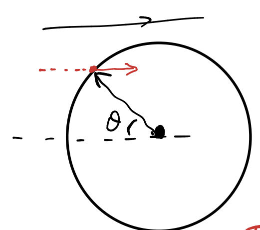
$a \ll r$. $R_0 \to R_0 + \Delta R_0$.

求 $\Delta R_0 = ?$

稳定: $\nabla \cdot \vec{j} = 0 \Leftrightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \boxed{\nabla^2 \varphi = 0}$ $j = \sigma E$

边界 $\sigma \cdot E_n = j_n = 0$. $\Rightarrow$ 唯一性定理

猜得: 小孔的偶极子. p. ←



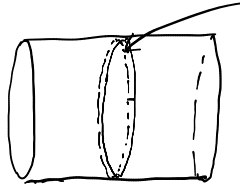
表面: $E_n = E_{r(a,\theta)} = -E\cos\theta + \frac{2p\cos\theta}{4\pi\varepsilon_0 a^3} = 0$.

$$\Rightarrow p = 2\pi\varepsilon_0 E_0 a^3.$$

(2). 用叠加原理. 找回对称性. ⇒ 平铺一层

$\sigma_e \pi r^2 h = p_0$.

$\Rightarrow$



$$\Delta u = \frac{\sigma_e}{\varepsilon_0} \cdot h = \frac{p_0}{\varepsilon_0 \pi r^2}$$

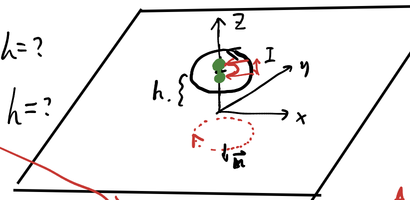
$$\Rightarrow \Delta R = \frac{\Delta U}{I} = \frac{\frac{2\pi\varepsilon_0 a^3 E_0}{\varepsilon_0 \pi r^2}}{I} = \frac{2a^3}{\pi\sigma r^4}$$

$I = \sigma E_0 \pi r^2$

例: 磁像

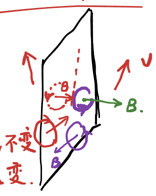
(1). I. r. $r \ll h$. 求 $h = ?$

(2) I, r, $\boxed{r \gg h}$ 求 $h = ?$



← 超导平面

直导线



A: $A_\parallel$ 不变, $A_\perp$ 变.

B: $B_\parallel$ 反转. $B_\perp$ 不变.

像电流 I, $-h$. $-\hat{z}$

$r \ll h$. $\Rightarrow \vec{m} = \pi r^2 I \cdot \hat{z}$

$$B(z) = \frac{\mu_0}{4\pi} \cdot \frac{2m \cdot \hat{z}}{(2h)^3}$$

$$\Rightarrow F = \left| m \cdot \frac{dB}{dz} \right|_{z=2h} = \boxed{\frac{\mu_0}{4\pi} \cdot \frac{3 \cdot 2m^2 \cdot 2}{(2h)^4} = mg} \text{ 对吗}$$

对场点操作

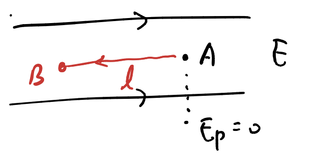
$$F = \left| m \frac{dB}{dl} \right|_{l=2h} = \left| \frac{\mu_0}{4\pi} \frac{2m\, d\frac{1}{l^3}}{dl} \right|_{l=2h} = \frac{\mu_0 \cdot 2m \cdot 3}{4\pi (2h)^4} = mg$$

$\uparrow$ 距离

$\Rightarrow h = \cdots \checkmark$

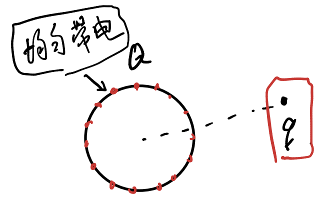
能量

1. 电势能: 对一个电荷. 固定外场下. 从某点 A → 某点 B.



$E_p = q \cdot E \cdot l$.

2. 相互作用能 (均匀带电)



总相互作用能.

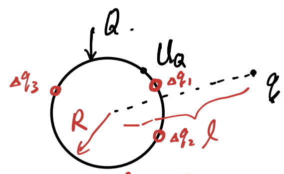
$$\Rightarrow \sum \frac{1}{2} \Delta q \cdot U_q$$

点荷

U_q: 除自己以外, 所有电荷在 q 处产生的电势. (点电荷)

q. Δq_1. Δq_2

练:



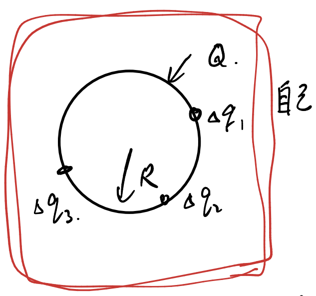
$$\Rightarrow \sum \frac{1}{2} \Delta q \cdot U_q$$

$$= \frac{1}{2} q \cdot U_q + \frac{1}{2} Q U_Q$$

$$U_Q = \frac{kq}{l} + \frac{kQ}{R}$$

$$= \frac{1}{2} q \cdot U_q + \frac{1}{2} Q \cdot \left(\frac{kq}{l} + \frac{kQ}{R}\right)$$

3. 自能.

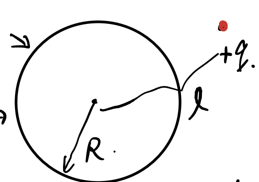


自己内部的相互作用能 ⇒ 自能

$$= \frac{1}{2} Q \cdot \frac{kQ}{R}$$

从 $+q$ 从 l 运到 ∞. 求外力做功.

4.



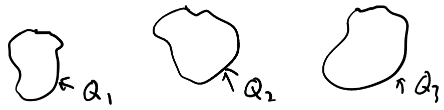
$+W$

$\leftarrow W(q, Q)$. $= W(0, Q)$

$\Rightarrow W_{外} = W(0, Q) - W(q, Q)$.

$$W(Q) = \frac{1}{2} \sum \Delta Q \cdot U_{\Delta Q}$$

多个金属总相互作用能.



