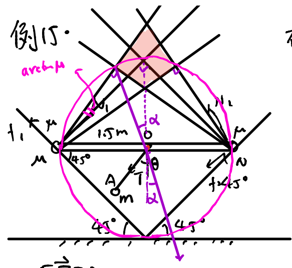
例15.



在平面内摆. θ=? ,不会滑

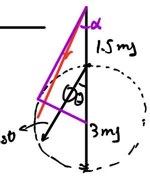
求 μ_{min}=?

$T = mg\cos\theta + m\cdot\frac{v^2}{l}$ θ< 90°

$= 3mg\cos\theta$. $\frac{1}{2}mv^2 = mgl\cos\theta$

$\Sigma\vec{F}=0$ ×.

$\Sigma\vec{M}=0$ ✓



α_{max} 切线

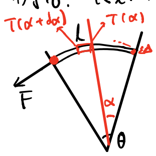
$\alpha_{max} = \frac{\pi}{6}$.

$\tan\frac{\alpha}{2} = \mu_{min}$

$= \tan\frac{\pi}{12}$

例16. 长l. m. 软绳子在地上盘θ角圆弧. μ.

求拉动 F_{min}=?



T(α)

① T(0)=0

② T(θ)=F

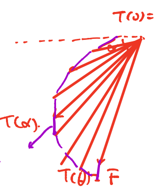
③ $(dT)^2 + (Td\alpha)^2 \le (df)^2$

$df = \frac{m}{\theta}d\alpha\cdot g\mu$ 临界取等

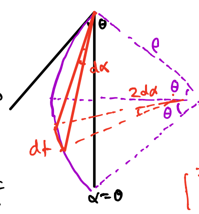
$T'^2 + T^2 = (\frac{mg\mu}{\theta})^2$

① $T = \frac{mg\mu}{\theta}$. T'=0

② $T = \frac{mg\mu}{\theta}\cos(\alpha+\gamma)$



$|df| \le dm\,g\mu$



$\rho = \frac{F}{2}\cdot\frac{1}{\sin\theta}$.

$df = \rho\cdot 2d\alpha$

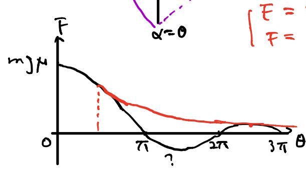
$df = \frac{F}{\sin\theta}d\alpha$.

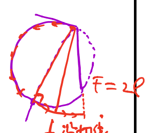
$\le dm\,g\mu$ $\frac{m}{\theta}\cdot d\alpha$

$F = mg\mu\frac{\sin\theta}{\theta}$ $\theta\le\frac{\pi}{2}$

$F = mg\mu/\theta$ $\theta > \frac{\pi}{2}$

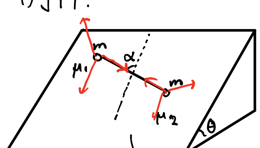
$\frac{\sin\theta}{\theta}\Big|_{\theta\to 0} = 1$





$F = \frac{1}{2}\cdot\frac{mg\mu}{2\theta}$

例17.

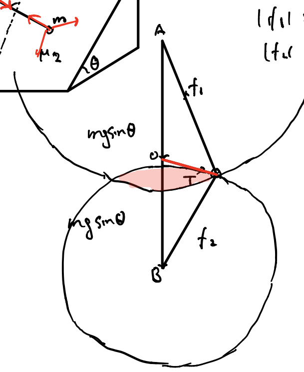


$\mu_1 > \tan\theta > \mu_2$

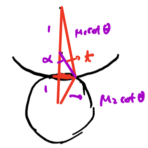
$\frac{\mu_1+\mu_2}{2} > \tan\theta$. 求 α_{max}=?

$|f_1| \le mg\cos\theta\,\mu_1$

$|f_2| \le mg\cos\theta\,\mu_2$



f_2小.



交点

$1^2 + t^2 + 2t\cos\alpha = \mu_1^2\cot^2\theta$ ①

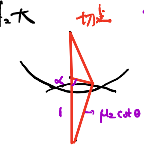
$1^2 + t^2 - 2t\cos\alpha = \mu_2^2\cot^2\theta$ ②

$t = \sqrt{\frac{\mu_1^2\cot^2\theta + \mu_2^2\cot^2\theta}{2} - 1}$

①-② $4t\cos\alpha = (\mu_1^2 - \mu_2^2)\cot^2\theta$

$\cos\alpha = \frac{(\mu_1^2-\mu_2^2)\cot^2\theta}{4\sqrt{\frac{\mu_1^2+\mu_2^2}{2}\cot^2\theta - 1}}$

f_2大 切线 $\sin\alpha = \mu_2\cot\theta$. 临界:



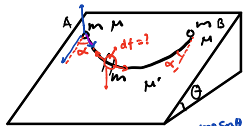
$\mu_2^2\cot^2\theta + \left(\frac{(\mu_1^2-\mu_2^2)\cot^2\theta}{16(\frac{\mu_1^2+\mu_2^2}{2}\cot^2\theta - 1)}\right)^2 = 1$

例18.

$\mu > \tan\theta > \mu'$

$\frac{2\mu+\mu'}{3} > \tan\theta$.

α_{max}=?

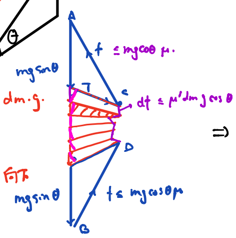


悬链线

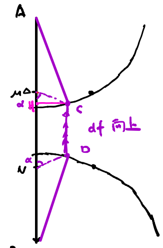
dmg sinθ, df 合力

dm. g(sinθ − μ'cosθ) 向下
g'

$|AB|_{max}$?



⇒



$|MH| = \frac{1}{3}(mg\sin\theta - mg\cos\theta\mu')$

$|AH| = mg\sin\theta + \frac{1}{2}mg\sin\theta - \frac{1}{3}mg\cos\theta\mu'$

$|AC| = mg\mu\cos\theta$

$|CH| = mg\sin\theta\sqrt{(\mu\cot\theta)^2 - (\frac{3}{2} - \frac{1}{2}\mu'\cot\theta)^2}$

$\alpha_{max} = \arctan\frac{|CH|}{MH}$

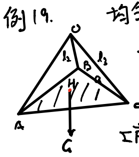
$= \arctan\frac{\sqrt{(\mu\cot\theta)^2 - (\frac{3}{2} - \frac{1}{2}\mu'\cot\theta)^2}}{\frac{1}{3} - \frac{1}{3}\mu'\cot\theta}$

例19. 均匀三角板ABC. 用OA. OB. OC

三根绳吊在O点. $|OA|=l_1$ $|OB|=l_2$ $|OC|=l_3$

$|AB|=c$ $|BC|=a$ $|CA|=b$

求 三绳上作用力 F_1. F_2. F_3.



$\Sigma\vec{F}=0$. $F_1\cdot\hat{OA} + F_2\hat{OB} + F_3\hat{OC} = -\vec{G}$

$\Sigma\vec{M}=0$ ⇒ H在O正下方. H心较点

$\vec{OH} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$

$F_1\hat{OA} + F_2\hat{OB} + F_3\hat{OC} = G\cdot\hat{OH} = G\frac{\vec{OA}+\vec{OB}+\vec{OC}}{3|OH|}$

$\hat{OA}\cdot\hat{OB}\cdot\hat{OC}$ 三个线性无关矢量.

⇒ $F_1 = \frac{G}{3}\cdot\frac{|OA|}{|OH|}$ $F_2 = \frac{G}{3}\frac{|OB|}{|OH|}$ $F_3 = \frac{G}{3}\frac{|OC|}{|OH|}$

$F_1:F_2:F_3 = l_1:l_2:l_3$.

