原子物理

1 卢瑟福.

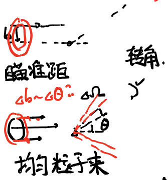
"点"粒子? atom 10^{-10}m nucleus 10^{-15}?

Z_1 α ，Z_2 Au 如何知道相互作用?

m_{He} m_{Au} 库仑 ⇌ 双曲线轨道 — 模型结果

$b \sim \theta$? → 分布.

单个核: 入射 $J = \frac{\Delta N}{\Delta t \Delta S}$.



瞄准距 $\Delta b \sim \Delta\theta$ $\Delta\Omega$ 均匀粒子束 弹性

出射 $I(\theta,\varphi) = \frac{\Delta N}{\Delta t \Delta \Omega}$ ← 立体角 可测

比例. $\frac{I(\theta,\varphi)}{J} = \frac{\frac{\Delta N}{\Delta t\Delta\Omega}}{\frac{\Delta N}{\Delta t \Delta S}} = \frac{\Delta S}{\Delta\Omega} = \sigma(\theta,\varphi)$ ✓

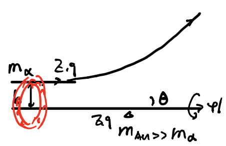
$N_入 = \Delta N_出$. 弹性. 微分散射截面.

$\Sigma = \int \sigma\, d\Omega$ $\Delta N_{\theta>0} = \Delta t \cdot J \cdot \Sigma$ | Σ有限. 短程 | Σ无限. 长程

总散射截面

计算 $\sigma(\theta,\varphi)$

例 78 算 $\sigma(\theta,\varphi)$? 库仑.

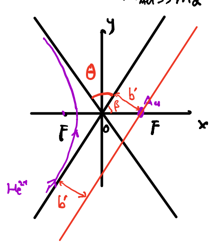


$\sigma(\theta) = \frac{\Delta S}{\Delta\Omega} = \frac{2\pi b\, db}{2\pi \sin\theta\, d\theta}$

$b' = |OF| \cdot \sin\beta$ $\tan\beta = \frac{b}{a}$.

$= \sqrt{a^2+b^2} \cdot \frac{b}{\sqrt{a^2+b^2}} = b$ 瞄准距

m_α $Z_1 q$ $Z_2 q$ $m_{Au} >> m_\alpha$



轨道能量 $(E = -\frac{GMm}{2a})$.

$E = \frac{KZ_1Z_2q^2}{2a} = \frac{1}{2}mU_0^2$; $a = \frac{KZ_1Z_2q^2}{mU_0^2}$

$\frac{1}{2}mU_0^2 = \frac{1}{2}mU_1^2 + \frac{KZ_1Z_2q^2}{a+c}$

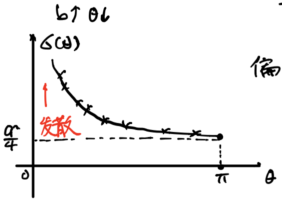
$mU_0 b = mU_1(a+c)$; $c = \sqrt{a^2+b^2}$

$\theta = 2 \cdot \arctan\frac{a}{b}$ $b = a \cdot \cot\frac{\theta}{2}$.

$\sigma(\theta) = \left|\frac{b\,db}{\sin\theta\, d\theta}\right| = \frac{a \cdot \cot\frac{\theta}{2} \cdot \frac{\frac{1}{2}d\theta}{\sin^2\theta/2} \cdot a}{2\sin\theta/2 \cos\theta/2 \quad d\theta} = \frac{a^2}{4} \cdot \frac{1}{\sin^4\theta/2}$.

$\propto \frac{1}{E_0^2}$.

$b\uparrow \theta\downarrow$



发散 偏差?

例 79 $\sigma(\theta) \sim \frac{1}{\sin^4\theta/2}$ 偏差原因.

核力? 最近距离 x_{min} 过小? 10^{-13}m ✓ (U大. 相对论)

E_0 →动能. He Au

$\frac{KZ_1Z_2q^2}{x_{min}} = \frac{1}{2} \cdot \frac{m_\alpha m_{Au}}{m_\alpha + m_{Au}} \cdot U_0^2$

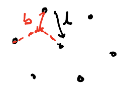
$= \frac{m_{Au}}{m_{Au}+m_\alpha} \cdot E_0$ N.R.

资用能 · $E_总 = m_\alpha c^2 + m_{Au}c^2 + E_0$

$P_总 = \sqrt{(E_0 + m_\alpha c^2)^2 - (m_\alpha c^2)^2}$

$E_s = \sqrt{E_总^2 - P_总^2 c^2}$ 代入 E_0 处.

几乎单层. 纤核作用.



$b \sim \frac{L}{5}$. 偏差. $\theta \sim 2\arctan\frac{a}{L}$.

U入射低. 量子?

8.2. 玻尔.

$P = ka^2$. 加速度 k常数. 辐射功率.

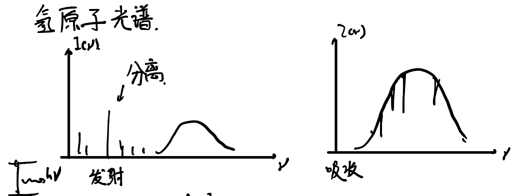
氢原子寿命 $E = -\frac{q^2}{8\pi\varepsilon_0 r}$

$\frac{dE}{dt} = -P$; $\frac{d\frac{q^2}{8\pi\varepsilon_0 r}}{dt} = -k \cdot (\frac{q^2}{4\pi\varepsilon_0 m r^2})^2$

$-\int_{r_0}^{0} r^2 dr = \int_0^{\tau} dt \cdot \frac{k \cdot q^2 \cdot 2}{4\pi\varepsilon_0 m^2}$

$\frac{1}{3}r_0^3 = \tau \cdot \frac{2kq^2}{4\pi\varepsilon_0 m^2}$. $\tau \to 10^{-8}$s.

氢原子光谱.



发射 吸收

$\lambda = R \cdot \frac{n^2m^2}{n^2-m^2}$, $n, m \in \mathbb{Z}$.

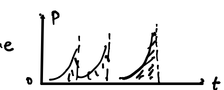
↳ $\frac{hc}{\lambda_{mn}} \sim \square \cdot (\frac{1}{n^2} - \frac{1}{m^2})$. 1.055×10^{-34}J s $= \frac{h}{2\pi}$

↳ $E_n \sim \square\frac{1}{n^2}$. ⇒ $r \sim r_0 n^2$ $mr \cdot U = \hbar \cdot n = L_n$

⇒ $U \sim U_0 \frac{1}{n}$.

① 分离. ② E_n, ν ③ $L_n = n\hbar$

DDL. Dead line



辐射? $[n \to n\pm1]$.

$E_n \to E_{n-1}$ $\sim \Delta E = h\nu$. $n=10^6 \to n=10^5$

$mr_n U_n = n\hbar$ ① $n = 1.2..$

$m\frac{U_n^2}{r_n} = \frac{KZq^2}{r_n^2}$ ②

两粒子体系, 相对运动 $L = L_c + L_r$ $L_r = n\hbar$ $m \to \frac{m_1m_2}{m_1+m_2}$

精细结构常数

$\frac{②}{①}$ $U_n = \frac{KZq^2}{n\hbar}$ 令 $\frac{Kq^2}{\hbar c} = \frac{q^2}{4\pi\varepsilon_0 \hbar c} = \alpha \sim \frac{1}{137}$ 3%

$\frac{U_n}{c} = \frac{Z}{n} \cdot \alpha$. $Z=1$. $n=1$.