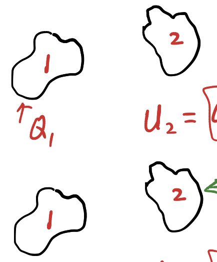
$\Rightarrow \frac{1}{2}Q_1\cdot U_1+\frac{1}{2}Q_2U_2+\frac{1}{2}Q_3U_3.$

$\Rightarrow W_总 = \frac{1}{2}\sum Q_i\cdot U_i.$ $\qquad U_i=\sum_j a_{ij}Q_j.$

$= \frac{1}{2}\sum Q_i\cdot\sum a_{ij}Q_j$

$= \frac{1}{2}\sum_{ij} a_{ij}Q_iQ_j.$

$a_{ij}=a_{ji}$. 格林互易.



$U_2=\boxed{a_{21}}Q_1$

$U_1=\boxed{a_{12}}Q_2.$

(1)	q_1	q_2	q_3	…	q_i
	U_1	U_2	U_3 …		$U_i.$
(2)	q_1'	q_2'	q_3'	…	q_i'
	U_1'	U_2'	U_3' …		$U_i'.$

格林互易定理: $\sum q_iU_i' = \sum q_i'U_i$

左: $\sum q_i\cdot U_i' = \sum_i q_i\sum_j a_{ij}q_j' = \sum_{ij} a_{ij}q_iq_j'$

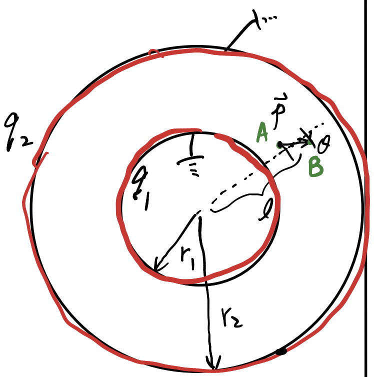
右: $\sum q_i'U_i = \sum_i q_i'\sum_j a_{ij}q_j = \sum_{ij} a_{ij}q_i'q_j$

$=\sum_{j,i} a_{ji}q_jq_i'$

例: 用格林互易定理求电荷

r_1、r_2. 接地, l, θ, $\vec{p}$.

求q_1、q_2. $\qquad p=q\cdot d.$



求q_1, q_2

	r_1	A	B	r_2
①: q:	q_1	$-q$	q	q_2
U:	0	φ_A	φ_B	0
②: q:	0	0	0	Q
U:	$\frac{kQ}{r_2}$	$\frac{kQ}{r_2}$	$\frac{kQ}{r_2}$	$\frac{kQ}{r_2}$
③: q:	Q	0	0	Q
U:	$\frac{kQ}{r_2}+\frac{kQ}{r_1}$	$\frac{kQ}{r_2}+\frac{kQ}{l}$	$\frac{kQ}{r_2}+\frac{kQ}{l+d\cos\theta}$	$\frac{kQ}{r_2}+\frac{kQ}{r_2}$

格林互易:

$(q_1-\cancel{q}+\cancel{q}+q_2)\cdot\frac{kQ}{r_2}=0$

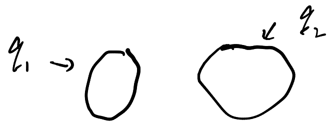
$\Rightarrow q_1+q_2=0$

$q_1\cdot\left(\frac{kQ}{r_2}+\frac{kQ}{r_1}\right)-q\cdot\left(\frac{kQ}{r_2}+\frac{kQ}{l}\right)+q\cdot\left(\frac{kQ}{r_2}+\frac{kQ}{l+d\cos\theta}\right)+q_2\left(\frac{kQ}{r_2}+\frac{kQ}{r_2}\right)=0$

格林互易

$\Rightarrow q_1\left(\frac{kQ}{r_1}-\frac{kQ}{r_2}\right)=q\cdot\left(\frac{kQ}{l}-\frac{kQ}{l+d\cos\theta}\right)$

$\Rightarrow q_1=\frac{r_1r_2}{r_2-r_1}\cdot\frac{(q\cdot d\cos\theta)}{l^2} = \frac{r_1r_2\cdot p\cos\theta}{l^2(r_2-r_1)}$



$W(q_1,q_2)$.

① $(0,0)\to(0,q_2)\to(q_1,q_2)$

② $(0,0)\to(q_1,0)\to(q_1,q_2)$

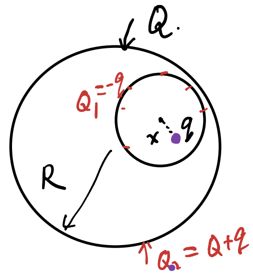
① $dW=\int U_1dq_1+\int U_2dq_2$ $\qquad W_{II}=\int_0^{q_2}a_{22}\cdot q_2\cdot dq_2=\frac{1}{2}a_{22}q_2^2.$

$+$

$W_I=\int_0^{q_1}(a_{12}\cdot q_2+a_{11}q_1)\cdot dq_1 = a_{12}q_1q_2+\frac{1}{2}a_{11}q_1^2$

② $\Rightarrow$ …… $\Rightarrow$ $\frac{1}{2}a_{11}q_1^2+a_{21}q_1q_2+\frac{1}{2}a_{22}q_2^2$ // $\Rightarrow a_{12}=a_{21}$

例:



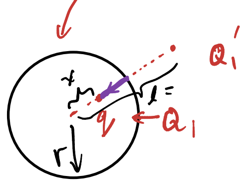
$Q\to R$、r. x处有一个q

(1). q的电势能: $q\cdot U_q$ ← 除q以外所有电荷产生的电势

(2) 求总相互作用能. → $\sum\frac{1}{2}q_iU_i$

(3) 把q运到∞ 求外力做功

$\Rightarrow$ (1) $U_q=\boxed{U_{Q_1\to q}}+U_{Q_2\to q}.$ $\qquad \frac{k(Q+q)}{R}$