$\vec{OH}\cdot\vec{OH} = \frac{1}{9}(\vec{OA}+\vec{OB}+\vec{OC})\cdot(\vec{OA}+\vec{OB}+\vec{OC})$

$= \frac{1}{9}\cdot(l_1^2+l_2^2+l_3^2 + 2\vec{OA}\cdot\vec{OB} + 2\vec{OB}\cdot\vec{OC} + 2\vec{OC}\cdot\vec{OA})$

$= \frac{1}{9}(3l_1^2+3l_2^2+3l_3^2 - a^2 - b^2 - c^2)$

$|\vec{OB} - \vec{OC}| = a$

$l_2^2 + l_3^2 - 2\vec{OB}\cdot\vec{OC} = a^2$

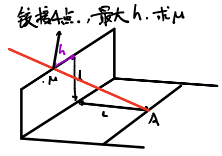
$F_1 = G\frac{l_1}{\sqrt{3l_1^2+3l_2^2+3l_3^2-a^2-b^2-c^2}}$

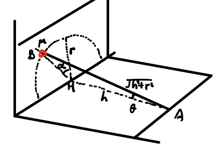
8.2 空间力矩.

铰接A点. 最大h. 求μ

均匀杆. 铰接A点. B在墙上. μ.

求 α_{max}=?





f方向. 允许区在平面上

判定 ① f⊥杆 ×

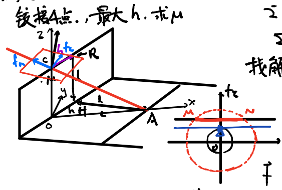
② f 沿 即将滑动方向的相反. ×

例20

铰接A点，最大h，求μ

$\sum\vec{M}=0$ $|\vec{f}|\le N\mu$

$\sum\vec{F}=0$



找解集，$\sum\vec{M}=0$ 不含有f_n ✓

← 线

← $|\vec{f}|\le N\mu$ 圆内 ✓

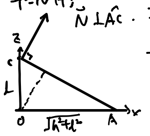
→ f_n 临界，相切。⇒ $f_n=0$

$\vec{f}\perp\vec{AC}$.

$\sum\vec{M}=0$. $l\cdot\hat{AC}\times\vec{G}+\vec{AC}\times(\vec{N}+\vec{f})=0$

$\hat{AC}\times(\vec{G}\,l+\square(\vec{N}+\vec{f}))=0$ $\vec{T}=\vec{N}+\vec{f}$ 方向。

$\vec{T}=\vec{N}+\vec{f}$; $\vec{N}\perp\hat{AC}$. $\vec{f}\perp\hat{AC}$. $\vec{T}\perp\hat{AC}$; $\vec{T}$ 沿 $\vec{G}$ 在垂直于杆的投影矢量



$\hat{T}:\left(\frac{l}{\sqrt{h^2+l^2}},0,\frac{\sqrt{h^2+l^2}}{\sqrt{h^2+2l^2}}\right)$.

$\hat{T}$ 与 $\hat{N}$ 夹角。

$\hat{N}\parallel\vec{OH}\times\vec{AC}$

$\vec{AC}:(-\sqrt{h^2+l^2},0,l)$ $\vec{OH}:(h,l,0)$ ↗

$$\vec{OH}\times\vec{AC}=\square\cdot\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-\sqrt{h^2+l^2}&0&l\\h&l&0\end{vmatrix}=\square\cdot(-l^2,lh,-l\sqrt{h^2+l^2})=\square\cdot\frac{(-l,h,-\sqrt{h^2+l^2})}{\sqrt{2h^2+2l^2}}$$

$$\hat{N}=\frac{(l,-h,\sqrt{h^2+l^2})}{\sqrt{2h^2+2l^2}}$$

$$\hat{N}\cdot\hat{T}=\cos\beta=\frac{l^2+h^2+l^2}{\sqrt{2h^2+2l^2}\cdot\sqrt{h^2+2l^2}}=\sqrt{\frac{h^2+2l^2}{2h^2+2l^2}}$$

$$\mu=\tan\beta=\frac{h}{\sqrt{h^2+2l^2}}$$

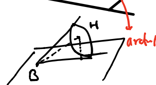
均匀杆，铰接A上，B在墙上，μ，求 α_{max}=?

$|\vec{f}|\le N\mu$. $\vec{T}=\vec{N}+\vec{f}$ 在 HA为轴 arctanμ 为角的圆锥内。

HA为轴，$\vec{T}$ 与 $\vec{G}$ 力矩平衡。

$\vec{AB}\times\vec{T}+\frac{1}{2}\vec{AB}\times\vec{G}=0$

$\vec{T}$ 在 $\vec{AB}$ 与 $\vec{G}$ 张平面中，是ABM平面。



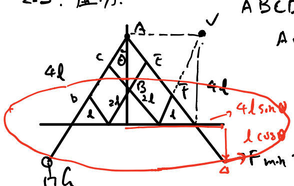
临界：圆锥切ABM平面

⇒ 切线沿杆。// AM.

$\frac{|MH|}{|AH|}=\mu=\frac{r\sin\alpha}{h}$. α_{max} ✓

$\vec{f}$ 沿 $\vec{MH}$，横向，$\vec{f}\perp\vec{AB}$. $\vec{f}$ 不沿圆切。

2.3. 虚功.



ABCDEF 铰接.

A、B 上下自由.

一个自由度

$E_p(\theta)$

$\vec{F}\cdot\delta\vec{S}=\delta E_p$

$E_p=-mgl\cos\theta$. $\vec{r}=(4l\sin\theta,-l\cos\theta)$

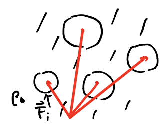
$\delta\vec{r}=(4l\cos\theta,l\sin\theta)\,\delta\theta$.

$\vec{F}\cdot(4l\cos\theta,l\sin\theta)\,\delta\theta=\delta E_p=mgl\sin\theta\,\delta\theta$

F_{min}. $\vec{F}\parallel(4l\cos\theta,l\sin\theta)$

$$F=\frac{mgl\sin\theta}{\sqrt{(4l\cos\theta)^2+(l\sin\theta)^2}}$$

例21.



N个塑料球，中空，外水ρ_0，只有之间万有引力。

每个施上一个外力 $\vec{F}_i$，保持平衡，求 $\vec{F}_i$=?

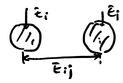
已知 $\vec{r}_i$，半径 r_i，G.

正负对应 → 引力势 → $p(\vec{r})$ → $\int p\,d\vec{S}=-\vec{F}_i$ → ...... ✓

单独，第i个球，相互作用能 E_i

$E=E_i+E_j+E_{ij}$ →之间作用

$E_{ij}=-\frac{Gm_im_j}{r_{ij}}$ $\frac{dE}{dr}=-F$

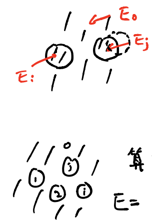


$E=E_i+E_j+E_0+E_{0i}+E_{0j}+E_{ij}$

0+j 视为一个物体.

$E=E_i+E_{0+j}+(E_i\sim(0+j))$ $dE_i\sim(0+j)=0$

与球所相对位移无关



算 $E_0(\vec{r}_1,\dots\vec{r}_i\dots\vec{r}_n)$. dE_0=?

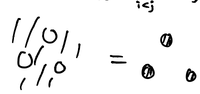
$$E=\sum_N E_i+E_0+\sum_N E_{0i}+\sum_{i<j}E_{ij}$$

$$=\sum_i E_i+E_0+\sum_i\left[\sum_{j\ne i}E_{ij}+E_{i0}\right]-\sum_{i<j}E_{ij}$$

$E_i(0+1+2\dots n)$ 不含i

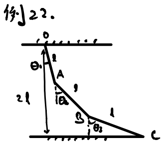
$dE=\sum dE_i+dE_0+\sum_i d(E_i-\text{起})-\sum_{i<j}dE_{ij}$

⇒ $dE_0=\sum_{i<j}dE_{ij}$



$$\vec{F}_i=\sum_{j\ne i}\frac{Gm_im_j}{r_{ij}^2}\hat{r}_{ji}$$ 外力

例22.



OA. AB. BC. 均匀杆.

O.A.B 铰接. C在地上. N.f.

求平衡 $\theta_1,\theta_2,\theta_3$=?