$r_n = \frac{n\hbar}{cmU_n/c} = \frac{n\hbar}{mc \cdot \frac{Z}{n}\alpha}$.

$E_n = -\frac{1}{2}\frac{KZq^2}{r_n} = -\frac{1}{2} \cdot \frac{KZ^2q^2\alpha}{n^2\hbar c} mc^2$

$m_H < m_{e^-} + m_{p^+}$

$= -\frac{1}{2}mc^2 \cdot \alpha^2 \frac{Z^2}{n^2}$

$E_n \to E_{n-1} = \frac{1}{2}mc^2\alpha^2(\frac{1}{n^2} - \frac{1}{(n-1)^2})$

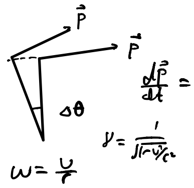
n大 $Z=1$. $\sim \frac{1}{2}mc^2\alpha^2 \cdot \frac{2}{n^3}$ $= h\nu$ $= \hbar\omega$

ω = 电子转动角速度 $\omega = \frac{v}{r}$.

回到经典辐射

例80. 十相对论 的玻尔

$$\begin{cases} m\gamma v_n \cdot \dfrac{v_n}{r_n} = \dfrac{KZq^2}{r_n^2} & ① \\ m\gamma v_n r = n\hbar & ② \end{cases}$$



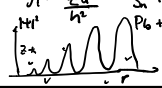
$\frac{d\vec{p}}{dt} = \vec{F}$ $\omega = \frac{v}{r}$ $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$m \to \gamma m$.

$\frac{②}{①}$: $v_n = \frac{Z}{n}\alpha$. $Z=140$. $n=1$. $\frac{v}{c} = \frac{140}{137}$ x.

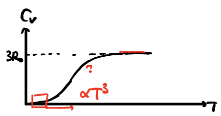
$$E = -\frac{1}{2}m\gamma c^2 \frac{\alpha^2 Z^2}{n^2} = -\frac{1}{2}mc^2 \frac{\alpha^2 Z^2}{n^2} \frac{1}{\sqrt{1-\frac{Z^2\alpha^2}{n^2}}}$$

$\mu \to \frac{m_1 m_2}{m_1+m_2}$ 同位素 Hg. 偏子低



8.3. 量子力学初步.

8.3.1. 黑体辐射.



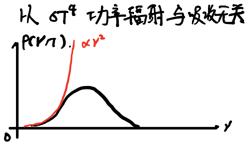
黑体: 吸收所有频率

反射率 $r(\nu)=0$.

N个原子. 3N个振动自由度

从 σT^4 功率辐射与吸收无关. 能均分. 分 $\frac{1}{2}k_BT \times 2$. $U = 3N k_BT$

$C_v = 3R$

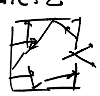


$\frac{dE}{dS\,dt\,d\nu} = P(\nu,T)$. $\int_0^\infty P(\nu,T)\,d\nu = \sigma T^4$

$= \frac{\square \nu^3}{e^{h\nu/kT}-1}$

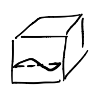
自由光子气 $u(\nu,T) = \frac{dE}{dV\,d\nu}$. 内能密度分布.

$\int_0^\infty u(\nu,T)\,d\nu = \frac{U}{V}$. ← 以后算



泄流 $P(\nu,T) = \frac{c}{4}u(\nu,T)$

$u(\nu,T) = g(\nu)\cdot \bar{\varepsilon}(\nu,T)$



$g(\nu)$. 从 $\nu \sim \nu+d\nu$. 共可能模式数量. (单位体积)

$dN(\nu) = g(\nu)\,d\nu$. 驻波上 × 2种偏振.

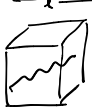
$N(\nu_0)$. $\nu \le \nu_0$ 的模式数量.

$\bar{\varepsilon}(\nu,T)$ 一个模式上的平均能量 (k_BT).

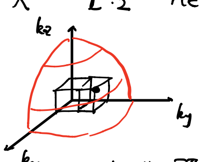
例81. 驻波个数.

全局

$\lambda = \frac{l\cdot 2}{n}$. $\nu = \frac{c}{\lambda} = \frac{c\cdot n}{l\cdot 2}$. $n \in N^*$.



$$\begin{cases} k_x = \frac{\pi}{l}\cdot n_x \\ k_y = \frac{\pi}{l} n_y \\ k_z = \frac{\pi}{l} n_z \end{cases}$$

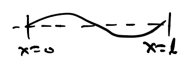


$\vec{k}$ 在 $(n_x,n_y,n_z)\cdot\frac{\pi}{l}$ 格点. $k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$

$$N(\nu_0) = 2\cdot\frac{\frac{1}{8}\cdot\frac{4}{3}\pi\left(\frac{2\pi\nu_0}{c}\right)^3}{V\cdot\left(\frac{\pi}{l}\right)^3} = \frac{8}{3}\pi\frac{\nu^3}{c^3}$$

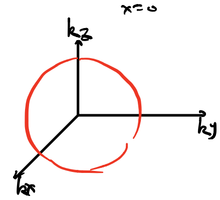
偏振 ← 单位体积 $g(\nu) = \frac{dN}{d\nu} = \frac{8\pi\nu^2}{c^3}$

循环边界



$\psi(x=0) = \psi(x=l)$. $k_x = \frac{2\pi}{l}\cdot n_x$. $n_x \in \mathbb{Z}$.

$\left(\frac{\pi}{l}\right) \to \left(\frac{2\pi}{l}\right)$ $n_x \in N^* \to \mathbb{Z}$.



例82. $\bar{\varepsilon}(\nu,T)$

能量可连续取值. $\varepsilon \sim \varepsilon + d\varepsilon$.

$dp = f(\varepsilon)\cdot d\varepsilon$. 概率. $f(\varepsilon) = Ae^{-\varepsilon/k_BT}$

$\bar{\varepsilon} = \int_0^\infty \varepsilon\,dp$. $1 = \int_0^\infty dp$ $x = \frac{\varepsilon}{k_BT}$

令 $Z(\beta) = \int_0^\infty e^{-\varepsilon/k_BT}\,d\varepsilon = \frac{1}{\beta}\cdot\int_0^\infty e^{-x}\,dx$ $\beta = \frac{1}{k_BT}$

$= \frac{1}{\beta}$. $\frac{dZ}{d\beta} = -\int_0^\infty \varepsilon e^{-\beta\varepsilon}\,d\varepsilon$

$$\bar{\varepsilon} = \frac{\int_0^\infty \cancel{A} e^{-\varepsilon/k_BT}\varepsilon\,d\varepsilon}{\int_0^\infty \cancel{A} e^{-\varepsilon/k_BT}\,d\varepsilon} = \frac{-\frac{dZ}{d\beta}}{Z}$$

$= \frac{1}{\beta} = k_BT$ 能均分.