$Q_1'=-\frac{r}{x}q$ $\qquad l=\frac{r^2}{x}$

$\Rightarrow U_{Q_1\to q}=\frac{k\cdot(-\frac{r}{x}\cdot q)}{(\frac{r^2}{x}-x)}=\frac{-kqr}{r^2-x^2}.$

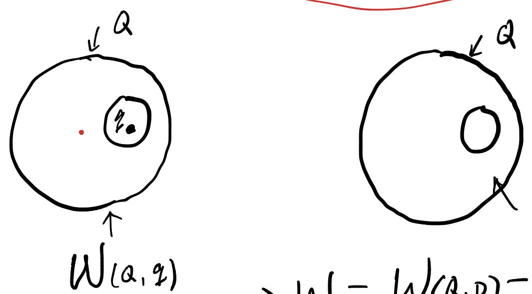
$q\cdot U_q=-\frac{kq^2r}{r^2-x^2}+q\cdot\frac{k(Q+q)}{R}$

(2). $\sum\frac{1}{2}q_i\cdot U_i=\frac{1}{2}q\cdot U_q+\frac{1}{2}Q_1U_1+\frac{1}{2}Q_2U_2.$ $\qquad =\frac{1}{2}Q\cdot\frac{k(Q+q)}{R}$

$U_1=U_2=U_{球壳}=\frac{k(Q+q)}{R}.$

$=-\frac{1}{2}\frac{kq^2r}{r^2-x^2}+\frac{1}{2}\cdot\frac{k(q+Q)^2}{R}.$

(3)

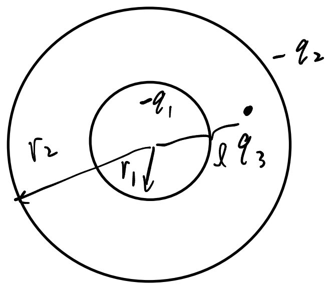


……

$\Rightarrow W=W(Q,0)-W(Q,q)$

总相互作用能.

例:



求总相互作用能

$\frac{1}{2}\sum q_iU_i$

$U_1=a_{11}q_1+a_{12}\cdot q_2+\boxed{a_{13}\cdot q_3}$

$U_2=a_{21}q_1+a_{22}q_2+\boxed{a_{23}q_3}$

$U_3=\boxed{a_{31}q_1}+\boxed{a_{32}\cdot q_2}$

只有q_1时: $U_1=a_{11}q_1=\frac{kq_1}{r_1}\Rightarrow a_{11}=\frac{k}{r_1}$. $\qquad U_3=a_{31}q_1=\frac{kq_1}{l}\Rightarrow a_{31}=\frac{k}{l}$ ✓

q_2时: $U_3=a_{32}q_2=\frac{kq_2}{r_2}\Rightarrow a_{32}=\frac{k}{r_2}$ ✓ $\qquad\Rightarrow$ ✓

有介质时的能量

各向同性、线性、均匀介质. $\vec D=\varepsilon_r\varepsilon_0\vec E$

场能密度：ω.

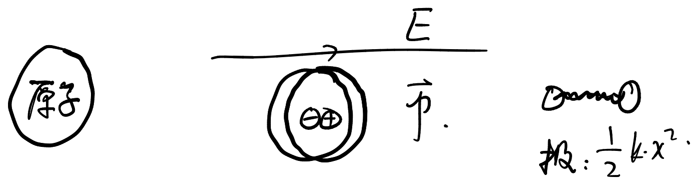
$w=\frac12\varepsilon_0E^2$. 场能. 极化

$w=\frac12\vec D\cdot\vec E$. 场能+极化能.

$\Rightarrow$ 极化能：$w_{极}=\frac12(\vec D\cdot\vec E)-\frac12\varepsilon_0\vec E\cdot\vec E$

$w_{极}=\frac12\vec P\cdot\vec E$

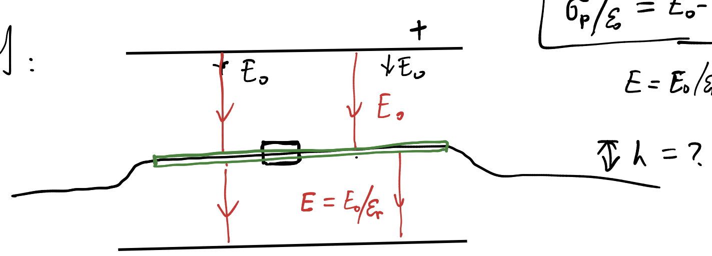
练：



极：$\frac12kx^2$.

$E_{极}=\frac12kx^2=\frac12x\cdot kx=\frac12(x)E(q)=\frac12\vec P\cdot\vec E$.

例：



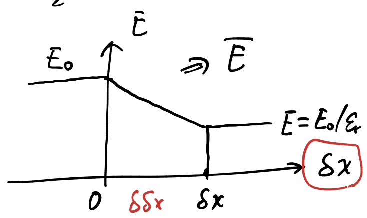
$\boxed{\sigma_p/\varepsilon_0=E_0-E_0/\varepsilon_r}$

$E=E_0/\varepsilon_r$

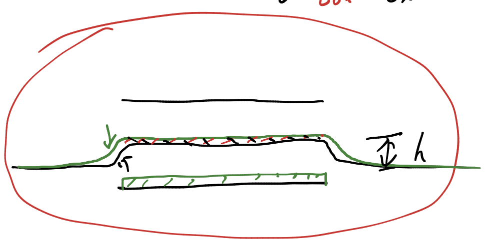
(1). $\frac{F}{S}=\sigma_p\cdot\bar E=\varepsilon_0(E_0-\frac{E_0}{\varepsilon_r})\cdot\frac{E_0+\frac{E_0}{\varepsilon_r}}{2}=\frac12\varepsilon_0E_0^2(1-\frac1{\varepsilon_r^2})=\rho gh$.