$\delta\theta_1$ 独立. $\delta\theta_2$, $\delta\theta_3$ 用 $\delta\theta_1$ 表达

约束

$df=f'dx=0$ dx任意

$\sin\theta_1+\sin\theta_2+\sin\theta_3=2$. ①

$\cos\theta_1+\cos\theta_2+\cos\theta_3=2$. ②

$-\frac{5}{2}\cos\theta_1-\frac{3}{2}\cos\theta_2-\frac{1}{2}\cos\theta_3=\frac{E_p}{mgl}$ ③

Table

①✓ ②✓ ③min

δ③: $\frac{5}{2}\sin\theta_1\,\delta\theta_1+\frac{3}{2}\sin\theta_2\,\delta\theta_2+\frac{1}{2}\sin\theta_3\,\delta\theta_3=0$

δ①: $\cos\theta_1\,\delta\theta_1+\cos\theta_2\,\delta\theta_2+\cos\theta_3\,\delta\theta_3=0$

δ②: $\sin\theta_1\,\delta\theta_1+\sin\theta_2\,\delta\theta_2+\sin\theta_3\,\delta\theta_3=0$

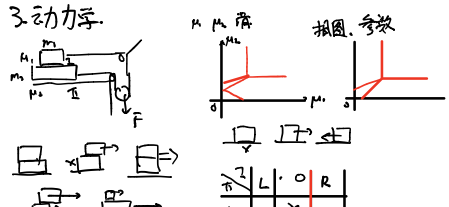
$\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\end{bmatrix}$ 是解

证明 $\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\end{bmatrix}$ 也是解

$[\;\vdots\;]\begin{bmatrix}\delta\theta_1\\\delta\theta_2\\\delta\theta_3\end{bmatrix}=0$ 有无穷组解.

⇒ $\det[\;.\;]=0$

3.动力学.



μ μ 摩 摆图、参数

下\上	L	0	R
L	×	×	✓
0	×	✓	✓
R	×	✓	✓

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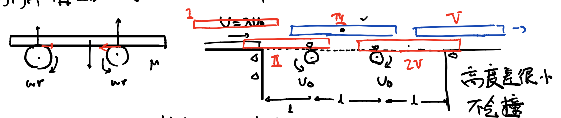
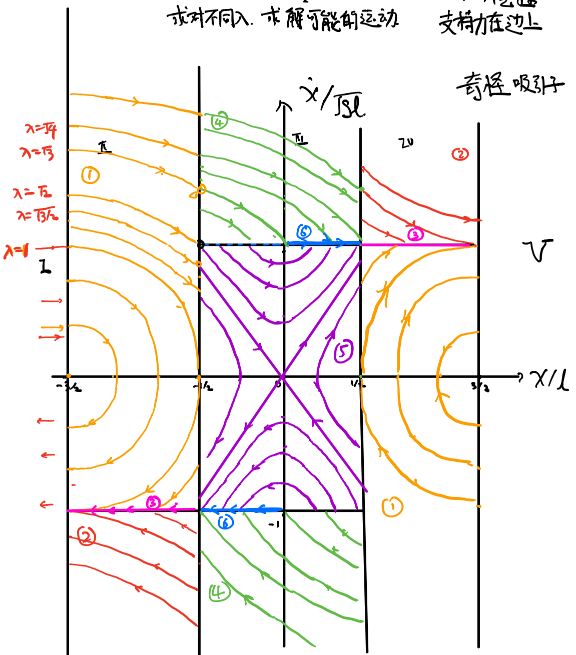
动摩擦图 不同运动状态. λ μ 　 2l 均匀板. $\mu=1$ 　 $u_0=\sqrt{gl}$.



求对不同入. 求解可能的运动 　 支持力在边上



① $-\frac{3l}{2}<x<-\frac{l}{2}$; $\dot{x}>-U_0$

$N_R=\frac{+x+\frac{3l}{2}}{l}mg$.

$m\ddot{x}=-N_R\mu$

$\ddot{x}=-g/l\cdot(x+\frac{3l}{2})$.

简振 $\omega^2=g/l$. 中心 $x=-\frac{3}{2}l$.

$\frac{1}{2}\dot{x}^2+\frac{1}{2}\frac{g}{l}(x+\frac{3l}{2})^2=C$.

以 $x=-\frac{3}{2}l$ 为中心 正圆.

② $-\frac{3l}{2}<x<-\frac{l}{2}$, $\dot{x}<-U_0$

$m\ddot{x}=+N_R\mu$, $\ddot{x}=\frac{g}{l}(x+\frac{3l}{2})$

$\frac{1}{2}\dot{x}^2-\frac{1}{2}\frac{g}{l}(x+\frac{3}{2}l)^2=C$. 双曲线 $x=-\frac{3}{2}l$ 为中心

③ $-\frac{3l}{2}<x<-\frac{l}{2}$, $\dot{x}=-U_0$. $f=?$

④ $-\frac{l}{2}<x<\frac{l}{2}$. $\dot{x}>U_0$

$\ddot{x}=-g$.

$\frac{1}{2}\dot{x}^2+gx=C$.

⑤ $-\frac{l}{2}<x<\frac{l}{2}$ 　 $-U_0<\dot{x}<U_0$

$N_R=\frac{\frac{l}{2}+x}{l}G$, $N_L=\frac{\frac{l}{2}-x}{l}G$

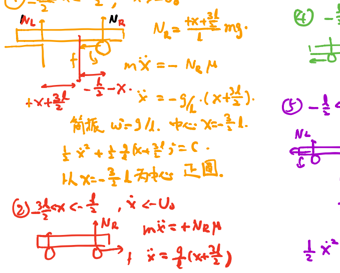
$m\ddot{x}=\mu N_R-\mu N_L$

$=2\frac{x}{l}mg$

$\ddot{x}=2\frac{g}{l}x$.

$\frac{1}{2}\dot{x}^2-\frac{1}{2}\cdot2\frac{g}{l}x^2=C$

⑥ $-\frac{l}{2}<x<\frac{l}{2}$. $\dot{x}=U_0$



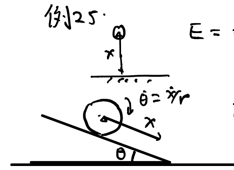
3.2. 单自由度

$E=\frac{1}{2}m\dot{x}^2+V(x)$ $\Longleftrightarrow$ $\frac{dE}{dt}=0$ 　 $m\ddot{x}\dot{x}+V'(x)\dot{x}=0$

$m\ddot{x}=-V'(x)$ 牛二.

能量的形式决定运动

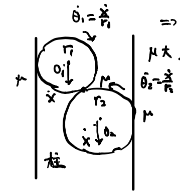
例25. 　 $E=\frac{1}{2}m\dot{x}^2+mgx$ $\Rightarrow$ $\ddot{x}=-g$.



$E=\frac{1}{2}m\dot{x}^2+\frac{1}{2}\cdot\frac{2}{5}mr^2(\frac{\dot{x}}{r})^2-mg\sin\theta x$

$=\frac{1}{2}\boxed{\frac{7}{5}m}\dot{x}^2-\frac{7}{5}m\boxed{(\frac{5}{7}g\sin\theta)}x$ 　 m^* 　 g^*

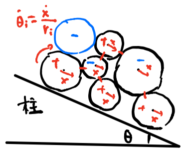
$\Rightarrow$ $\ddot{x}=+g^*=+\frac{5}{7}g\sin\theta$.



$E=\frac{1}{2}m_1\dot{x}^2+\frac{1}{2}m_2\dot{x}^2$
$+\frac{1}{2}\frac{1}{2}m_1r_1^2(\frac{\dot{x}}{r_1})^2+\frac{1}{2}\frac{1}{2}m_2r_2^2(\frac{\dot{x}}{r_2})^2$
$-m_1gx-m_2gx$

$=\frac{1}{2}\frac{3}{2}(m_1+m_2)\dot{x}^2-(m_1+m_2)gx$ 　 m^* 　 m^*g^*

$\ddot{x}=\frac{2}{3}g$.

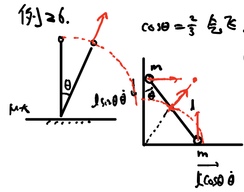


滚动. $\ddot{x}=?$

$E=\frac{1}{2}\sum m_i\dot{x}^2+\frac{1}{2}\sum\frac{1}{2}m_ir_i^2\frac{\dot{x}^2}{r_i^2}$
$-\sum m_ig\sin\theta x$

$\ddot{x}=\frac{2}{3}g\sin\theta$.

例26. 　 $\cos\theta=\frac{2}{3}$ 离开.



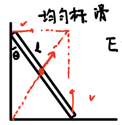
$\theta=0$. 静止放

$\theta=?$ 会离开?