$\Rightarrow u(\nu,T) = \frac{8\pi\nu^2}{c^3}\cdot k_BT$ $\int_0^\infty u(\nu,T)\,d\nu \to \infty$

若能量 $\varepsilon = nh\nu$ $\beta = \frac{1}{k_BT}$.

$P_n = Ae^{-nh\nu\cdot\beta}$

$\sum P_n = 1$. $\sum P_n\varepsilon_n = \bar{\varepsilon}$.

$$Z(\beta) = \sum_{n=0}^\infty e^{-h\nu n\beta} = \frac{1}{1-e^{-\beta h\nu}}$$

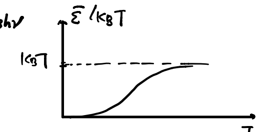
$$\frac{dZ}{d\beta} = -\sum_{n=0}^\infty nh\nu\,e^{-h\nu\beta n}$$

$$\bar{\varepsilon} = -\frac{1}{Z}\cdot\frac{dZ}{d\beta} = -(1-e^{-\beta h\nu})\frac{d\frac{1}{1-e^{-\beta h\nu}}}{d\beta} = \frac{h\nu}{e^{\beta h\nu}-1}$$

$\left(\bar{n} = \frac{1}{e^{h\nu/k_BT}-1}\right)$

高温. $\beta\to 0$. $\bar{\varepsilon} = \frac{h\nu}{1+\beta h\nu-1+\dots} \simeq \frac{1}{\beta} = k_BT$.

低温. $\beta\to\infty$ $\bar{\varepsilon} \sim h\nu e^{-\beta h\nu}$



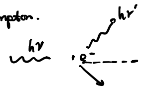
$$u(\nu,T) = \frac{8\pi\nu^2}{c^3}\cdot\frac{h\nu}{e^{h\nu/k_BT}-1}.$$

$P = \frac{c}{4}\int_0^\infty u(\nu,T)\,d\nu$ 令 $x = \frac{h\nu}{k_BT}$.

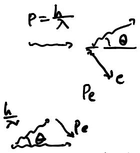
$$= \frac{c}{4}\cdot\frac{8\pi h}{c^3}\cdot\int\frac{\nu^3}{e^{h\nu/k_BT}-1}\,d\nu$$

$$= \frac{c\cdot 8\pi h}{4\cdot c^3}\cdot\frac{k_B^4\boxed{T^4}}{h^4}\cdot\boxed{\int_0^\infty\frac{x^3\,dx}{e^x-1}}\ \frac{\pi^4}{15} = \sigma T^4$$

8.3.2. 光的粒子性和粒子的波动性

Compton.  $p=\frac{h\nu}{c}$ $E=h\nu$

$\lambda'-\lambda=\frac{h}{mc}(1-\cos\theta)$



$p=\frac{h}{\lambda}$ $p'=\frac{h}{\lambda'}$ $\lambda'(\theta)$?

$\Delta E=0$. $\frac{hc}{\lambda}+m_ec^2=\frac{hc}{\lambda'}+\sqrt{p_e^2c^2+m_e^2c^4}$

$\Delta p=0$ $p_e^2=(\frac{h}{\lambda})^2+(\frac{h}{\lambda'})^2-2\cos\theta\frac{h}{\lambda}\frac{h}{\lambda'}$

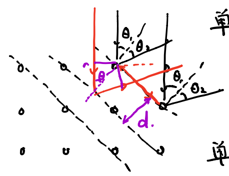
$\cancel{(\frac{hc}{\lambda})^2}+\cancel{m_e^2c^4}+\cancel{(\frac{hc}{\lambda'})^2}+2m_ec^2\frac{hc}{\lambda}-2\frac{hc}{\lambda}\frac{hc}{\lambda'}-2m_ec^2\frac{hc}{\lambda'}$

$=\cancel{m_e^2c^4}+c^2[\cancel{(\frac{h}{\lambda})^2}+\cancel{(\frac{h}{\lambda'})^2}-2\cos\theta\frac{h}{\lambda}\frac{h}{\lambda'}]$.

$m_ec^2\,hc(\frac{1}{\lambda}-\frac{1}{\lambda'})=(1-\cos\theta)\cdot\frac{h^2c^2}{\lambda\lambda'}$

$\lambda'-\lambda=\frac{h}{mc}(1-\cos\theta)$

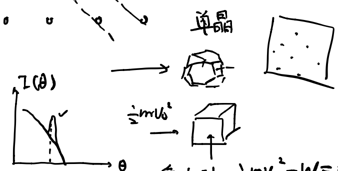
例. 83. Bragg. 衍射. e^-.



单层晶面 $\theta_i=\theta_r$ 最强.
等光程.

多层晶面 $d\cdot2\cos\theta=n\lambda$. 加强.

单晶



$\frac12mv_0^2$

逸出功 $\frac12mv_1^2-W=\frac12mv_0^2$ 代入

例84. Bose-Einstein Condensate

$T\downarrow$ $\frac12mv^2\sim\frac32kT\downarrow$ $P\downarrow$ $\lambda\uparrow$

粒子间距 $l\sim\lambda$ 相当 一干涉. 同一状态.

估计温度. $p=\frac{h}{\lambda}=\hbar\cdot k$ $k=\frac{2\pi}{\lambda}$. $\hbar=\frac{h}{2\pi}$

n 数密度. $n^{-1/3}=l=\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mE}}=\frac{h}{\sqrt{3k_BTm}}$

不确定性关系, x. p_x.

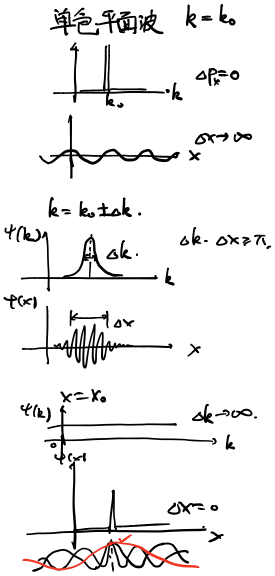
$\Delta x=\sqrt{\overline{x^2}-\bar{x}^2}$ $\Delta p_x=\sqrt{\overline{p^2}-\bar{p}^2}$

$\Delta x\cdot\Delta p_x\ge\frac{\hbar}{2}$

估算 $\Delta x\to\Delta p\to E\downarrow$

$\overline{v^2}\sim\bar{v}^2$; $\overline{\frac1r}\sim\frac{1}{\bar r}$;

单色平面波 $k=k_0$



例85. 氢原子. 估算 $E\sim k.q.m.\hbar$. 正确.

$\bar x=0$. $\Delta x=\overline{x^2}=\frac13\overline{r^2}\sim\frac13\bar r^2$

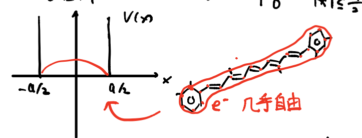
$\bar p_x=0$. $\Delta p_x=\sqrt{\overline{p_x^2}}\ge\frac{\hbar}{2\Delta x}$

$\overline{p_x^2}\ge(\frac{\hbar}{2\Delta x})^2$; $E=-\frac{kq^2}{r}+3\cdot\frac{\overline{p_x^2}}{2m}$.

$E\ge-\frac{kq^2}{\bar r}+3\frac{1}{2m}\cdot\frac{\hbar^2}{4\frac13\bar r^2}$ E_{min}?