表面附近. E-深度

$\Delta E\propto\frac{\rho_p(\delta\delta x)}{\varepsilon_0}$



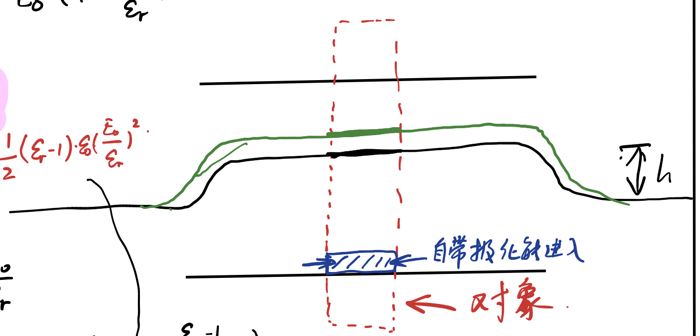
错误的能量：



$+\rho dVgh=(+\frac12\varepsilon_0E_0^2+\frac12\varepsilon_0E_0\frac{E_0}{\varepsilon_r})dV$

$\rho gh=\frac12\varepsilon_0E_0^2(1-\frac1{\varepsilon_r})$.

正确的能量：



$\frac12(\varepsilon_r-1)\varepsilon_0(\frac{E_0}{\varepsilon_r})^2$

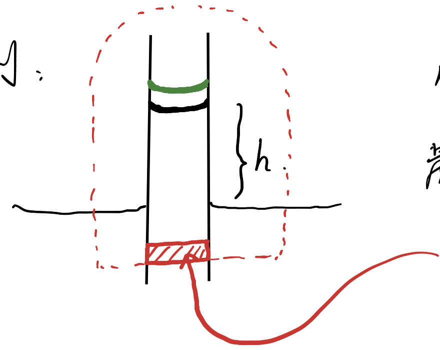
$-\rho gh\Delta V+\frac12\vec P\cdot\vec E$

$=-\frac12\varepsilon_0E_0^2+\frac12\varepsilon_0E_0\frac{E_0}{\varepsilon_r}$

$\Rightarrow\rho gh=\frac12\varepsilon_0E_0^2(1-\frac1{\varepsilon_r}+\frac{\varepsilon_r-1}{\varepsilon_r^2})$

$=\frac12\varepsilon_0E_0^2(1-\frac1{\varepsilon_r^2})$.

例：



受力算的：全是边界.

能量是对的.

没有自带极化能.

能量：$+\rho dVgh=(+\frac12\varepsilon_0E_0^2+\frac12\varepsilon_0E_0\frac{E_0}{\varepsilon_r})\cdot dV$

保角变换. 解(猜) = 准静电问题

(一). 可用一个复函数 $f(z)=U(x,y)+iV(x,y)$.

$Z=x+iy$. 来表示二维. 稳. 场的分布.

$U(x,y)\to\varphi(x,y)$.

二维静电问题：$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=\rho$ 或 $-\sigma/\varepsilon_0$ + 边界条件.

可微 $\boxed{df}=\boxed{f'(z)}\boxed{dz}=f'(\,)\,d(\,)$ $\boxed{dz=dx+idy}$

$df=\frac{\partial u}{\partial x}dx+i\frac{\partial v}{\partial x}dx+\frac{\partial u}{\partial y}dy+i\frac{\partial v}{\partial y}dy$

$=(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x})dx+(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y})\cdot idy$

柯西黎曼条件. CR条件：

$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$. $\frac{\partial v}{\partial x}=-\frac{\partial u}{\partial y}$.

$\Rightarrow\frac{\partial^2u}{\partial x^2}=\frac{\partial^2v}{\partial x\partial y}$, $\frac{\partial^2u}{\partial y^2}=-\frac{\partial^2v}{\partial y\partial x}$ $\Rightarrow\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$

$\Rightarrow$ 可以用 u. 表示 φ.

构造了一个可微复函数 $f(z)=U(x,y)+iV(x,y)$

U: 电势. $\nabla^2U=0$.

$\nabla u=(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y})$. $\nabla v=(\frac{\partial v}{\partial x},\frac{\partial v}{\partial y})$ $\Rightarrow\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}=0$

$\Rightarrow$ u.v 梯度方向⊥. $\Rightarrow$ u, v. 等值线相互垂直

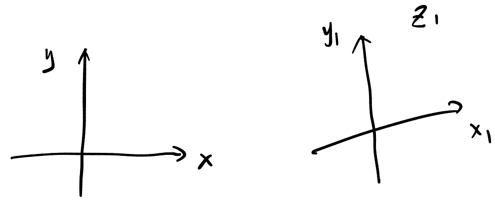
U: ⇔ φ: 等势线

V: 电场线：等电通线 ⇔ V

(二) 可将此二维空间变换到另一个二维空间.

仍满足拉普拉斯方程. ← 边界一样.

令 $z_1 = g(z)$ $\Updownarrow$ $z = g^{-1}_{(z_1)}$



$z_1 = x_1 + i y_1$ $\quad x_1:(x,y)\quad y_1:(x,y)$.

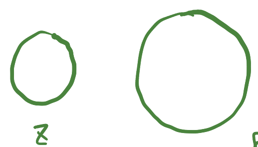
f(z) ↖ $f(g^{-1}_{(z_1)}) = u(x_1,y_1) + i v(x_1,y_1)$.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \iff \frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial y_1^2}=0$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x_1}\cdot\frac{\partial x_1}{\partial x} + \frac{\partial u}{\partial y_1}\cdot\frac{\partial y_1}{\partial x}$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x_1}(\quad)\frac{\partial x_1}{\partial x} + \frac{\partial}{\partial y_1}(\quad)\frac{\partial y_1}{\partial x}.$$

再 $\frac{\partial^2 u}{\partial y^2} = \vdots$



$$\left(\frac{\partial x_1}{\partial x}\right)^2+\left(\frac{\partial x_1}{\partial y}\right)^2=\left(\frac{\partial y_1}{\partial x}\right)^2+\left(\frac{\partial y_1}{\partial y}\right)^2=|z_1'(z)|^2 \qquad \boxed{(z_1(z))'}$$

$z'_{1(z)}=2$. $\quad\boxed{\frac{dz_1}{dz}=2}$

$$\Rightarrow |z'_{1(z)}|^2\cdot\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial y_1^2}\right)=0. \qquad -\rho/\varepsilon_0$$

即在新空间 z_1 中 u 也满足拉普拉斯

u 在新空间中仍可表示电势. + 边界条件.