$E=\frac{1}{2}ml^2\sin^2\theta\dot{\theta}^2+2mg\frac{l}{2}\cos\theta$
$+\frac{1}{2}ml^2\cos^2\theta\dot{\theta}^2$

$=\frac{1}{2}ml^2\dot{\theta}^2+mgl\cos\theta$

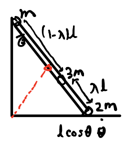
均匀杆 滑



$E=\frac{1}{2}m(\frac{l}{2})^2\dot{\theta}^2+\frac{1}{2}\frac{1}{12}ml^2\dot{\theta}^2$
$+mg\frac{l}{2}\cos\theta$

$=\frac{1}{2}\cdot\boxed{\frac{1}{3}m}l^2\dot{\theta}^2+\boxed{\frac{1}{3}m}l\cdot\boxed{\frac{3}{2}g}\cos\theta$.

$g\to\frac{3}{2}g$. 　 $\theta_c=\arccos\frac{2}{3}$



要求 $\theta=\arccos\frac{2}{3}$ 飞.

$\lambda=?$.

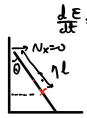
$E_k=\frac{1}{2}2m(l\cos\theta\dot{\theta})^2$
$+\frac{1}{2}m(l\sin\theta\dot{\theta})^2$
$+\frac{1}{2}\cdot3m\cdot((1-\lambda)l\cos\theta\dot{\theta})^2$
$+\frac{1}{2}3m\cdot(\lambda l\sin\theta\dot{\theta})^2$

$=\frac{1}{2}\boxed{?}\dot{\theta}^2$ 　 $\cos\theta$: $2+3(1-\lambda)^2$
无 θ. 　 $\sin\theta$. $1+3\lambda^2$
$\lambda=\ldots$

$E_k=\frac{1}{2}f(\theta)\dot{\theta}^2$, $E_p=V(\theta)$

$=\frac{1}{2}(m+m\sin^2\theta)l^2\dot{\theta}^2$. $E_p=mg\cos\theta l$

$\frac{dE}{dt}=0$ 　 $\frac{1}{2}m(1+\sin^2\theta)l^2 2\dot{\theta}\ddot{\theta}+\frac{1}{2}m\cdot2\sin\theta\cos\theta\dot{\theta}^3-mg\sin\theta\dot{\theta}l=0$



$\ddot{\theta}=\frac{mg\sin\theta l-m\sin\theta\cos\theta\dot{\theta}^2l}{m(1+\sin^2\theta)l^2}$.

$N_x=0$ 　 $a_{cx}=0$ 　 $x_c=\eta l\sin\theta$.

$\dot{x}_c=\eta l\cos\theta\dot{\theta}$ 　 $\ddot{x}_c=\eta l(\cos\theta\ddot{\theta}-\sin\theta\dot{\theta}^2)=0$

$\ddot{\theta}=\frac{\sin\theta}{\cos\theta}\dot{\theta}^2=\frac{g\sin\theta}{(1+\sin^2\theta)l}-\frac{\sin\theta\cos\theta\dot{\theta}^2}{}$

$\Delta E=0$ 　 $\dot{\theta}^2=f(\theta)$ ∫

2.3 两个、多个自由度

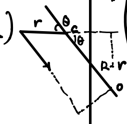
n个自由度，n个守恒量 ✓

找守恒量，⇒ 对称性。

$E(q_1, q_2, \dot q_1, \dot q_2)$

$\frac{\partial E}{\partial q} = 0$ ⇒ $\frac{\partial E_k}{\partial \dot q}$ ⇒ 守恒。

$\left(\frac{\partial L}{\partial q} = 0 \Rightarrow \frac{\partial L}{\partial \dot q} = P_1 \text{ 守恒}\right)$

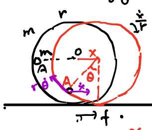


$\frac{1}{2}m\dot x^2 + \frac{1}{2}m\dot y^2 + V(y)$

$\frac{\partial E}{\partial x} = 0 \Rightarrow \frac{\partial E}{\partial \dot x} = m\dot x = P_x$ 　x平移 ⇒ P_x 守恒

例. $\frac{1}{2}m(\dot\rho^2 + \rho^2\dot\theta^2) + V(\rho)$

$\frac{\partial E}{\partial \theta} = 0 \Rightarrow \frac{\partial E_k}{\partial \dot\theta} = m\rho^2\dot\theta = L$ 　θ转动 ⇒ L守恒



例28. 空心圆柱壳，地上滚，m.r

内光滑质点 m，水平静止释放。

求A轨迹。 $\Delta P_x \neq 0$

x, θ 独立坐标，x平移不变，"P_x"

$E = \frac{1}{2}m\dot x^2 + \frac{1}{2}mr^2\left(\frac{\dot x}{r}\right)^2 + \frac{1}{2}m(\dot x^2 + r^2\dot\theta^2 - 2\cos\theta\,\dot x r\dot\theta) - mgr\cos\theta$

$\frac{\partial E}{\partial x} = 0$ 　$\frac{\partial E}{\partial \dot x} \Rightarrow m\dot x + m\dot x + m\dot x - m\cos\theta r\dot\theta = C$

$\Sigma P_x = f$. 　$f = \frac{d}{dt}[m\dot x + m\dot x - m\cos\theta r\dot\theta]$ ①

$L = rf$. 对筒，ILO转：$-fr = mr^2\ddot{\varphi}$ ②

①+②/r 　$0 = \frac{d}{dt}[\Sigma P_x + \frac{L}{r}]$. 　$-\cos\theta\,\frac{d\theta}{dt}$

$L_0 = 0$

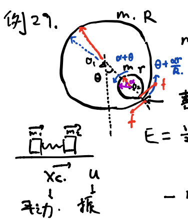
$3m\dot x - m\cos\theta r\dot\theta = 0$

⇒ $\frac{d}{dt}[3x - r\sin\theta] = 0$ 　$3x - r\sin\theta = -r$.

A: $x_A = x - r\sin\theta = \frac{-r + r\sin\theta}{3} - r\sin\theta$ ⇒ 椭圆

$y_A = r - r\cos\theta = r - r\cos\theta$

例29.



复查第一题，m.R 定轴O点。

m.r 圆盘 滚动。 振动周期

2个自由度。

整体转θ + 小盘α / 大 $\frac{\alpha r}{R}$.

$E = \frac{1}{2}m(R-r)^2\dot\theta^2 + \frac{1}{2}mR^2\cdot\left(\dot\theta + \frac{\dot\alpha r}{R}\right)^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2(\dot\alpha + \dot\theta)^2 - mg(R-r)\cos\theta$

$\frac{\partial E}{\partial \alpha} = 0 \Rightarrow \frac{\partial E_k}{\partial \dot\alpha} \Rightarrow L?$ 　$L_小$, I.LO转

$mR^2\cdot(\dot\theta + \frac{\dot\alpha r}{R})\cdot\frac{r}{R} + \frac{1}{2}mr^2(\dot\alpha + \dot\theta)$. 守恒。$= Const. = 0$

$L_大\cdot\frac{r}{R}$. ILO转

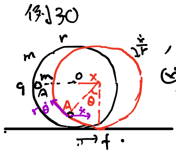
把α用θ表达 → 单自由度

对大ILO，$\frac{d}{dt}L_大 = -fR$. ①

对小ILO₂，$\frac{d}{dt}L_小 = fr$ ②

$\frac{①}{R}r + ② = 0$

例30 　+ B 　A带电q。



x平移不变 "P_x" 正则动量

$\left[L = E_k - E_p - q\varphi + q\vec v\cdot\vec A;\ \frac{\partial L}{\partial x} = 0 \Rightarrow \ldots\right]_x$

$\frac{d\Sigma P_x}{dt} = f - q\,r\dot\theta\sin\theta\cdot B$ ①

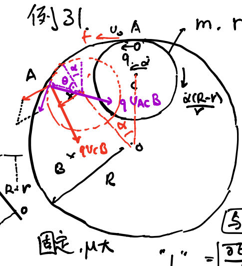
$\frac{dL}{dt} = -rf$. ②

①+②/r. $\frac{d}{dt}[\Sigma P_x + \frac{L}{r}] = -qr\dot\theta\sin\theta\,B$. → 也是全微分。

$\frac{d}{dt}(qr\cos\theta B)$

$\Sigma P_x + \frac{L}{r} - qr\cos\theta B = Const.$

例31



m, r 环，初态静止 B.