$\frac{1}{\bar r}=\frac{kq^2}{2\cdot\frac{3}{2m}\cdot\frac{\hbar^2}{4/3}}$ 代入 E_{min}

方势阱 $V(x)=\begin{cases}\infty & |x|>a\\0 & |x|\le\frac a2\end{cases}$

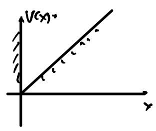


e^- 几乎自由

$\overline{x^2}\sim a^2$ $\Delta x\sim a$. $\Delta p_x\sim\frac{\hbar}{2a}$.

$E\sim\frac{(\Delta p_x)^2}{2m}=\frac{1}{2m}\cdot\frac{\hbar^2}{4a^2}$

重力势. $V(x)=\begin{cases}mgx & x>0\\0 & x<0\end{cases}$



Δx ; $\Delta p_x\ge\frac{\hbar}{2\Delta x}$.

$E_p\sim mg\Delta x$.

$E_k\sim\frac{\Delta p_x^2}{2m}=\frac{1}{2m}\cdot\frac{\hbar^2}{4\Delta x^2}$

$E\sim mg\Delta x+\frac{1}{2m}\frac{\hbar^2}{4\Delta x^2}$

$\ge3\cdot\sqrt[3]{\frac{mg\Delta x}{2}\cdot\frac{mg\Delta x}{2}\cdot\frac{\hbar^2}{2m\cdot4\Delta x^2}}$. $E\sim g^{\frac23}m^{1/3}\hbar^{\frac23}$

例86. 卢瑟福.

$b=a\cot\theta/2$. $\theta\cdot1$. $\Delta\theta$

$b\tan\theta/2=a\Rightarrow b\cdot\theta\sim\frac a2$. Δb.

$\Delta\theta=\frac{\Delta p_y}{p}$. $\Delta b\sim\Delta y$. $\Delta p_y\cdot\Delta y\ge\frac{\hbar}{2}$.

代入 $a=\frac{kZ_1Z_2q^2}{mv_0^2}$, $b\sim\Delta b$. $\Delta\theta\sim\theta$.

$\frac{kZ_1Z_2q^2}{2mv_0^2}\sim\Delta y\cdot\frac{\Delta p_y}{p}\ge\frac1p\frac{\hbar}{2}$

$\frac{kZ_1Z_2q^2}{\hbar v_0}\ge1$. $\longrightarrow$ U_1 玻尔原子基态速度. $\frac{Z_1Z_2U_1}{U_0}\ge1$.

U_0 较小.

8.3.3.

2018深圳三期决赛
扫一扫二维码，加入该群。

A. 假设. 哥本哈根解释.

$\psi(x,t)$ 描述体系状态.

$x(t)$. $p(t)$....

ψ 连续. 平方可积 的复函数

$dp=|\psi|^2dx$ 在 $x\sim x+dx$ 范围 找到粒子

缺点. $\psi(x,t)$ $e^{i\alpha(x)}\psi(x,t)$, 不同状态.

$\psi(x,t)\leftrightarrow e^{i\alpha}\psi(x,t)$ 相同态.

$\int|\psi|^2dx=1$

B. 叠加原理.

对 ψ. 测量力学量 A. 只得一个结果 a.

称 ψ 是 A 的本征态. 记 $|a\rangle$. 复共轭 $\langle a|$.

$\psi = \lambda_1|a\rangle + \lambda_2|b\rangle$ 也是状态.

ψ 测量 A 之后的结果 只能是 a. b. 概率之比为 $|\lambda_1|^2 : |\lambda_2|^2$

E_1 $\sqrt{E_1}$ ⋮ 量子. E_1 或 E_2.

$+$ E_2 $\sqrt{E_2}$

$(\lambda_1\sqrt{E_1}+\lambda_2\sqrt{E_2})^2$ ← 经典. $\lambda_1\sqrt{E_1}+\lambda_2\sqrt{E_2}$

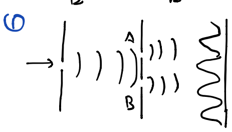
测量改变结果 测A 再测

$\boxed{\psi = \lambda_1|a\rangle+\lambda_2|b\rangle}$ — a $|a\rangle$ — $|a\rangle$; b $|b\rangle$ — $|b\rangle$

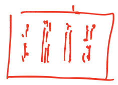
不同 问

$\frac{1}{\sqrt2}|\uparrow\rangle + \frac{1}{\sqrt2}|\downarrow\rangle$. — $|\uparrow\rangle \to |\uparrow\rangle$; $|\downarrow\rangle \to |\downarrow\rangle$

$\frac{1}{\sqrt2}|\uparrow\downarrow\rangle + \frac{1}{\sqrt2}|\downarrow\uparrow\rangle$

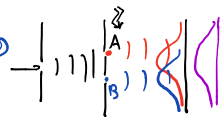
① 

① 明暗是概率



② 不是粒子间作用. 单粒 也有.

③. 不对A.或B 路径测量

② 

在A.B处 测量 粒子状. "拍照". ⇒ 结果变为非相干.

光子与电子作用.

C. 测量力学量. 算符. 力学量对应 厄米算符, 把一个 ψ 变成 ψ 的操作.

$\bar{A} = \int \psi^* \hat{A}\, \psi \, dv$

位置算符. x. $\bar{x} = \int \psi^*(x)\, x\, \psi(x)\, dx$

动量. p_x. 平面波 $e^{ik_x x - i\omega t}$.

$-i\hbar \frac{\partial}{\partial x} e^{ik_x x - i\omega t} = k_x\hbar\, e^{ik_x x - i\omega t}$, $= p_x$

$-i\hbar\frac{\partial}{\partial x} |k_x\rangle = p_x\, |k_x\rangle$.

动量算符. $\bar{p}_x = \int \psi^*(-i\hbar\frac{\partial}{\partial x})\psi\, dx$

p_x^2 $(-i\hbar\frac{\partial}{\partial x})(-i\hbar\frac{\partial}{\partial x})|k_x\rangle = p_x^2 |k_x\rangle$

$-\hbar^2 \frac{\partial^2}{\partial x^2}$

$E_k = \frac{p_x^2}{2m} \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

$E_p = \frac{\alpha}{r} \Rightarrow \int \psi^*(x)\, \frac{\alpha}{r}\, \psi(x)\, dx = \bar{E}_p$

$L_z = x p_y - y p_x = x(-i\hbar\frac{\partial}{\partial y}) - y(-i\hbar\frac{\partial}{\partial x}) = i\hbar\frac{\partial}{\partial\varphi}$

对易关系. $\hat{A}\hat{B}\,|\psi\rangle \stackrel{?}{=} \hat{B}\hat{A}|\psi\rangle$

$x(-i\hbar\frac{\partial}{\partial x}\psi) - (-i\hbar\frac{\partial}{\partial x}(x\psi)) = i\hbar\,\psi$.

$\hat{x}\hat{p}_x - \hat{p}_x\hat{x} = i\hbar \stackrel{记}{=} [\hat{x},\hat{p}_x]$. ✓

$|\Delta A|^2|\Delta B|^2 \geq \frac14 |[\hat{A},\hat{B}]|^2$

$\hat{C} = \hat{A} + i\beta\hat{B}$

$\hat{C}^*\hat{C}$

D. Schrödinger Equation.

$\psi(x,t) = e^{ik_x x - i\omega t}$ $i\hbar\frac{\partial}{\partial t} e^{ik_x x - i\omega t} = \boxed{\hbar\omega}\, e^{\cdots}$ ↓ E.