(三). 在空间中复杂或丑电势.

⇒ z_1 空间中, 简单或美丽电势分布.

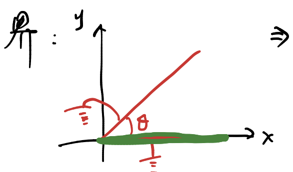
⇒ 换回 z. ⇒ 原始电势分布.

注意. 变换时换成无量纲数.

练: 换边界. ① 令 $z_1 = (z/A_0)^n$.

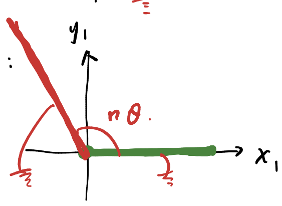
原边: ① $z = A$. $A\in[0,+\infty)$ ② $z = Ae^{i\theta}$. $A\in[0,+\infty)$

原边界:



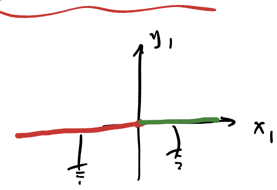
⇒ z_1: ① $z_1 = (A/A_0)^n \to [0,+\infty)$ ② $z_1 = (\frac{A}{A_0})^n e^{in\theta}$, $[0,+\infty)$

新空间:



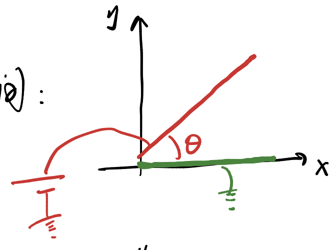
令 $n\theta = \pi \Rightarrow n = \frac{\pi}{\theta}$

⇒



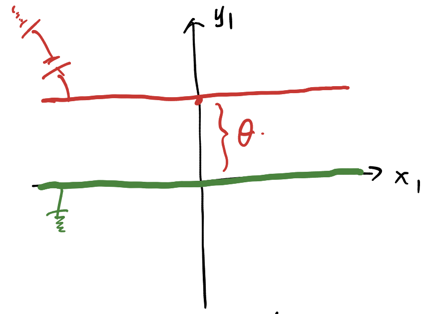
② 对数变换 $\quad$ 对数: 令 $z_1 = \ln(\frac{z}{A_0})$

原空间:



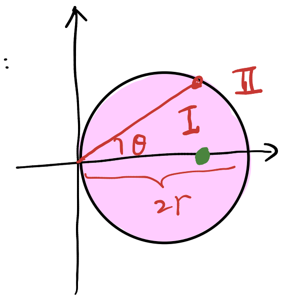
原本: ① $z = A$. $A\in[0,+\infty)$. ② $z = Ae^{i\theta}$. $A\in(0,+\infty)$

新: ① $z_1 = \ln\frac{A}{A_0} \in(-\infty,+\infty)$ ② $z_1 = \ln(\frac{A}{A_0}\cdot e^{i\theta}) = \ln\frac{A}{A_0} + i\theta$.

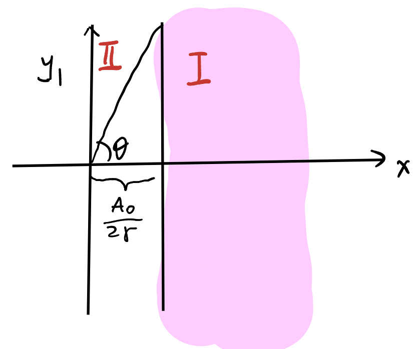


③. 令 $z_1 = \frac{A_0}{z}$. $\quad z = Ae^{i\theta}$ ⇒ $A\to\frac{A_0}{A}$ $\quad\theta\to-\theta$

原:



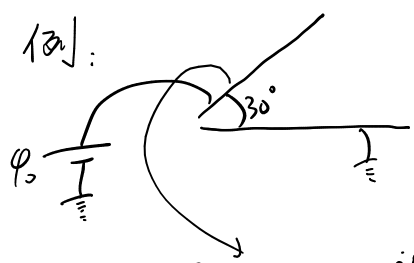
边界: $z = 2r\cdot\cos\theta\cdot e^{i\theta}$. ⇓ $z_1 = \frac{A_0}{2r\cos\theta}\cdot e^{-i\theta}$.



I: $A \le 2r\cos\theta$. ⇓ $A_1 \ge \frac{A_0}{2r\cos\theta}$

④. 令 $z_1 = \cos z = \frac{e^{iz}+e^{-iz}}{2}$ ⇒ …… ⇒ 平直 ⇒ 椭圆,双曲线

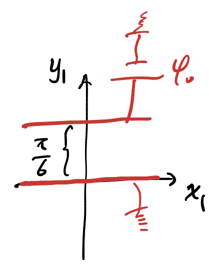
例:



求电势分布. 猜. $\varphi = \varphi_0\frac{\theta}{\frac{\pi}{6}}$

用保角变换.