问什么角度分离。 α转动不变

ΔE ✓

$L = E_k - E_p - (q\varphi - \vec v\cdot\vec A q)$ 　$\frac{\partial L}{\partial \alpha} = 0$

$E_k = \frac{1}{2}m(R-r)^2\dot\alpha^2 + \frac{1}{2}mr^2\cdot\left(\frac{\dot\alpha(R-r)}{r}\right)^2 + \frac{1}{2}m\left\{[(R-r)\dot\alpha]^2 + (r\dot\theta + r\dot\alpha)^2 + 2\cos\theta(r\dot\theta + r\dot\alpha)(R-r)\dot\alpha\right\}$

与α无关

固定，M大

"L" $= \frac{\partial E_k}{\partial \dot\alpha} = m(R-r)^2\dot\alpha + m(R-r)^2\dot\alpha + m(R-r)^2\dot\alpha + m\cos\theta\,r\dot\theta(R-r)$

从O点转，整体L_O

从C点转，OC，L_C

$\frac{dL_O}{dt} = f\cdot R + q v_C B\, r\sin\theta - q\cdot B\cdot r(\dot\theta + \dot\alpha)(R-r)\sin\theta$. ① 　$(R-r)$ 　$-qB\,r(R-r)\sin\theta\dot\theta$

$\frac{dL_C}{dt} = f\cdot r$. ②

$\frac{①}{R} - \frac{②}{r}$. 　$\frac{d}{dt}\left[\frac{L_O}{R} + \frac{L_C}{r}\right] = \frac{d}{dt}\left[\frac{qBr(R-r)\cos\theta}{R}\right]$.

$\begin{cases}\frac{L_O}{R} + \frac{L_C}{r} - \frac{qBr(R-r)}{R}\cos\theta = Const. \\ \Delta E = c\end{cases}$

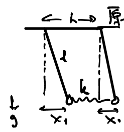
3.3 多自由度振动

小振动。

$E = \frac{1}{2}m\dot x^2 + \frac{1}{2}kx^2$ ← $V(x) = V(x_0) + V'(x_0)(x - x_0) + \frac{1}{2}V''(x_0)(x - x_0)^2 + \ldots$

$E_k = \frac{1}{2}a_{11}\dot x_1^2 + \frac{1}{2}a_{22}\dot x_2^2 + a_{12}\dot x_1\dot x_2$

$E_p = E_0 + \frac{1}{2}b_{11}x_1^2 + b_{12}x_2x_1 + \frac{1}{2}b_{22}x_2^2$



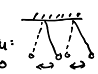
原长

$E_k = \frac{1}{2}m\dot x_1^2 + \frac{1}{2}m\dot x_2^2$

$E_p = \frac{1}{2}\frac{g}{l}mx_1^2 + \frac{1}{2}\frac{gm}{l}x_2^2 + \frac{1}{2}k(x_1 - x_2)^2$ 　正定二次

猜得：$u = \frac{\sqrt2}{2}(x_1 + x_2)$ 　$v = \frac{\sqrt2}{2}(x_1 - x_2)$? 　$x_1 \leftrightarrow x_2$

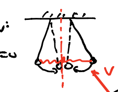
$x_1 = \frac{\sqrt2}{2}(u + v)$ 　$x_2 = \frac{\sqrt2}{2}(u - v)$



ω_u: $v = 0$

$E_k = \frac{1}{2}m\dot u^2 + \frac{1}{2}m\dot v^2$

$E_p = \frac{1}{2}m\cdot\frac{g}{l}u^2 + \frac{1}{2}m\frac{g}{l}v^2 + \frac{1}{2}\cdot 2kv^2$

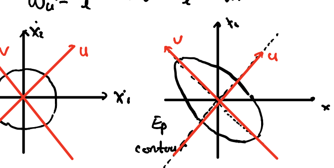


ω_v: $u = 0$

$\omega_u^2 = \frac{g}{l}$ 　$\omega_v^2 = \frac{g}{l} + 2\frac{k}{m}$

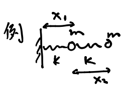
$E_k = \Sigma\frac{1}{2}m_i\dot q_i^2$

$E_p = \Sigma\frac{1}{2}k_iq_i^2$

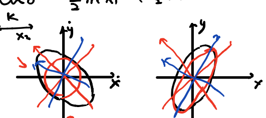


E_k等值线 contour 　E_p contour

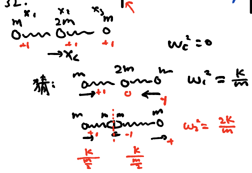
例



$\frac{1}{2}m\dot x_1^2 + \frac{1}{2}m(\dot x_1 + \dot x_2)^2$



例32.



$\omega_c^2 = 0$

猜：$\omega_1^2 = \frac{k}{m}$

$\omega_2^2 = \frac{2k}{m}$

$x_c = \frac{x_1 + 2x_2 + x_3}{4}$. 　$x_u = x_3 - x_1$ 　$x_v = x_1 - x_2 + x_3$

例33

$m\ddot{x}_1 = -kx_1 - k(x_1-x_2)$

$m\ddot{x}_1 = -2kx_1 + kx_2$

$m\ddot{x}_2 = +kx_1 - kx_2$

猜解得 $\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}a_1\\a_2\end{bmatrix} e^{i\omega t}$ 代入 $\frac{k}{m}=\omega_0^2$

$$\begin{bmatrix} -\frac{\omega^2}{\omega_0^2}+2 & -1 \\ -1 & -\frac{\omega^2}{\omega_0^2}+1 \end{bmatrix} \cdot \begin{bmatrix}a_1\\a_2\end{bmatrix} = 0 \quad ①②$$

要求: $\begin{bmatrix}a_1\\a_2\end{bmatrix}$ 有无穷组解. $①+\lambda② = 0$, $\det[\ ]=0$

$$\frac{-\frac{\omega^2}{\omega_0^2}+2}{-1} = \frac{-1}{-\frac{\omega^2}{\omega_0^2}+1}$$

$\left(\frac{\omega^2}{\omega_0^2}-2\right)\left(\frac{\omega^2}{\omega_0^2}-1\right)=1 \qquad \left(\frac{\omega^2}{\omega_0^2}\right)^2 - 3\frac{\omega^2}{\omega_0^2}+1=0$

$\frac{\omega^2}{\omega_0^2} = \frac{3\pm\sqrt5}{2} \qquad \omega_\pm = \omega_0\sqrt{\frac{3\pm\sqrt5}{2}}$

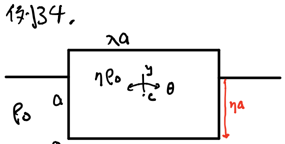
代入 ω_+: $\left(\frac{1-\sqrt5}{2}\right)a_1 - a_2 = 0$; $a_1:a_2 = 1:\frac{1-\sqrt5}{2}$

代入 ω_-: $\left(\frac{1+\sqrt5}{2}\right)a_1 - a_2 = 0$; $a_1:a_2 = 1:\frac{1+\sqrt5}{2}$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = A_+\begin{bmatrix}1\\\frac{1-\sqrt5}{2}\end{bmatrix}\cos(\omega_+t+\varphi_+) + A_-\begin{bmatrix}1\\\frac{1+\sqrt5}{2}\end{bmatrix}\cos(\omega_-t+\varphi_-)$$

代入初态 $x_1, x_2, \dot{x}_1, \dot{x}_2$

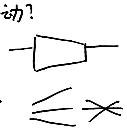
例34.



求振动. 忽略水的动能 ← 不对

是否独立? ✓

摆动 质心不动=浮心不动?



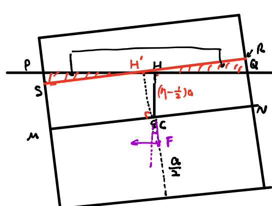
$E_k = \frac12 m\dot{y}^2 + \frac12\left(\frac1{12}ma^2 + \frac1{12}ma^2\lambda^2\right)\dot{\theta}^2$

$E_p(y,\theta) = □ + □y + □\theta + □y^2 + □y\theta + □\theta^2$

$E_p(y,\theta) = E_p(y,-\theta)$ θ偶, 独立.

