

第八屆泛珠三角物理奧林匹克暨中華名校邀請賽
力學基礎試答案

選擇題 (50 分)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	C	D	E	B	E	D	B	C	F	F	F	A	E	D	C	E	B	A

21. (11 分)

(1) $mv_0 = mv + (m_1 + m_2)v_L \Rightarrow v_L = 1\text{m/s};$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}(m_1 + m_2)v_L^2 + \mu mgL \Rightarrow L = 10\text{m}.$$

(2) $mv + m_2v_L = (m + m_2)v_R \Rightarrow v_R = 2\text{m/s};$

$$\frac{1}{2}mv^2 + \frac{1}{2}m_2v_L^2 = \frac{1}{2}(m + m_2)v_R^2 + mgR \Rightarrow R = 0.6\text{m}.$$

22. (13 分)

(1) $v_0 = 7,676 \text{ m/s}. T_0 = 5,542 \text{ s}.$

(2) $E_k = \frac{1}{2}m(0.99v_0)^2 = 5.774 \times 10^{10} \text{ J} \quad E_p = -\frac{GMm}{r} = -mv_0^2 = -11.783 \times 10^{10} \text{ J}.$

(3) $E = E_k + E_p = -6.00889 \times 10^{10} \text{ J} = -\frac{GMm}{2a}, \quad a = -\frac{GMm}{2E} = 6.638 \times 10^6 \text{ m}; \quad T = 2\pi\sqrt{\frac{a^3}{GM}} = 5\,380\text{s}.$

(4) $t = T_0 - T - 72 = 90\text{s}.$

23. (13 分)

(1) $x_C = 0.4865\text{m}; I = 0.2025 \text{ kg}\cdot\text{m}^2; \quad T = 2\pi\sqrt{\frac{I}{mgx_C}} = 1.49 \text{ s}.$

(2) 角動量守恆 $\Sigma l_i = \Sigma l_f. \quad mv_0(r + L) = mv(r + L) + I\omega, \quad \omega = \frac{m(v_0 - v)(R + L)}{I} = 4 \text{ rad/s}.$

機械能守恆 $\frac{1}{2}I\omega^2 = Mg x_C(1 - \cos\theta); \quad \cos\theta = 1 - \frac{I\omega^2}{2Mg x_C} = 0.55 \Rightarrow \theta \approx 56.63^\circ, \text{ or } 1\text{rad}.$

24. (13 分)

(1) $l_C = \frac{L}{2}, I = \frac{mL^2}{3}, \quad \omega^2 = \frac{mgl_C}{I} = \frac{3g}{2L}, \quad \omega = \sqrt{\frac{3g}{2L}}.$

(2) 桿重力 $G = \rho g V$ 和浮力 $f = \rho_0 g(L - l)A.$

$\Sigma \tau_o \leq 0$ 時傾斜現象, 即 $G \times \frac{L}{2} \sin\phi - f \times \frac{L+l}{2} \sin\phi \leq 0 \Rightarrow l_{\max} = \sqrt{1 - \frac{\rho}{\rho_0}} L.$

(3) $\Sigma \tau_o = \frac{\rho_0 l^2 - (\rho_0 - \rho)L^2}{2} g A \phi = -I\ddot{\phi} \Rightarrow \omega = \sqrt{\frac{3g}{2L} \cdot \frac{\rho_0}{\rho} \left(\frac{\rho}{\rho_0} + \left(\frac{l}{L}\right)^2 - 1 \right)}.$

(4) 當 $l=L$ 時, $\omega = \omega_0.$