令 $z_1 = \ln(z/A_0)$

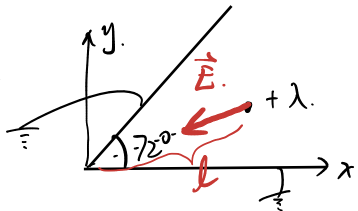


边界 $z = Ae^{i\theta}$ ⇒ $z_1 = \ln\frac{A}{A_0} + i\theta$. $\quad z_1 = x_1 + iy_1$

⇒ 新空间中: $\varphi = \varphi_0\cdot\frac{y_1}{\frac{\pi}{6}}$

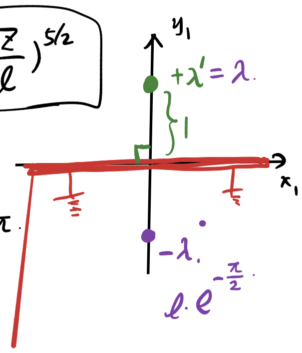
⇒ 变回新空间. $y_1 = \theta$ ⇒ $\varphi_{(\theta)} = \varphi_0\cdot\frac{\theta}{\pi/6}$

例:



$\frac{360°}{72°} = 5$ $\quad$ 求+λ单位长度受力.

令 $\boxed{z_1 = (\frac{z}{l})^{5/2}}$



边界: ①. $z = Ae^{i0}$ ⇒ $z_1 = (\frac{A}{l})^{\frac{5}{2}}$

② $z = Ae^{i\frac{2}{5}\pi}$ ⇒ $z_1 = (\frac{A}{l})^{\frac{5}{2}}e^{i\pi}$

+λ: $z = l\cdot e^{i\frac{1}{5}\pi}$ ⇒ $z_1 = 1\cdot e^{\frac{\pi}{2}}$

$\Rightarrow$ 新空间中: $\vec{E}' = \dfrac{\lambda}{2\pi\varepsilon_0 \cdot 2} \cdot e^{-i\frac{\pi}{2}}$

电场如何被拉伸. 保电势(边界)

原: $\vec{E}\cdot d\vec{l} = du$. 新: $\boxed{\vec{E}'} d\vec{l}' = du$.

$d\vec{l} = dz$. $d\vec{l}' = dz'$

$\Rightarrow \boxed{\vec{E}\cdot dz = \vec{E}' dz'}$ $z_1 = z' = \left(\frac{z}{l}\right)^{\frac{5}{2}}$

$\dfrac{dz'}{dz} = \dfrac{5}{2}\cdot\dfrac{1}{l^{\frac{5}{2}}}\cdot z^{\frac{3}{2}}$

$\Rightarrow \vec{E} = \dfrac{5}{2}\cdot\dfrac{z^{\frac{3}{2}}}{l^{\frac{5}{2}}}\cdot\vec{E}' = \dfrac{5}{2}\cdot\dfrac{(l e^{i\cdot\frac{1}{5}\pi})^{\frac{3}{2}}}{l^{\frac{3}{2}}}\cdot\dfrac{\lambda}{4\pi\varepsilon_0} e^{-i\frac{\pi}{2}}$

$= \dfrac{5\lambda}{8\pi\varepsilon_0\cdot l}\cdot e^{-i\frac{1}{5}\pi}$

$\omega = \frac{1}{2}\varepsilon_0 E^2$. (场) $\boxed{\omega = \frac{1}{2}\vec{D}\cdot\vec{E}}$ (总) $\frac{1}{2}\vec{D}\cdot\vec{E} - \frac{1}{2}\varepsilon_0\vec{E}\cdot\vec{E} = \frac{1}{2}\vec{P}\cdot\vec{E}$ 极化能.

$\vec{A}\cdot(\nabla f) = \nabla(\vec{A} f) - f(\nabla\cdot\vec{A})$

$E = \int \frac{1}{2}\varepsilon_0\vec{E}\cdot\vec{E}\cdot dV = -\int\frac{1}{2}\varepsilon_0\vec{E}\cdot(\nabla\varphi)\cdot dV$

$= -\int\frac{1}{2}\varepsilon_0\nabla(\vec{E}\cdot\varphi)\,dV + \int\frac{1}{2}\varepsilon_0\varphi(\nabla\cdot\vec{E})\cdot dV$ $\boxed{\rho/\varepsilon_0}$

$= -\oint\frac{1}{2}\varepsilon_0(\vec{E}\cdot\varphi)\cdot d\vec{S} + \int\frac{1}{2}\varepsilon_0\cdot\varphi\cdot\rho/\varepsilon_0\cdot dV$

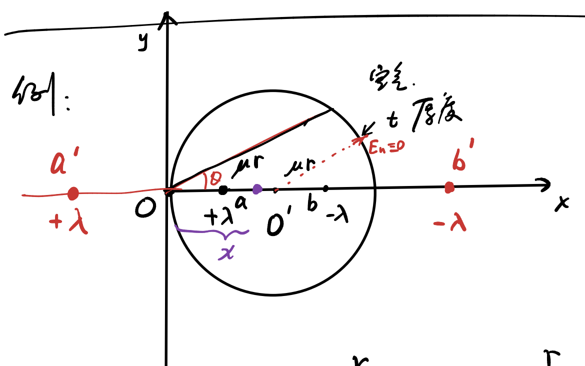
$= -\int\frac{1}{2}\varepsilon_0\vec{E}\cdot d\vec{S}\cdot\varphi$ $\rho_总: (\rho_f + \rho_p)$

$= \frac{1}{2}\int\sigma\cdot d\vec{S}\cdot\varphi + \int\frac{1}{2}\rho\varphi\cdot dV$

总电荷. 全部自由电荷. 不包含极化能.

上题: 也可以先求新空间(z_1)中电势分布 → 原空间 → 梯度得电场 → 受力.

例:



r. 正负电流 $j = \sigma E$

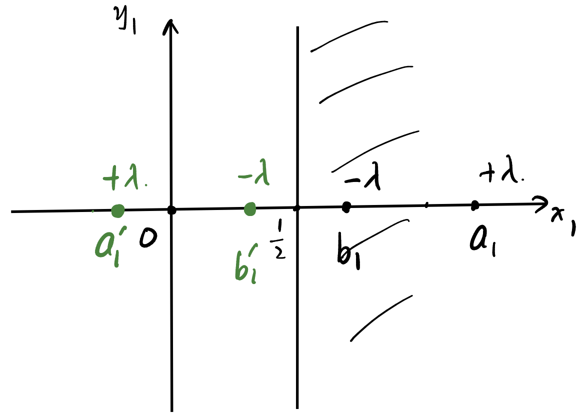
求 x轴上电流分布.