E_p, E_k 2阶 牛二 1阶 $y \to y + \theta^2 x$

质心 浮心



$|PH| = \frac{\lambda}{2}a\frac{1}{\cos\theta} + \tan\theta\cdot(\eta-\frac12)a$

$|QH| = \frac{\lambda}{2}a\frac{1}{\cos\theta} - \tan\theta(\eta-\frac12)a$

$|PM| = (\eta-\frac12)a\frac{1}{\cos\theta} + \tan\theta\cdot\frac12\lambda a$ $\quad \frac{1}{\cos\theta}\sim 1+\theta^2$

$|QN| = (\eta-\frac12)a\frac{1}{\cos\theta} - \tan\theta\frac12\lambda a$

F浮力 $F_{\text{⊥}}\to O(1)$ 力矩 $O(1)$

浮心移动 $\Delta H'QR \to \Delta H'SP$ 相差 $O(2)$ ✓

$\Delta x_F = -|CF|\cdot\theta + \frac{S_\Delta}{S_□}\cdot\Delta x_\Delta$

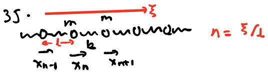
$= -\left(\frac a2 - \frac\eta2 a\right)\theta + \frac12\frac{|QR|}{\eta a}\frac{\lambda}{2}\cdot\frac23\lambda a$

$= -\frac12(1-\eta)a\theta + \frac12\cdot\frac12\frac{\lambda^2}{\eta}\cdot\frac12\cdot\frac23\ a\ \theta$

$I\ddot{\theta} = -\left(\frac{1}{12}\frac{\lambda^2}{\eta}a - \frac12(1-\eta)a\right)\theta\cdot mg$

$\lambda^2 = 6\eta(1-\eta)$

例35.



$m\ddot{x}_n = -k(x_n - x_{n-1}) - k(x_n - x_{n+1})$

猜得. $x_n = Ae^{in\varphi}e^{i\omega t}$ 代入

$x(\xi,t) = Ae^{i(\frac{\varphi}{l}\xi-\omega t)}$ 行波 $k=\frac{\varphi}{l}$. ω.

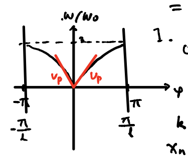
$u_p = \frac{\omega}{\varphi}\cdot l$

$-\omega^2 mA = -k(2A - Ae^{i\varphi} - Ae^{-i\varphi})$ $\quad e^{i(n+1)\varphi}$

$\frac{k}{m}=\omega_0^2$

$\frac{\omega^2}{\omega_0^2} = 2 - 2\cos\varphi = 2(1-\cos(kl))$

$= \left(2\sin\frac{\varphi}{2}\right)^2$



I. $0\le\omega\le2\omega_0$. 行波解.

$\frac12\frac{\omega}{\omega_0} = \pm\sin\frac{\varphi}{2} \doteq \sin\frac{kl}{2}$

$\omega\to 0$ $\quad \frac12\frac{\omega}{\omega_0} = \frac{k}{2}\cdot l \Rightarrow u_p = \frac{\omega}{k} = \omega_0 l$

$x_n = A_+\cos(k\xi-\omega t+\varphi_+) + A_-\cos(-k\xi-\omega t+\varphi_-)$

$kl=\varphi$. $\quad \sin\frac{\varphi}{2} = \pm\frac12\frac{\omega}{\omega_0}$

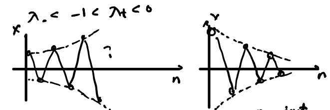
II. $\omega > 2\omega_0$ φ有虚部. k有虚部

$\left(\frac{\omega}{\omega_0}\right)^2 = 2 - e^{i\varphi} - e^{-i\varphi}$ $\quad e^{i\varphi}=\lambda$

$\lambda^2 - \left(2 - \left(\frac{\omega}{\omega_0}\right)^2\right)\lambda + 1 = 0$

$\lambda_\pm = \frac{2-\left(\frac{\omega}{\omega_0}\right)^2 \pm \sqrt{\left(2-\left(\frac{\omega}{\omega_0}\right)^2\right)^2-4}}{2}$ $\quad \omega>2\omega_0$ λ实 $\quad \lambda_+,\lambda_-$ $\quad \lambda_+\cdot\lambda_-=1$

$\lambda_- < -1 < \lambda_+ < 0$

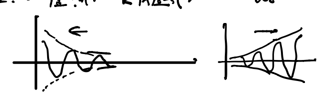


$|\lambda_+|<1$

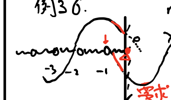
$x_n = A_+\lambda_+^n e^{i\omega t} + A_-\lambda_-^n e^{i\omega t}$

$n\to+\infty$. $A_-=0$ $\quad n\to-\infty$ $A_+=0$

III. ω虚部. k有虚部. $\quad \frac{\omega^2}{\omega_0^2} = 2-2\cos\varphi$.



例36. $n\le -1$. $\quad \xi = nl$.



$x(\xi,t) = A_+\cos(k\xi-\omega t) + A_-\cos(-k\xi-\omega t)$.

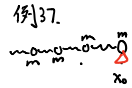
要求 $x(0,t)=0$ $\quad A_+\cos(\omega t) + A_-\cos(\omega t) = 0$

$\Rightarrow A_- = -A_+$

$x(\xi,t) = A_+\cos(k\xi-\omega t) + A_+\cos(-k\xi-\omega t+\pi)$

半波损失

例37.



$m\ddot{x}_0 = -k(x_0 - x_{-1})$

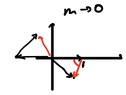
$x = A_+\cos(k\xi-\omega t) + A_-\cos(k\xi-\omega t)$

$\frac{\omega^2}{\omega_0^2}(A_+ + A_-) = \ .(A_+ + A_- - A_+e^{i\varphi} - A_-e^{i\varphi})$

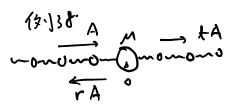
$A_- = A_+\frac{-\frac{\omega^2}{\omega_0^2}+1-e^{-i\varphi}}{\frac{\omega^2}{\omega_0^2}-1+e^{i\varphi}}$ $\quad \frac{1}{\omega_0^2} = \frac{m}{k} \to \infty$ $\Rightarrow A_- = -A_+$

$m\to0$ $\quad A_- = A_+\cdot\frac{1-e^{-i\varphi}}{-1+e^{i\varphi}}$

$= A_+e^{i\alpha}$



例38



$\xi\ge0$. $\quad x = tAe^{i(k\xi-\omega t)}$

$\xi\le0$ $\quad x = Ae^{i(k\xi-\omega t)} + Are^{i(-k\xi-\omega t)}$

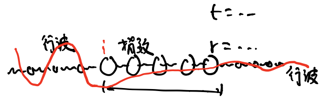
连续. $\xi\to0^+, 0^-$ $\quad t = 1+r$

牛二: $\quad -M\omega^2 t = -k\cdot(t - te^{i\varphi})$

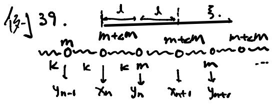
$m a \uparrow$ $\quad -k(t - e^{i\varphi} - re^{i\varphi})$.

$t = \ldots$

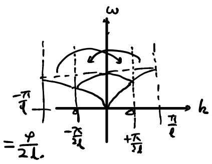
$r = \ldots$



例39.



$$\begin{cases}(m+\alpha m)\ddot{x}_n = -k(x_n-y_n)-k(x_n-y_{n-1})\\ m\,\ddot{y}_n = -k(y_n-x_{n+1})-k(y_n-x_n)\end{cases}$$



$\begin{bmatrix}x_n\\y_n\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix}e^{i(n\varphi-\omega t)}$. $n=\frac{x}{2l}$. $k=\frac{\varphi}{2l}$

$\varphi\in[-\pi,\pi]$ $k\in[-\frac{\pi}{2l},\frac{\pi}{2l}]$.