$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi = (\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}_{E_k} + V(r))\,\psi$

E. 全同粒子. $\psi(x_1,x_2)$ $\psi(x_2,x_1)$ 得同一个态.

$\psi(x_1,x_2) = e^{i\alpha}\psi(x_2,x_1) = e^{i2\alpha}\psi(x_1,x_2)$

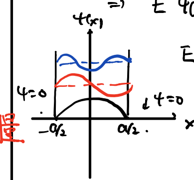
$e^{i2\alpha} = 1$ $e^{i\alpha} = \pm 1$ $\begin{cases}\psi(x_1,x_2) = \psi(x_2,x_1) & \text{Boson}\\ \psi(x_1,x_2) = -\psi(x_2,x_1) & \text{Fermion}\end{cases}$

例.87. 一维方势阱 $V(x) = \begin{cases}\infty & |x|\geq \frac a2\\ 0 & |x|<\frac a2\end{cases}$

定态解. 也是能量本征态.

$\hat{H}\psi = E\psi = -i\hbar\frac{\partial}{\partial t}\psi$. $\psi(x,t) = \psi(x)\, e^{-i\frac{E}{\hbar}t}$

$\Rightarrow E\,\psi(x) = (-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x))\psi(x)$ 定态......



E不发散 连续 $\psi(x)\big|_{x\to\pm\frac a2} = 0$

中间: $-\frac{\hbar^2}{2m}\psi''(x) = E\psi$

$\psi(x) = Ae^{ikx} + Be^{-ikx}$ 代入边界

$k^2\cdot\frac{\hbar^2}{2m} = E$.

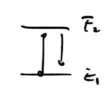
$Ae^{ik(\pm\frac a2)} + Be^{ik(\pm\frac a2)} = 0 \Rightarrow \cdots$

偶. $\psi(x) = A\cos kx$. $k\frac a2 = (n+\frac12)\pi$.

奇 $\psi(x) = A\sin kx$. $k\cdot\frac a2 = n\pi$

$k = \frac{\pi}{a}(2n+1)$ 或 $k = \frac{\pi}{a}\cdot 2n \Rightarrow k_n = N\cdot\frac{\pi}{a}$.

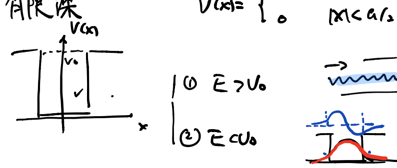
$E_N = \frac{p^2}{2m} = \frac{(\frac{\pi}{a}N)^2\hbar^2}{2m} = \frac{\pi^2\hbar^2}{a^2\cdot 2m}\cdot N^2$



基: $N=1$. $E_1 = \frac{\pi^2\hbar^2}{a^2 2m}$.]→ΔE ✓

第一: $N=2$ $E_2 = \frac{\pi^2\hbar^2}{a^2\cdot 2m}\cdot 4$

有限深 $V(x) = \begin{cases}V_0 & |x|\geq a/2.\\ 0 & |x|<a/2\end{cases}$



① $E > V_0$ 行波 非束缚

② $E < V_0$ 束缚.

$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = E\psi$

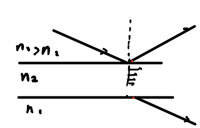
$E > V_0$. $-\frac{\hbar^2}{2m}\psi'' = (E-V_0)\,\psi$. >0.

$E < V_0$. $-\frac{\hbar^2}{2m}\psi'' = (E-V_0)\,\psi$ <0

代入 ψ'连续 定 E_n ...

ψ连续.

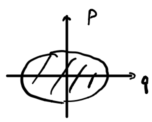
波动有

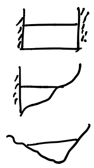


一级玻恩近似 无视 $V'''(x)$ 以上. λ: 0, $\frac{1}{2}$, $\frac{1}{4}$. 不要求.

一维点粒子. $\oint p\,dq = (n+\lambda)h$. 束缚

$p\,dq$. 一维粒子经典周期运动中. 动量—坐标图的面积



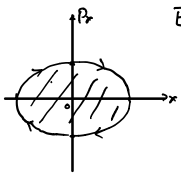


$\lambda=0$

$\lambda=\frac{1}{4}$ 已知条件

$\lambda=\frac{1}{2}$

例88. 简振 $V(x)=\frac{1}{2}kx^2$. 量子化.



$E=\frac{1}{2}kx^2+\frac{p^2}{2m}$. $E=E_0$.

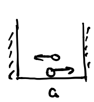
$x_m=\sqrt{\frac{2E_0}{k}}$, $P_m=\sqrt{2mE_0}$

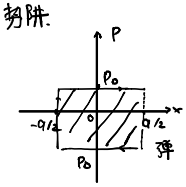
$\pi\cdot x_m\cdot P_m=\pi\sqrt{\frac{2E_0}{k}}\cdot\sqrt{2mE_0}=(n+\frac{1}{2})h$.

$E_0=(n+\frac{1}{2})\hbar\omega$. $\hbar=\frac{h}{2\pi}$.

$1\hbar\omega$

例89. 方势阱



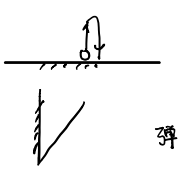


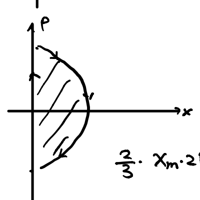
$a\cdot 2P_0=nh$

$P_0=\frac{nh}{2a}$

$E_n=\frac{P_0^2}{2m}=\frac{n^2h^2}{8a^2m}$.

弹





弹

$\frac{p^2}{2m}+mgx=E$

$x_m=\frac{E}{mg}$. $P_m=\sqrt{2mE}$

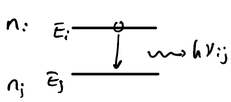
$\frac{2}{3}\cdot x_m\cdot 2P_m=(n+\frac{1}{4})h$.

$\frac{4}{3}\cdot\frac{E}{mg}\cdot\sqrt{2mE}=(n+\frac{1}{4})h$ $E_n=\ldots$

自发辐射?

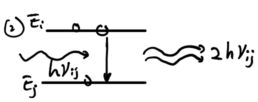
例90. Einstein A.B系数模型

① 光场与原子作用. $h\nu_{ij}=E_i-E_j$ 自发辐射.

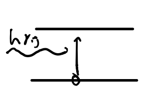


$\frac{dn_i}{dt}=-\frac{dn_j}{dt}=-A\cdot n_i$

② 受激辐射/吸收



$\frac{dn_i}{dt}=-\frac{dn_j}{dt}=-B\cdot u(\nu_{ij},T)\,n_i$



$-\frac{dn_i}{dt}=\frac{dn_j}{dt}=-B'u(\nu_{ij},T)\,n_j$

一定温度平衡态 $\frac{dn_i}{dt}=\frac{dn_j}{dt}=0$

$n_i[A+B\,u(\nu_{ij},T)]=B'\,u(\nu_{ij},T)\cdot n_j$

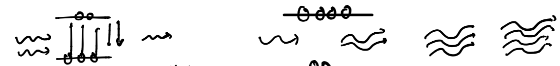
T平衡 $\frac{n_i}{n_j}=e^{-h\nu_{ij}/k_BT}$.