边界: $z = A\cdot e^{i\theta}$

$A = 2r\cdot\cos\theta$.

令 $z_1 = \dfrac{r}{z}$: 边 $\dfrac{r}{2r\cos\theta\, e^{i\theta}} = \dfrac{1}{2\cos\theta}\cdot e^{-i\theta}$

$+\lambda, -\lambda$: 电流: $\lambda\cdot\dfrac{1}{2\pi l}$



a: $+\lambda$: $(1-\mu)r$.

a_1: $\boxed{+\lambda}$: $\dfrac{r}{(1-\mu)r} = \dfrac{1}{1-\mu}$

$\mu\in(0,1)$.

b: $-\lambda$: $(1+\mu)r$

b_1: $-\lambda$: $\dfrac{r}{(1+\mu)r} = \dfrac{1}{1+\mu}$

$x'_{a_1} = \dfrac{1}{2} - \left(\dfrac{1}{1-\mu} - \dfrac{1}{2}\right) = 1 - \dfrac{1}{1-\mu} = \dfrac{-\mu}{1-\mu}$

$x'_{b_1} = \dfrac{1}{2} - \left(\dfrac{1}{1+\mu} - \dfrac{1}{2}\right) = \dfrac{\mu}{1+\mu}$

$\Rightarrow$ 先变回原空间:

$x_{a_1} = \dfrac{r}{x'_{a_1}} = -\dfrac{1-\mu}{\mu}\cdot r$

$x_{b_1} = \dfrac{r}{x'_{b_1}} = \dfrac{1+\mu}{\mu}\cdot r$

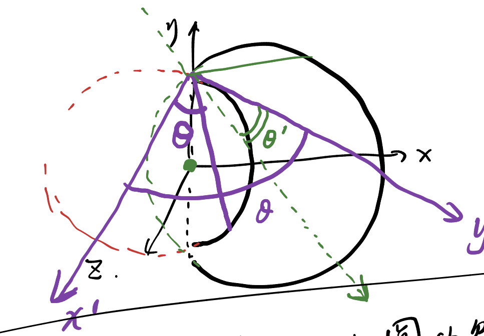
x处: 电流密度: $j_a + j_{a'} + j_b + j_{b'}$

$= \dfrac{+\lambda}{2\pi(x-(1-\mu)r)\cdot t} + \dfrac{+\lambda}{2\pi(x+\frac{1-\mu}{\mu}r)\cdot t} + \dfrac{\lambda}{2\pi((1+\mu)\cdot r - x)\cdot t}$

$+ \dfrac{\lambda}{2\pi(\frac{1+\mu}{\mu}r - x)\cdot t}$

$= \cdots$

例:



求电容. 保角变换.

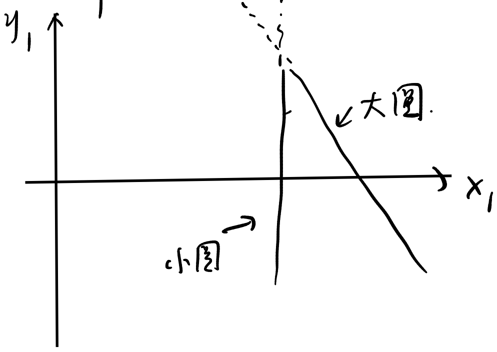
单位长度电容

(1). 平移. (x', y') $\Rightarrow$ 小圆边界: $2r\cdot\cos\theta\cdot e^{i\theta}$

z'

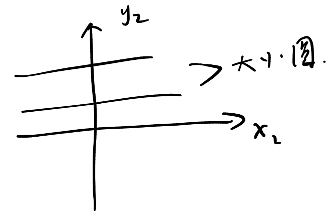
大圆边界: $2R\cdot\cos(\theta-\theta_0)e^{i(\theta-\theta_0)}$

(2) 令 $z_1 = \dfrac{r}{z'}$ $\Rightarrow$

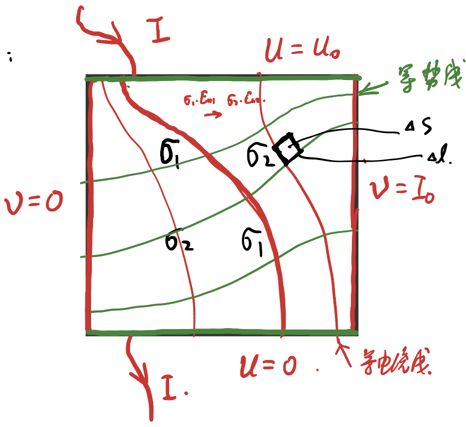


(3) 令 $z_2 = \ln z_1$

$\Rightarrow$



例：



$t\to 0$. 厚度. 电导率.

$R=\frac{U}{I}=?$

$R=\frac{1}{t\sqrt{\sigma_1\sigma_2}}$

U: 等电势线. V: 等电流线

$j=\sigma\cdot E$

$\frac{\Delta U}{\Delta S}=E$. $\frac{\Delta V}{\Delta l\cdot t}=j\Rightarrow\frac{\Delta V}{\Delta l}=j\cdot t$

$j\cdot\Delta l=\sigma\cdot E\cdot\Delta l=\Delta I/t$.