$\omega_1^2=\frac{k}{m+\alpha m}$, $\omega_2^2=\frac{k}{m}$

$-\frac{\omega^2}{\omega_1^2}a=-2a+b+be^{i\varphi}$

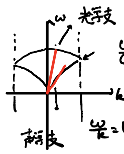
$-\frac{\omega^2}{\omega_2^2}b=-2b+a+ae^{-i\varphi}$ 要求 a.b 无穷组解.

$\Rightarrow\begin{bmatrix}2-\frac{\omega^2}{\omega_1^2} & -1-e^{-i\varphi}\\ -1-e^{i\varphi} & 2-\frac{\omega^2}{\omega_2^2}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=0$ det[ ]=0

$(2-\frac{\omega^2}{\omega_1^2})(2-\frac{\omega^2}{\omega_2^2})=(-1-e^{i\varphi})(-1-e^{-i\varphi})$

① $\omega_1^2=\omega_2^2$ $\alpha m\to 0$ $(2-\frac{\omega^2}{\omega_2^2})^2=2+2\cos\varphi$. $\cos\varphi=-1+2\cos^2\frac{\varphi}{2}$

$2-\frac{\omega^2}{\omega_2^2}=\pm2\cos\frac{\varphi}{2}$



$\frac{\omega^2}{\omega_2^2}=2\pm2\cos\frac{\varphi}{2}$

$\frac{\omega^2}{\omega_2^2}=4$

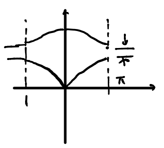
② $\omega_1^2=\frac{k}{m+\alpha m}$ $\omega_2^2=\frac{k}{m}$

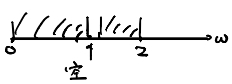
$\varphi\to\pi$. $(2-\frac{\omega^2}{\omega_1^2})(2-\frac{\omega^2}{\omega_2^2})=0$

$\omega\to\omega_1\sqrt{2}$

$\omega\to\omega_2\sqrt{2}$.

$\Delta\omega=|\omega_1-\omega_2|$



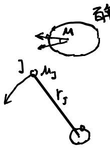


3.4 微扰 无量纲数.

例40. 水星进动

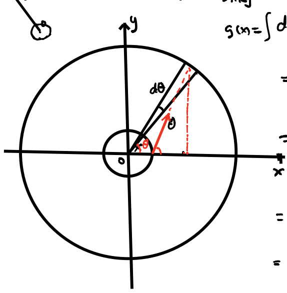
进动. 近日点移动.

百年. 600″ 大行星 + S.R. + C.R.



$\omega_M\gg\omega_J$. 对径 椭引力等效"椭环"

M_J 一周 $\lambda=\frac{M_J}{2\pi R_J}$ $g(x)=?$ $\theta\sim\theta+d\theta$



$g(x)=\int dg(x)=\int_0^{2\pi}\frac{G\lambda R_J\,d\theta\cdot(R_J\cos\theta-x)}{(x^2+R_J^2-2\cos\theta\, x R_J)^{3/2}}$

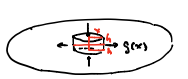
$=\frac{G\lambda}{R_J}\int_0^{2\pi}(\cos\theta-\frac{x}{R_J})\cdot(1-2\cos\theta\frac{x}{R_J}+(\frac{x}{R_J})^2)^{-3/2}d\theta$

$=\frac{G\lambda}{R_J}\int_0^{2\pi}(\cos\theta-\frac{x}{R_J})(1+(-\frac{3}{2})\cdot(-2\cos\theta\frac{x}{R_J}+\frac{x^2}{R_J^2})+\frac{(-\frac{3}{2})(-\frac{5}{2})}{2}\cdot(-2\cos\theta\frac{x}{R_J})^2)\,d\theta$ $+O(x^2)$

$=\frac{G\lambda}{R_J}\int_0^{2\pi}(-\frac{x}{R_J}+3\cos^2\theta\frac{x}{R_J})\,d\theta$.

$=\frac{G\lambda}{R_J}\frac{x}{R_J}\cdot(-2\pi+\frac{3}{2}\cdot2\pi)$.

$=\frac{1}{2}\cdot\frac{GM_J x}{R_J^3}$ $2\pi R_J\lambda=M_J$.

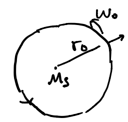


$\nabla\cdot\vec{g}=0$

$\int\vec{g}\cdot d\vec{s}=G\Sigma M$.

$g_\perp=\frac{GM_J}{R_J^2}\cdot\frac{h}{R_J}$ 高斯

$g(x)\cdot2\pi x\cdot2h=2\cdot g_\perp\cdot\pi x^2\Rightarrow g(x)=\frac{1}{2}\frac{GM_J}{R_J^3}\cdot x$



"平衡态" 微扰前.

$-m\omega_0^2r_0=-\frac{GM_sm}{r_0^2}+\frac{GM_Jm}{R_J^3}\cdot r_0$

$\frac{M_J}{R_J^3}/\frac{M_s}{r_0^3}=\varepsilon$.

有扰动 $r=r_0(1+x)$, $x\ll1$ $r^2\dot\theta=r_0^2\omega_0$ 径向

$\ddot r-r\dot\theta^2=-\frac{GM_s}{r^2}+\frac{GM_J}{R_J^3}\cdot r$

$r_0\ddot x-\frac{\omega_0^2r_0}{(1+x)^3}=-\frac{GM_s}{r_0^2}(1+x)^{-2}+\frac{GM_s}{r_0^2}\varepsilon\cdot(1+x)$

验算 $x=0$ $-\omega_0^2r_0=-\frac{GM_s}{r_0^2}(1-\varepsilon)$ $\frac{GM_s}{r_0^3}=\frac{\omega_0^2}{1-\varepsilon}$.

$\ddot x-\omega_0^2(1+x)^{-3}=-\frac{GM_s}{r_0^3}(1+x)^{-2}+\frac{GM_s}{r_0^3}\varepsilon(1+x)$

两个小量. x. ε 无关小量. 逐个展. 先x.

$\ddot x-\omega_0^2(1-3x)=-\frac{GM_s}{r_0^3}(1-2x)+\frac{GM_s}{r_0^3}\varepsilon(1+x)$

L-R-L. Vector

$\ddot x-\omega_0^2(1-3x)=-\omega_0^2\frac{1}{1-\varepsilon}(1-2x)+\frac{\varepsilon}{1-\varepsilon}\omega_0^2(1+x)$

$\ddot x=-3\omega_0^2x+\frac{\omega_0^2 2x}{1-\varepsilon}+\frac{\omega_0^2\varepsilon}{1-\varepsilon}x$

$=-\omega_0^2(3-\frac{2}{1-\varepsilon}-\frac{\varepsilon}{1-\varepsilon})x$.

再展ε. $\ddot x=-\omega_0^2(3-2(1+\varepsilon)-\varepsilon)x$

$\ddot x=-\omega_0^2(1-3\varepsilon)x$

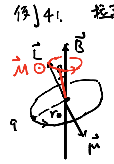
$x=A\cos(\omega_x t+\varphi)$ $\omega_x=\sqrt{\omega_0^2(1-3\varepsilon)}=\omega_0(1-\frac{3\varepsilon}{2})$

$\Omega=\omega_0\frac{3\varepsilon}{2}$.

3.5 刚体.

(进动).

例41. 拉莫尔进动. 经典 . 玻尔

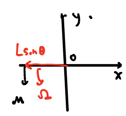


几乎圆周.

$\vec\mu=\frac{q}{2m}\cdot\vec L$ $q<0$

$\frac{d\vec L}{dt}=\vec M=\vec\mu\times\vec B$

$|\vec\mu\times\vec B|=\frac{q}{2m}L\cdot B\cdot\sin\theta$.