$T\to\infty$ $n_i=n_j$ u大. $B=B'$.

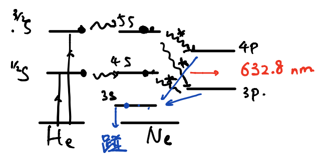
T一定 $e^{-h\nu/k_BT}(A+Bu)=Bu$.

$u(\nu,T)=\frac{A/B}{e^{h\nu/k_BT}-1}$ A.B 只是ν函数 $A\neq 0$.

激光. 粒子数反转. $n_i>n_j$ $E_i>E_j$



光传. 吸收多. 吸收<辐射. (917)



电场 共振转移

例91. 致冷.

原子上看 $\nu'=\nu\cdot\sqrt{\frac{1+v/c}{1-v/c}}\simeq\nu(1+\frac{v}{c})$

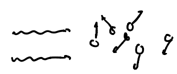
T=300k. $v\sim10^4$ m/s $\frac{\Delta\nu}{\nu}\sim10^{-4}$. Doppler

① ⇒ $p=\frac{h\nu}{c}$ ... $\frac{h\nu}{c}$ ⇒ $\frac{p^2}{2m}+h\nu=\Delta E$.

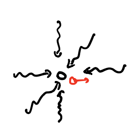
$(\frac{h\nu}{c})^2\frac{1}{2m}+h\nu=\Delta E$. $\frac{\Delta\nu}{\nu}\sim\frac{\Delta E}{2mc^2}$ → 10eV / → 1MeV 反冲 $\sim10^{-5}$

$h\nu=\Delta E(1-\varepsilon)$.

只有 $v_x=c\cdot\varepsilon$ 时, 吸收.



$v_x\downarrow$ 吃动量

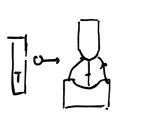


迎光的粒子吃动量 $v\downarrow$ T↓

mK.

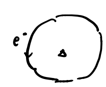
8.4 精细结构和超精细结构

施特恩-盖拉赫实验



收到?

例91. 氢原子磁矩. $\mu=?$ 玻尔

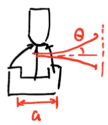


$\mu=Is=\frac{q\omega}{2\pi}\cdot\pi r^2=\frac{q}{2m}\omega r^2m=\frac{q}{2m}\cdot L$.

$\vec{\mu}=\frac{-q}{2m}\vec{L}$, $L=n\hbar$. $\mu=n\cdot\mu_B$. 玻尔磁子.

$\mu_B=\frac{q}{2m}\cdot\hbar$.

例1. H 入 $\frac{\Delta B_z}{\Delta z}$?　　$\varepsilon = -\vec{\mu}\cdot\vec{B}$.



$F = -\frac{d\varepsilon}{dz} = \mu\cdot\frac{dB_z}{dz}$.

$\theta = \frac{F\cdot a}{mv}$

$\bar{v} \sim \sqrt{\frac{kT\,8}{m\pi}}$

$\theta = \frac{\mu\cdot\frac{dB_z}{dz}\cdot a}{mv^2}$

经典 ⇒ θ连续

量子. $\mu_z = m\cdot\mu_B$　$L=0$　$\mu_z=0$

$\mu_z \le \mu$.　$L=\hbar$.　$\mu_z = \pm\mu_B$. 0.

$L=2\hbar$　$\mu_z = \pm 2\mu_B$. $\pm 1\mu_B$. 0

实验.H.　$+\mu_B$　$-\mu_B$

$\mu_z = L_z\frac{q}{2m} = m\mu_B$.

$\Delta\theta = \frac{\mu_B\frac{dB_z}{dz}}{mv^2}\times 2$

多的磁矩? 乌伦贝克 自旋?　$r<10^{-15}m$　$\omega r > c$ ×

$g = 2.00001\ldots$

$\mu_s = \pm\mu_B$.　$L_z = \pm\frac{\hbar}{2}$.　$\mu = \frac{q}{2m}\cdot g\cdot L$.

1/4后知.

点粒子. 内

例1.93. 精细结构.

$\varepsilon = \pm\mu_B\cdot B$.

从电子系

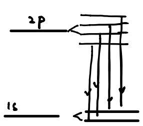
$\Delta\varepsilon = 2\mu_B\cdot B = 2\mu_B\cdot\frac{\mu_0 I}{2r} = \mu_B\cdot\frac{\mu_0}{r}\frac{q\omega}{2\pi}$

$= \frac{q}{2m}\cdot\hbar\cdot\frac{\mu_0}{r}\frac{q\,\omega}{2\pi}$

$\frac{\Delta\varepsilon}{|\varepsilon_0|} = \ldots = 2\cdot\frac{\hbar\omega}{mc^2} = 2\alpha^2$

$\frac{\hbar\omega}{mc^2} = \frac{\hbar\, v/c}{mc\cdot r} = \alpha\cdot\frac{\hbar}{mc\cdot r}$

$\frac{\Delta\varepsilon}{|\varepsilon_0|} \sim \alpha^2$.



2p→1s 两条光谱.

$\pm\mu_B\cdot B$.

$\Delta\varepsilon \sim \varepsilon\cdot\alpha^2$

Na. 589.3nm < 589.0nm, 589.6nm　$\frac{\Delta\nu}{\nu}\sim 10^{4}$

例1.94. 超精细结构.

原子核自旋.　$\mu_n = \frac{q}{2m_n}\cdot L_n\cdot g$.

$m_n : m_e \sim 1800\times A$.　$\mu_n \ll \mu_e$

$\frac{\mu_n}{\mu_e}\sim\frac{m_e}{m_n}\sim\frac{1}{1800A}$.

$\sim 10^{4}$　$\sim 10^{3}$.

MRI. CT.

$\mu\cdot B$ 大. $\vec{\mu}$ 稍有B.

$h\nu = 2\mu B$. 受激吸收

主要光场. 自发辐射

分子磁场 → $h\nu = 2\mu B$ 不同.

3:1

8.5 · 半经典 光与物质作用　(非弹性. 吸收)　弹性

Tight Bound.

$F = -kx$.　$U(x) = U_0 + U'x + \frac{U''}{2}x^2$

辐射阻尼 等效阻力 $f = -\beta v$.　电场

$m\ddot{x} = -kx - \beta\dot{x} - eE\cos\omega t$

受迫振动 $\tilde{x} = \tilde{A}e^{i\omega t}$

$-\omega^2 m\tilde{A} = -k\tilde{A} - i\omega\beta\tilde{A} - eEe^{i\omega t}$

$\tilde{A} = \frac{-eE\,e^{i\omega t}}{k - m\omega^2 + i\omega\beta}$.

$\tilde{p} = -e\tilde{A} = \frac{e^2E\,e^{i\omega t}}{k - m\omega^2 + i\omega\beta}$.　$\tilde{p}\propto E$. 相位差.