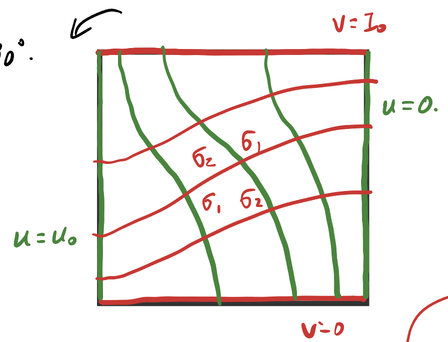
$j=\sigma E\Rightarrow\boxed{\frac{\Delta V}{\Delta l}=t\cdot\sigma\cdot\frac{\Delta U}{\Delta S}}$ (*)

u: 拉普拉斯方程 u⊥v.

边界: ✓. $\frac{\Delta u}{\Delta s}\sigma_1=\frac{\Delta u}{\Delta s}\sigma_2$

复变函数. $f(x,y)=u+iv$ u: 势. v.

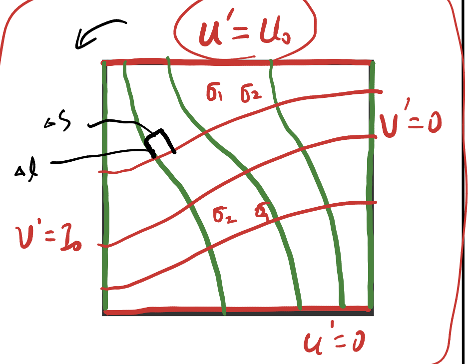
转 90°.



令: $u'=\frac{u_0}{I_0}V$

令: $V'=\frac{I_0}{u_0}u$

令: $\sigma_1\Leftrightarrow\sigma_2$



边界: u'. ✓.

使 $j'=\sigma'E'$.

$\frac{\Delta V'}{t\Delta S}=\sigma'\frac{\Delta u'}{\Delta l}$

$\frac{I_0}{u_0 t}\frac{\Delta u}{\Delta S}=\sigma'\frac{u_0}{I_0}\frac{\Delta V}{\Delta l}$

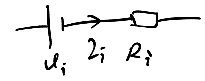
$\frac{I_0}{u_0}\cancel{\frac{\Delta u}{\Delta S}}=\sigma'\frac{u_0}{I_0}\sigma t^2\cancel{\frac{\Delta u}{\Delta S}}$

$\sigma'\sigma=\sigma_1\sigma_2$.

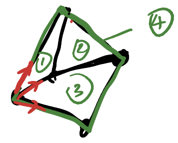
$\Rightarrow\frac{I_0}{u_0}=\sigma_1\sigma_2\frac{u_0}{I_0}t^2\Rightarrow R_2\ \frac{u_0}{I_0}=\frac{1}{t\sqrt{\sigma_1\sigma_2}}$

电路: 基尔霍夫. 节点: $\oint\sigma\vec{E}\cdot d\vec{S}=0\Rightarrow\sum I=0$.

回路: $\oint\vec{E}\cdot d\vec{l}=0\Rightarrow$



网络:



V个顶点 E条边 F个区域.

I: 有(V-1)个独立电压: U_i. $i=1,2,\cdots(V-1)$.

⇒ 有(V-1)个独立节点方程. $\sum_j I_{ij}=0$.

II: 设圈电流: (F-1)个圈电流.

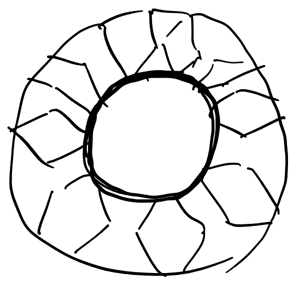
基尔霍夫回路: $\sum U_{ij}=0$ (F-1)个回路方程

III: 设E个电流. 约束: 节点 $\sum_j I_{ij}=0$. (V-1)个.

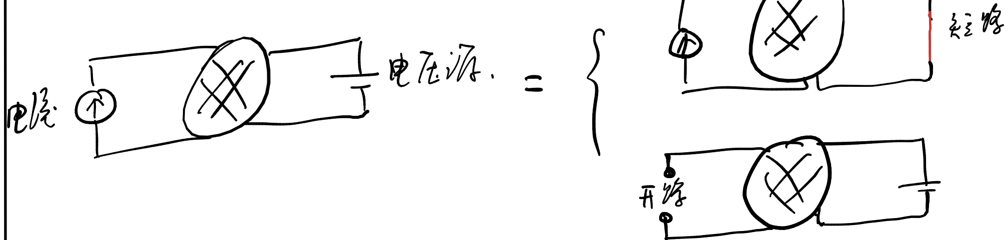
回路方程: (F-1)个. $\sum U_{ij}=0$

$\boxed{E=V-1+F-1=V+F-2}$ ← 欧拉公式

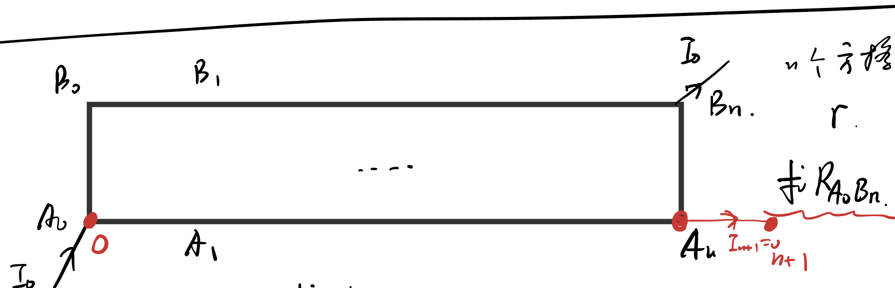
简单多面体



叠加.



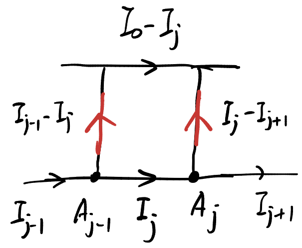
例:



n个方程. r.

求 $R_{A_0B_n}$.

加入 I_0. 求 U. I_j.



⇒ 回路:

$I_j+(I_j-I_{j+1})-(I_0-I_j)-(I_{j-1}-I_j)=0$

⇒ $-I_{j-1}+4I_j-I_{j+1}=I_0$. ← 非齐次...