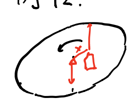
$\Omega\cdot L\sin\theta=\frac{q}{2m}LB\sin\theta$

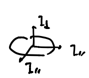
$\Omega=\frac{qB}{2m}$. ✓

多 $\vec\mu_\Omega=\square\cdot\frac{qB}{2m}$ $(-\frac{q}{2})$

例42. $M=k\cdot\omega$. 摩擦力矩

求出各参数关系

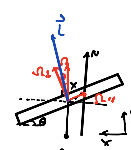




例: $\Omega_\perp=\Omega\cos\theta$. $\Omega_{//}=\Omega\sin\theta$

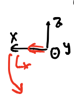
圆盘 $I_\perp=\frac{1}{2}mr^2$. $I_{//}=\frac{1}{4}mr^2$

$\begin{cases}L_\perp=\Omega_\perp I_\perp=\frac{1}{2}mr^2\Omega\cos\theta\\ L_{//}=\Omega_{//}I_{//}=\frac{1}{4}mr^2\Omega\sin\theta\end{cases}$



$L_y=\Omega_\perp\cos\theta+\Omega_{//}\sin\theta$

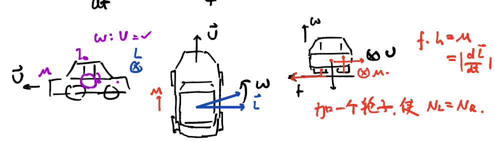
$=\frac{1}{2}mr^2\Omega\cos\theta+\frac{1}{4}mr^2\Omega\sin\theta$



$L_x=\Omega_\perp\sin\theta-\Omega_{//}\cos\theta$

$=\frac{1}{4}mr^2\Omega\sin\theta\cos\theta$

$\frac{d\vec L}{dt}=\vec M$ $\frac{1}{4}\Omega r^2\sin\theta\cos\theta\cdot\Omega=M=mg\,x\cdot\cos\theta$



4. 狭义相对论

事件 $A(t,x,y,z)$

变系. $ct'(t,x,y,z) = \square ct + \square x + 0y + 0z + \square$ ← 零点. 对齐止

$x'(t,x,y,z) = \square ct + \square x + 0y + 0z + \square$

$y'(t,x,y,z) = 0ct + 0x + 1\cdot y + 0z + \square$

$z'(t,x,y,z) = 0ct + 0\cdot x + 0y + 1\cdot z + \square$.

沿x轴 boost. $y=y'$ $z=z'$

$$\begin{cases} ct' = a_{00}\,ct + a_{01}\,x \\ x' = a_{10}\,ct + a_{11}\,x \end{cases}$$

光速不变 $(ct')^2 - x'^2 = (ct)^2 - x^2$

模方不变: $(ct)^2 - x^2 - y^2 - z^2$ 不变

$(a_{00}ct + a_{01}x)^2 - (a_{10}ct + a_{11}x)^2 = (ct)^2 - x^2$

$a_{00}^2 - a_{10}^2 = 1.$ $a_{11}^2 - a_{01}^2 = 1.$ $a_{00}a_{01} = a_{10}a_{11}$

$a_{00} = \mathrm{ch}k$

$ct' = \mathrm{ch}k\; ct - \mathrm{sh}k\; x$

$x' = -\mathrm{sh}k\; ct + \mathrm{ch}k\; x$

从U向右 点在U右的系中静止 $\frac{\mathrm{sh}k}{\mathrm{ch}k} = \frac{v}{c}$

$\mathrm{ch}k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. $\mathrm{sh}k = -\beta\gamma$. $\beta = \frac{v}{c}$. $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

记: $\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \Rightarrow X^{\mu'} = \Lambda^{\mu}{}_{\nu} X^{\nu}$ (求和)

$X^{\mu} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$ $x^0 = ct$ $x^i \Rightarrow \vec{x}$

$X_{\mu} = \begin{bmatrix} ct \\ -x \\ -y \\ -z \end{bmatrix}$. $\sum_{\mu} X^{\mu} X_{\mu} = (ct)^2 - x^2 - y^2 - z^2$

$X^{\mu}X_{\mu}$ 不变 $\Leftrightarrow$ $X^{\mu'} = \Lambda^{\mu}{}_{\nu} X^{\nu}$ $\Rightarrow$ 4D Vector 定义.

四维标量 $\phi' = \phi$. 静…相位… m_0, τ_0,
q. ϕ. 相位.

四维矢量 $A^{\mu'} = \Lambda^{\mu}{}_{\nu} A^{\nu}$. · x^{μ} 位移 $x_1^{\mu} - x_2^{\mu}$

$\frac{x^{\mu}(\tau + d\tau) - x^{\mu}(\tau)}{d\tau} = U^{\mu}$ 四维速度

$U^{\mu} = \frac{dt}{d\tau}\frac{dx^{\mu}}{dt} = \gamma \begin{bmatrix} c \\ u_x \\ u_y \\ u_z \end{bmatrix}$ $\frac{dU^{\mu}}{d\tau} = a^{\mu}$, $m_0 U^{\mu} = P^{\mu}$

$P^{\mu} = m_0\gamma \begin{bmatrix} c \\ \vec{u} \end{bmatrix} = \begin{bmatrix} mc \\ m\vec{u} \end{bmatrix} = \begin{bmatrix} \frac{E}{c} \\ \vec{p} \end{bmatrix}$. $F^{\mu} = \frac{dP^{\mu}}{d\tau} = \frac{dt}{d\tau}\frac{dP^{\mu}}{dt} = \gamma \begin{bmatrix} \frac{W}{c} \\ \vec{F} \end{bmatrix}$.

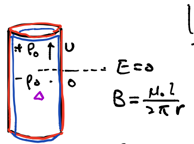
$J^{\mu} = \begin{bmatrix} \rho c \\ \vec{j} \end{bmatrix}$. $k^{\mu} = \begin{bmatrix} \frac{\omega}{c} \\ \vec{k} \end{bmatrix}$

$\rho_0 U^{\mu} = \rho_0\gamma \begin{bmatrix} c \\ \vec{u} \end{bmatrix} = \rho \begin{bmatrix} c \\ \vec{u} \end{bmatrix}$ $k^{\mu}\hbar = \begin{bmatrix} \frac{\hbar\omega}{c} \\ \hbar\vec{k} \end{bmatrix} = P^{\mu}$

↓ 固有电荷密度 $= \begin{bmatrix} \rho c \\ \vec{j} \end{bmatrix}$.

$[\vec{L} = \vec{r} \times \vec{p}]$?: $[\vec{E}, \vec{B}]$ × 张量 $(F^{\mu\nu})' = \Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta} F^{\alpha\beta}$ ×

$\phi = \omega t - \vec{k}\cdot\vec{x} = (\frac{\omega}{c}, -\vec{k}) \cdot \begin{bmatrix} ct \\ \vec{x} \end{bmatrix}$



$\begin{bmatrix} \vec{E} \\ \vec{B} \end{bmatrix} \rightarrow \begin{bmatrix} \vec{E}' \\ \vec{B}' \end{bmatrix}$
换向上U系中 $\Longrightarrow$

$ct' = \gamma[ct - \beta x)$

$x' = \gamma[x - \beta ct]$

$\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} \rho c \\ j_x \\ j_y \\ j_z \end{bmatrix}$

$\rho' c = \gamma(\rho c - \beta j_x)$

$j_x' = \gamma(j_x - \beta \rho c)$

$j_y' = j_y.$

$j_z' = j_z$

$+$: $J_+^{\mu} = \begin{bmatrix} \rho_0 c \\ \rho_0 u \\ 0 \\ 0 \end{bmatrix}$ $J_-^{\mu} = \begin{bmatrix} -\rho_0 c \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$J^{\mu} = J_+^{\mu} + J_-^{\mu} = \begin{bmatrix} 0 \\ \rho_0 u \\ 0 \\ 0 \end{bmatrix} \Rightarrow$ $\rho' c = \gamma(0 - \beta\cdot\rho_0 u)$ $\Rightarrow \rho' = -\gamma\beta\rho_0$

$j_x' = \gamma(\rho_0 u - \beta\cdot 0)$ $\Rightarrow j_x' = \gamma\rho_0 u$

不要求: $E_{/\!/}' = E_{/\!/}$

$\vec{E}_\perp' = \gamma(\vec{E}_\perp + \vec{u} \times \vec{B})$

$B_{/\!/}' = B_{/\!/}$

$\vec{B}_\perp' = \gamma(\vec{B}_\perp - \frac{\vec{u}}{c^2}\times\vec{E})$

← 验算

$$\begin{cases} E' = \frac{-\gamma\beta\rho_0\cdot S}{2\pi r\varepsilon_0} \\ B' = \frac{\mu_0}{2\pi r}\cdot\gamma\rho_0 u\cdot S \end{cases}$$