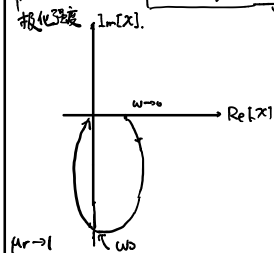
$= \frac{e^2E\frac{1}{m}e^{i\omega t}}{\sqrt{(\omega_0^2-\omega^2)^2+\frac{\omega^2\beta^2}{m^2}}}e^{i\alpha}$.　$\tan\alpha = \frac{-i\,\beta\omega/m}{\omega_0^2-\omega^2}$

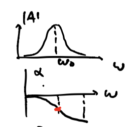
数密度

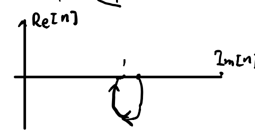
$P = n\cdot\tilde{p} = \boxed{\frac{\frac{e^2}{m}n}{\sqrt{(\omega_0^2-\omega^2)^2+\frac{\omega^2\beta^2}{m^2}}}e^{i\alpha}}E\,e^{i\omega t}$　$P = \varepsilon_0\chi E$.

极化强度　$\varepsilon_0\chi$ 极化率

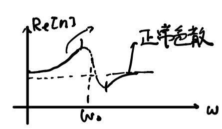
$\omega_0^2 = \frac{k}{m}$

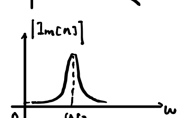






$n^2 = \varepsilon_r = 1+\chi$; $n = \sqrt{1+\chi}$





$e^{i(kx-\omega t)}$　$k = n\frac{\omega}{c}$

$e^{i[Re[n]\frac{\omega}{c}x-\omega t]}\cdot e^{-Im[n]\cdot\frac{\omega}{c}x}$　$Re[n]<1$

$U_p = \frac{c}{Re[n]}$.　衰减　$Re[n]<0$

远离 $\omega=\omega_0$ 区.　$n^2\simeq 1+\frac{n_{数}e^2/m}{\omega_0^2-\omega^2}$　↳ k/m

例95. X射线 $\omega \gg \omega_0$, $n^2 \simeq 1 - K\lambda^2$

$$n^2 \simeq 1 + \frac{n_{核} e^2/m}{\omega_0^2 - \omega^2} \simeq 1 - \frac{n_{核} e^2/m}{\omega^2} \qquad \omega = c \cdot k_0 = c \cdot \frac{2\pi}{\lambda}$$

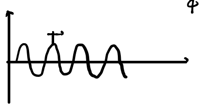
$\hookrightarrow k/m$

$K = \frac{n_{核} e^2}{m \cdot c^2 4\pi^2}$; 证明 $U_p \cdot U_g = c^2$

相速. 群速

单色. $E = E_0 \cos(\omega t - kx + \varphi)$ 要求稳定波形

$\hat{E} = E_0 \, e^{i\underbrace{(\omega t - kx + \varphi)}_{\phi}}$



$\phi = \omega t - kx + \varphi$

$\Rightarrow x = \boxed{\frac{\omega}{k}} t + \square$ 等相位面

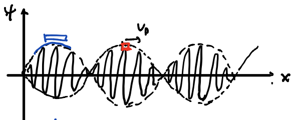
$v_p = \frac{\omega}{k}$.

群速度. $\omega - \Delta\omega \sim \omega + \Delta\omega$.

相邻两个频率. $\cos[(kx - \omega t)] + \cos[(k + \Delta k)x - (\omega + \Delta\omega)t]$.

慢变. 振幅变化

$$\psi = \boxed{2\cos\left(\frac{\Delta k}{2} x - \frac{\Delta\omega}{2} t\right)} \cos\left[\left(k + \frac{\Delta k}{2}\right) x - \left(\omega + \frac{\Delta\omega}{2}\right) t\right]$$



$\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t = 2\pi \cdot n$ 等幅面移动

$x = \boxed{\frac{\Delta\omega}{\Delta k}} t + \square \rightarrow U_g = \frac{d\omega}{dk}$. 群速度

$n^2 = 1 - \frac{A}{\omega^2}$ $\qquad v_p = \frac{\omega}{k} \qquad U_g = \frac{d\omega}{dk}$

$k = n \cdot k_0 = n \cdot \frac{\omega}{c} = \sqrt{1 - \frac{A}{\omega^2}} \cdot \frac{\omega}{c}$ $\qquad k(\omega)$

$c^2 k^2 = \omega^2 - A. \Rightarrow c^2 k dk = \omega d\omega.$

$c^2 = \frac{\omega}{k} \cdot \frac{d\omega}{dk} = U_p \cdot U_g$

物质波. 相对论粒子 $\quad p^2 c^2 + m_0^2 c^4 = E^2$

$p = \hbar k$. $\quad E = \hbar\omega$ $\quad \mathcal{L}(-E_0)$ 零点

$k^2 c^2 + \frac{m_0^2 c^4}{\hbar^2} = \omega^2 \quad \Rightarrow c^2 k \, dk = \omega \, d\omega$

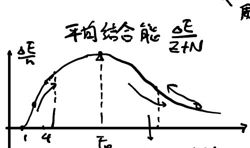
$\Rightarrow U_p \cdot U_g = c^2$

8.6 原子核.

例96. 结合能 $\Delta E = Z m_p c^2 + N m_n c^2 - m_N c^2 \quad > 0$

原子数 $\quad$ 中子数

平均结合能 $\frac{\Delta E}{Z+N}$



$A = Z + N$

$$\Delta E = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_{sym}(Z-N)^2 A^{-1} + E_{Bp} + E_{壳}$$

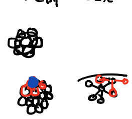
A大

$a_v = 15.8\,MeV$ 强相互作用 饱和. 短程

$a_s = 18.3\,MeV$ 表面. $A^{1/3} \sim r \quad A^{2/3} \sim S$.

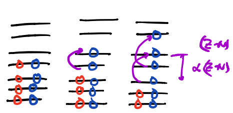
$a_c = 0.72\,MeV$ 库仑 $\sum \frac{1}{2} U_i q_i \quad \propto Z^2$

$\hookrightarrow \sim \frac{KZq}{r}$



$a_{sym} = 23.2\,MeV$

$E_{B.p} = \begin{cases} +a_p A^{-1/2} & 偶偶 \\ 0 & 偶奇. \\ -a_p A^{-1/2} & 奇奇 \end{cases}$ $\quad {}^{12}C \quad {}^{14}C \quad {}^{13}C \quad {}^{14}N$



2018寒深圳冲决-1-007

1. 运动学

1.1. 描述

Degree of freedom. 原自由度 - 约束

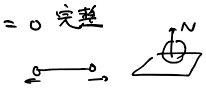
• 3D x, y, z $\quad d.o.f = 3$.

2个3D. $\quad d.o.f = 6$.

-1. $\Delta x^2 + \Delta y^2 + \Delta z^2 = l^2$ $\quad -1$. $\quad d.o.f = 5$

3×3. $- 3 \quad = 6$. $\quad d.o.f = 6$

$f(x, y, z) = 0$ 完整



静力. 约束 ↔ 约束力.

独立自由度 — 平衡类型 $\quad E_p(q_1, q_2, \ldots q_n)$

$f = 0$?

$f < 0$. 过约束 , 无穷组解

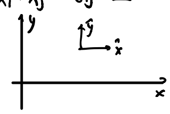
动力. 方程数最少 $= d.o.f.$

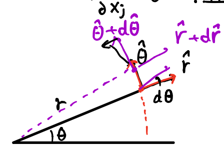
坐标. 直角坐标.?

$\vec{r} = (x, y, z) = x \cdot \hat{x} + y \hat{y} + z \hat{z}$.

$\vec{r} = \sum x_i \cdot \hat{x}_i$ $\quad$ 直角. $\hat{x}_i \cdot \hat{x}_j = \delta_{ij}$ 正交. $\quad i = j \Rightarrow 1$, $i \neq j \Rightarrow 0$

$\frac{\partial \hat{x}_i}{\partial x_j} = 0$ 平直





非直角坐标.

$\vec{r}(r, \theta)$, $\frac{\partial \vec{r}}{\partial r} \parallel \hat{r}$. $\frac{\partial \vec{r}}{\partial \theta} \parallel \hat{\theta}$.

$\hat{r}(r, \theta) \qquad \hat{\theta}(r, \theta)$

$r \rightarrow r + dr$; $d\hat{r} = 0 \qquad d\hat{\theta} = 0$

$\theta \rightarrow \theta + d\theta$; $d\hat{r} = 1 \cdot d\theta \, \hat{\theta}$. $\quad d\hat{\theta} = -1 \cdot d\theta \cdot \hat{r}$

$\hat{r} \cdot \hat{\theta} = 0 \qquad (\hat{r} + d\hat{r}) \cdot (\hat{\theta} + d\hat{\theta}) = 0$

$d\hat{r} \cdot \hat{\theta} + \hat{r} \cdot d\hat{\theta} = 0$

$\vec{r} = r \cdot \hat{r}$

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}(r \cdot \hat{r}) = \dot{r} \hat{r} + r \frac{d\hat{r}}{dt}$$

$$= \dot{r} \hat{r} + r \cdot \frac{d\theta}{dt} \hat{\theta}$$

$$= \underbrace{\dot{r}}_{v_r} \hat{r} + \underbrace{r\dot{\theta}}_{v_\theta} \hat{\theta}$$

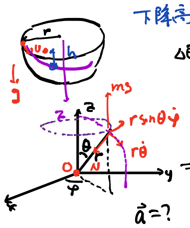
$$\frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}(\dot{r} \hat{r}) + \frac{d}{dt}(r\dot{\theta} \hat{\theta})$$

$$= \ddot{r} \hat{r} + \dot{r} \dot{\theta} \hat{\theta} + \dot{r} \dot{\theta} \hat{\theta} + r\ddot{\theta} \hat{\theta} + r\dot{\theta}(-\dot{\theta} \hat{r})$$

$$= \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r} \hat{r} + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{a_\tau} \hat{\theta}$$

例1. 碗内的质点的加速度.

下降高为h时. $\dot\theta=?$ $\dot\varphi=?$ $\ddot\theta=?$ $\ddot\varphi=?$



$\Delta E=0$

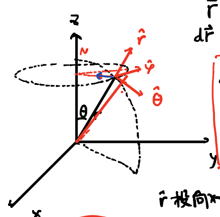
$\frac{1}{2}mv_0^2=\frac{1}{2}m(r\dot\theta)^2+\frac{1}{2}m(r\sin\theta\dot\varphi)^2-mgh.$

$\Delta L_z=0$ H.O力矩

$m r v_0 = m r\sin\theta\dot\varphi\cdot r\sin\theta$

$\Rightarrow \dot\varphi=✓$ $\dot\theta=✓$

$\vec a=?$

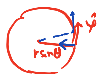


$\vec r = r\hat r$

$d\vec r = dr\,\hat r + r\,\underline{d\hat r}$? ✓

$$\boxed{\begin{aligned} d\hat r &= 0\cdot dr + \underline{1\cdot\hat\theta}\,d\theta + \underline{\boxed{\sin\theta}\,\hat\varphi}\,d\varphi \\ d\hat\theta &= 0\cdot dr + \underline{-1\cdot\hat r}\,d\theta + \underline{\boxed{\cos\theta}\,\hat\varphi}\,d\varphi \\ d\hat\varphi &= 0\,dr + 0\cdot d\theta + \underline{\boxed{-\cos\theta}\,\hat\theta\ \boxed{-\sin\theta}\,\hat r}\,d\varphi \end{aligned}}$$

$\hat r$投向x-y. $\sin\theta$.



$$\frac{d\vec r}{dt}=\dot r\,\hat r+r\dot\theta\,\hat\theta+r\sin\theta\dot\varphi\,\hat\varphi$$

$$\vec a=\frac{d\vec r}{dt}= \ddot r\,\hat r+\dot r\,(r\dot\theta\hat\theta+r\sin\theta\dot\varphi\,\hat\varphi)$$

$$+\frac{d}{dt}(r\dot\theta)\,\hat\theta+r\dot\theta\,(-\dot\theta\,\hat r+\cos\theta\dot\varphi\,\hat\varphi)$$

$$+\frac{d}{dt}(r\sin\theta\dot\varphi)\,\hat\varphi+r\sin\theta\dot\varphi\,(-\cos\theta\,\hat\theta\,\dot\varphi-\sin\theta\,\hat r\,\dot\varphi)$$

$$\begin{cases} a_r=\ddot r-r\dot\theta^2-r\sin^2\theta\dot\varphi^2 \\ a_\theta=r\ddot\theta+2\dot r\dot\theta-\underline{r\sin\theta\cos\theta\dot\varphi^2} \\ a_\varphi=2\dot r\sin\theta\dot\varphi+2r\dot\theta\cos\theta\dot\varphi+r\sin\theta\ddot\varphi \end{cases}$$

不要背.

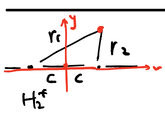
碗: $\dot r=0$ $\ddot r=0$

$a_r=-r\dot\theta^2-r\sin^2\theta\dot\varphi^2 = N-mg\cos\theta.$

$a_\theta=r\ddot\theta=-g\sin\theta.$

$a_\varphi=\underline{2r\dot\theta\cos\theta\dot\varphi}+r\sin\theta\ddot\varphi=\frac{F_\varphi}{m}=0$

N✓. $\ddot\theta$✓, $\ddot\varphi$✓



$x=c\cdot ch\xi\cdot\cos\eta$ $\qquad sh\xi=\frac{e^\xi-e^{-\xi}}{2}$

$y=c\cdot sh\xi\cdot\sin\eta$. $\qquad ch\eta=\frac{e^\eta+e^{-\eta}}{2}$

$$\begin{cases} ch\,iu=\frac{e^{iu}+e^{-iu}}{2}=\cos u \\ sh\,iu=i\sin u. \end{cases}$$

$$r_1=\sqrt{(x+c)^2+y^2}=\sqrt{c^2ch^2\xi\cos^2\eta+2c^2ch\xi\cos\eta+c^2+c^2sh^2\xi\sin^2\eta}$$

$$=c\cdot\sqrt{ch^2\xi+2ch\xi\cos\eta+\cos^2\eta}=c\cdot(ch\xi+\cos\eta)$$

$r_2=c\,(ch\xi-\cos\eta)$

$$\begin{cases} r_1+r_2=2c\,ch\xi.=Const \\ r_1-r_2=2c\cos\eta \end{cases}$$

$\hat\xi$. $\hat\eta$...